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Mathematical modeling of the impacts of the nanoparticle in River-Aquatic system with convective cooling

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Abstract : In this study, a mathematical model to examine the combined effects of nanoparticles of Copper (Cu) and Alumina (Al_2O_3) in unsteady channel flow of water (river) based nanofluids was established. Both first and second laws of thermodynamics were employed to analyse the model. Using the discretization finite difference method together with the Runge-Kutta integration scheme, the governing partial differential equations were solved numerically. Numerical simulations of the effects of parameter variation on concentration and temperature were graphically presented and quantitatively discussed. Results show that the nanofluid concentration increases with an increase in thermophoresis parameter (Nt) and nanoparticle volume fraction (η) and decreases with the increase in the Biot number (Bi). While nanofluid temperature increases with Biot number (Bi), Schmidt number (Sc) and the Brownian motion parameter (Nb), and decreases with an increased nanoparticle volume fraction (η). From the study, it was recommended that in cooling systems and industrial processes nanofluids were found significant to be used for the effective operation of machines and cooling systems. Using the same geometry of the channel flow and the same problem, the Brownian motion in any fluid system has a great impact also.

Keywords : Modeling; Flow; Aquatic; Copper; Alumina
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1 Introduction

In the development of energy-efficient heat transfer equipment, the thermal conductivity of the heat transfer fluid plays a vital part. However, traditional heat transfer fluids such as water, oil, and ethylene glycol mixtures are inherently poor heat transfer fluids. In increasing global competition, industries have a strong need to develop advanced heat transfer fluids with significantly higher thermal conductivities than the currently available [1].

Technological growths in material processing technologies are driven by demands of sustainable manufacturing and higher performances. Altering factors in the manufacturing environment and current developments in nanotechnology unlocked a new area for triggering technological potential of nano fluids [12].

Nanofluids are dilute liquid suspensions of nanoparticles with at least one of their principal dimensions smaller than 100 nm. In some investigations, nano fluids have been originated to possess improved

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thermophysical properties such as thermal conductivity, thermal diffusivity, viscosity and convective heat transfer coefficients compared to those of base fluids such as oil or water [11].

Nanofluids are suspensions of nanoparticles in fluids that show important enhancement of their properties at modest nanoparticle concentrations. Nanoparticles of material like metallic oxide (Al_2O_3, CuO) ,nitride ceramics (AlN, SiN) carbide ceramics (SiC, TiC), metals (Cu, Ag), semiconductors (TiO, SiO), alloyed particles (Al70Cu30) single ,double or multi walled carbon nanotubes (SWCNT, DWCNT, MWCNT). have been used for the preparation of nanofluids. These nanofluids have been originated to possess improved thermal conductivities [2].

figures In the study by [8], a numerical investigation on Second law analysis of bouyancy unsteady

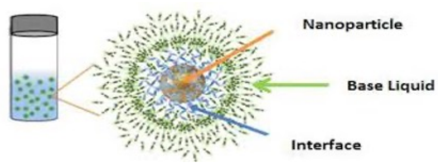

Figure 1: Nanoparticle; source: slideshare.net

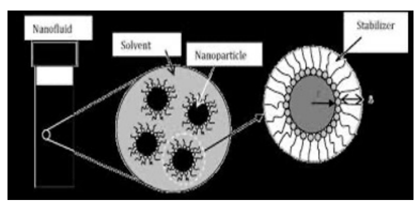

Figure 2: Nanofluid; source: slideshare.net

channel flow of nanofuids with convective cooling was conducted and the results show that, the Al2O3-water nanofluid rises higher than Cu-water nano fluid, the Cu-water nano fluid produces high skin friction than Al2O3-water nanofluid and the Al2O3-water nano fluid produces higher entropy than Cu-water nanofluid.

A model on water pollution and self-purification of the river Ganges was developed by [5] with the results that river has its self-cleaning ability which permits integration and treatment of industrial waste in the river, but if releasing of waste in river remains unstopped then self-cleaning process will not remain effective.

[4] developed the model of water quality alteration in Eastern Europe river and had the findings that recent changes in agriculture could be a major reason for the increase of the river pollution.

[3] on their study on water pollution and its impacts on human health had found that the native peoples are suffering from a variety of health difficulties that could be direct or indirect related to waste water released. The problems of diarrhea and dysentery are unlike to be caused directly by the industrial waste, as they are typically the outcome of bacterial contamination.

[18] investigated entropy generation and found that the optimal volume concentration of nano particles to reduce the entropy generation increases when the Reynolds number decreases. Furthermore, the thermal entropy generation increases with the increase of nano particle size while the frictional entropy generation decreases. Thus ZrO_2 -water provides the highest entropy generation compared to other nano fluids like Al_2O_3, SiO_2 and CuO nanoparticles in water.

[7] developed the model on water quality in river systems and came out with the results that waste materials from industries are the major source of river pollution. The use of polluted water for domestic purpose has led to water-borne diseases spread from man to man. People living in areas around the river also are suffering from respiratory problems.

[15] developed the mathematical modeling of river water quality and the results were found that pollution in the rivers is due to eutrophication by the increased nutrients in the water surface. This might be due to agricultural activities if take place around the rivers. Chemicals and dissolved nutrients will be accumulated into the river during raining and thus changing the chemistry of water.

[13] highlighted in their non-modeling study that water quality is affected by pollution originating from salinization, organic matters, pathogenic microorganisms, eutrophication, agrochemical contaminants and industrial use. Therefore if effective action is to be initiated to combat the threats to agricultural production and human health posed by various forms of pollution, then up to date information on water quality is essential.

The investigation of the entropy generation in a variable viscosity channel flow of nanofluids with convective cooling was conducted by [9]. The results show that, the mixture of parameter values, the entropy production within the channel flow of a variable-viscosity water-based nanofluid in the presence of convective cooling can be minimized. These parameters are Eckert number (Ec), Biot number (Bi), thermophoresis parameter (Nt) and so on.

[16] in their study of approaches to evaluate water quality model parameter indicated that water quality is highly influenced by the management factor; proper management of resources will enable maximum benefits. If water resources are properly managed then pollution will highly be overcame and water quality is maintained.

The networks for the quality drinking water distribution model was developed by [10]. Their results show that in order to increase the efficiency in the drinking water then water should be always be treated to overcome the effects of the pipes as they become subjected to different climatic condition like increase temperature which may lead to deformation and blockage of pipes as the results the water will no longer be safe for drinking.

[17] in their study of assessing of ground quality and hydro geological profile of Kavala area in Northern Greece showed that management of a required water quality for human consumption needs not only monitoring of the physicochemical parameters of the source of water and the portable water but also involvement of the environmetric methods as standard procedure for real and low-cost assessment of the ground water sources in a certain region.

The study by [6] investigated the effect of variable viscosity on unsteady couette flow of nanofluids with convective cooling and concluded that, while other parameters remain constant, the velocity profile increases with the increase of A or variable viscosity parameter β. Temperature increase with the increase of β, Nb reduce the rate of decrease of nanoparticle volume fraction distribution while Nt show the reverse effect. Skin friction increases with the increase of viscosity. The Nusselt number and convective cooling increase with the increase of Bi.

Despite all cited works, few of them highlight on the modeling of the impacts of nanoparticles in the river type aquatic systems and most of them investigated the impacts of temperature and velocity in the channel. That gave rise for this study to be done. The study aimed at investigating the impact of nanoparticles in a river-type aquatic system.

2 Mathematical model

Supposed that you have unsteady laminar flow of viscous incompressible nano fluids containing Alumina (Al_2O_3) and Copper (Cu) as nanoparticles through channel with no wall permeability. It is assumed that the channel walls exchange heat with the ambient surrounding following the Newtons law of cooling.

Consider (x, y) coordinate where x lies along the flow direction of the channel, y is the distance measured in the normal direction as shown in 3. Using the Boussinesq approximation, all the fluid properties are

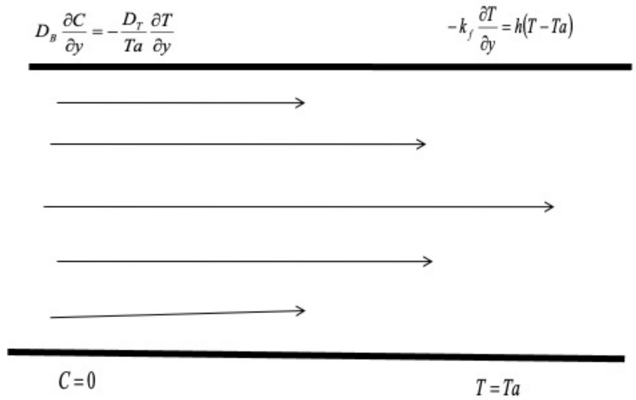

Figure 3: Schematic diagram of the problem under consideration

supposed to be fixed except the temperature and concentration varies with time in the buoyancy force term. The Navier-Stokes nanofluids momentum and energy balance equation in one dimension under the Boussinesq approximation for the transient channel flow can be written as follows;

$$\frac{\partial T}{\partial t} = \alpha_f \frac{\partial^2 T}{\partial y^2} + \tau \left\{ D_B \frac{\partial T}{\partial y}\frac{\partial C}{\partial y}\frac{D_T}{Ta}\left(\frac{\partial T}{\partial y}\right)^2 \right\} \tag{2.1}$$

$$\frac{\partial C}{\partial t} = D_B \frac{\partial^2 C}{\partial y^2} + \frac{D_T}{Ta}\frac{\partial^2 T}{\partial y^2} \tag{2.2}$$

The initial and boundary conditions are given as;

$$C(y, 0) = 0 \tag{2.3}$$

$$C(0, t) = 0 \tag{2.4}$$

$$T = Ta \tag{2.5}$$

$$D_B \frac{\partial C}{\partial y} = -\frac{D_T}{Ta}\frac{\partial T}{\partial y} \tag{2.6}$$

$$-k_f \frac{\partial T}{\partial y} = h(T - Ta) \tag{2.7}$$

Table 1: Thermo-physical properties of water, alumina and copper at specified temperature

Physical properties	Fluid phase (water)	Copper (Cu)	Alumina (Al_2O_33)
$C_p(J/kgK)$	4179	385	765
$p(kg/m3)$	997.1	8933	3970
$k(W/mK)$	0.613	401	40

Where, $k_f$is the thermal conductivity of fluid fraction and is heat transfer coefficient. Introduce the following dimensionless variables and parameters into equations 2 - 2, where T is the temperature of the nanofluid, t is the time, a is the channel width, Ta is the ambient temperature, $\alpha_f$is thermal diffusion of the nanofluid, D_B is the Brownian diffusion coefficient, D_T is the thermophoretic diffusion coefficient and τ is the ratio of heat capacitance of the solid particles to that of the nanofluid.
The dimensionless variables and parameters are introduced as follows:

$$\begin{cases} \theta = \dfrac{T - Ta}{Ta}, \eta\dfrac{y}{a}, \bar{t} = \dfrac{t\nu_f}{a^2}, N_b = \tau\dfrac{D_B C_0}{\alpha_f} \\ P_r = \dfrac{\mu_0 (C_p)_f}{k_f}, Sc = \dfrac{\nu_f}{D_B}, H = \dfrac{C}{C_0} \\ N_t = \tau\dfrac{D_T}{\alpha_f}, \tau\dfrac{(\rho C_p)_s}{(\rho C_p)_f}, Bi = \dfrac{ha}{k_f}, \nu = \dfrac{\mu_0}{\rho_f} \end{cases} \tag{2.8}$$

The dimensionless leading equations together with the appropriate initial and boundary conditions can be descritized as follows;

$$Pr\frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial \eta^2} + N_b \frac{\partial \theta}{\partial \eta}\frac{\partial H}{\partial \eta} + N_t \left(\frac{\partial \theta}{\partial \eta}\right)^2 \tag{2.9}$$

$$Sc\frac{\partial H}{\partial t} = \frac{\partial^2 H}{\partial \eta^2} + \frac{N_t}{N_b}\frac{\partial^2 \theta}{\partial \eta^2} \tag{2.10}$$

Considering the boundary conditions we have the following equations;

$$\Rightarrow \frac{\partial H}{\partial \eta} = \frac{-N_t}{N_b}\frac{\partial \theta}{\partial \eta}$$
$$\Rightarrow \frac{\partial \theta}{\partial \eta} = \frac{ha}{-k_f}\theta$$
$$\Rightarrow \frac{\partial \theta}{\partial \eta} = -Bi\theta$$

Where Bi is the Biot number, Pr is the Prandtl number, N_t is the thermophoresis parameter, N_b is the Brownian motion parameter and Sc is the Schmidt number.

3 Numerical procedure

Applying a discretization finite difference method of lines (Na, 1979) the nonlinear initial boundary value problem (IBVP) in equations can be solved numerically. We partition the spatial interval $0 \leq \eta \leq 1$ into N equal parts with grid size $\Delta\eta = 1/N$ and grid points $\eta_i = (i-1)\Delta\eta$, for $1 \leq i \leq N+1$. Let $H_i(t), \theta_i(t)$

be approximation of $H(;t), (;t)$ and then the discrete system for the problem becomes as:

$$\frac{\partial \theta_i}{\partial t} = \frac{1}{Pr}\left[\frac{\theta_{i+1} - 2\theta + \theta_{i-1}}{(\Delta\eta)^2} + N_b\left(\frac{\theta_{i+1} - \theta_{i-1}}{2\Delta\eta}\right)\left(\frac{H_{i+1} - H_{i-1}}{2\Delta\eta}\right) + N_t\left(\frac{\theta_{i+1} - \theta_{i-1}}{2\Delta\eta}\right)^2\right] \tag{3.1}$$

$$\frac{\partial H_i}{\partial t} = \frac{1}{Sc}\left[\frac{H_{i+1} - 2H + H_{i-1}}{(\Delta\eta)^2} + \frac{N_t}{N_b}\left(\frac{\theta_{i+1} - 2\theta + \theta_{i-1}}{(\Delta\eta)^2}\right)\right] \tag{3.2}$$

Boundary conditions yield the following ordinary differential equations;

$$\frac{\theta_{i+1} - \theta_{i-1}}{2\Delta\eta} = -Bi\theta \tag{3.3}$$

$$\frac{H_{i+1} - H_{i-1}}{2\Delta\eta} = \frac{-N_t}{N_b}\left(\frac{\theta_{i+1} - \theta_{i-1}}{2\Delta\eta}\right) \tag{3.4}$$

Equations (xiv)-(xv) is a system of first order ordinary differential equations with known initial conditions and can be easily solved iteratively using Runge-Kutta Fehlberg integration technique implemented on computer using Matlab.

4 Results and Discussions

Recently, pure water has been taken as the base fluid and the nanoparticles used are alumina (Al_2O_3) and copper (Cu). The Prandtl number of the base fluid (water) is kept constant at 6.2 and the effect of solid volume fraction is investigated in the range of $0 \leq \phi \leq 0.3$. Intensive discussion for the problem has been done with great focus on concentration profiles and temperature profiles and effects of variation of parameters in different profiles.

4.1 Effects of parameter variation on concentration profiles

In figure 4, it is observed that as the time increases the concentration also increases. More interestingly, it is clearly observed that increase in space leads to concentration increase with increase in time from the lower plate towards the upper plate of the channel. Observing the figure 5 the results shows that as space increases the concentration also increases, but there is no change in concentration from $\eta = 0$ up to $\eta = 0.925$ and soon from there, there is a significance increase concentration from the lower plate to the upper plate of the channel. However observations show that as time is increased, the concentration of nanoparticle also increase but there is no any marked concentration change when $\eta = 1$ and therefore the increase in space is limited to $\eta = 0.98$.

From figure 6, shows the significant slight increase in concentration with increase of nanoparticle volume fraction (ϕ), That change is observed from $\eta = 0.64$ to $\eta = 0.8$ because concentration increases slightly with increase in space towards the upper plate, however there is no any marked significant change from $\eta = 0$ to $\eta = 0.6$.

Figure 7 shows the impact of the Biot number (Bi) in the nanofluids concentration profile. The results show that with an increase in Biot number the nanofluids concentration decreases. It is also noted that there is increase in concentration towards the upper plate of the channel; also the observation shows an increase in concentration as the space increases.

Considering figure 8 it is observed that the concentration is constant on the lower plate from $\eta = 0$ to $\eta = 0.5$ but as it reaches the space $\eta = 0.6$ it is observed to have significant increase in concentration with the increase in space. More interestingly, the results indicate that, the concentration increases with increase in thermophoresis parameter (Nt) towards the upper plate while other factors are kept constant.

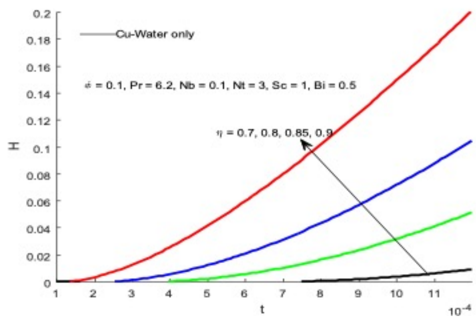

Figure 4: Nanofluids concentration profile with increasing time

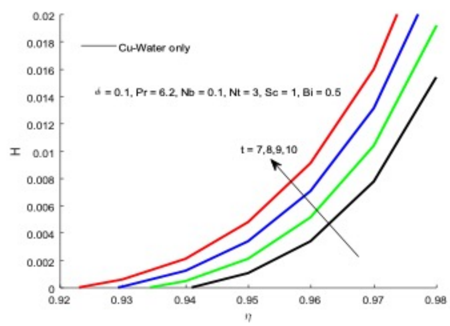

Figure 5: Nanofluids concentration profile with increasing space

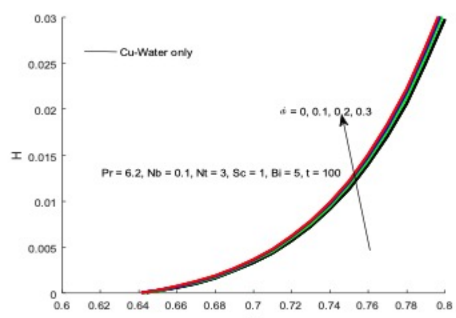

Figure 6: Nanofluids concentration profile with increasing ϕ

4.2 Effects of parameter variation on temperature profiles

It has been observed from figure 9, the temperature decreases with increase in time, though there is a significant decrease of the temperature to the freezing point towards the upper plate as the space increases due to convective heat loss by the ambient surroundings.

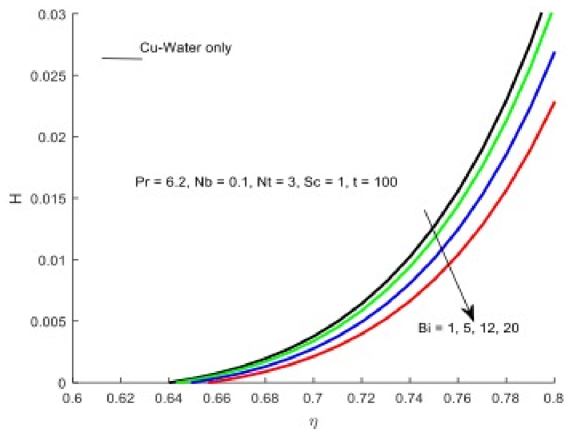

Figure 7: Nanofluids concentration profile with increasing Biot number, (Bi)

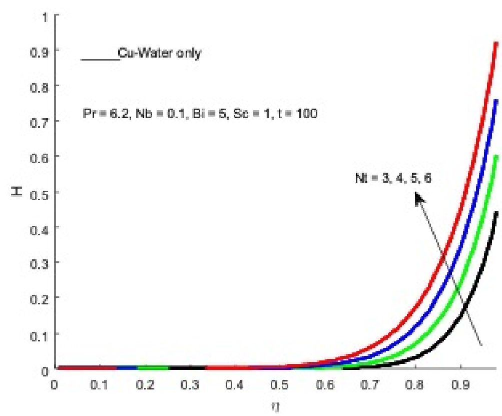

Figure 8: Nanofluids concentration profile with increasing Nt

The results from figure 10 shows that decrease with increase in space, the space from the observation is limited from $\eta = 0.8$ to $\eta = 0.99$ where significant change is observed. However, there is a marked impact of time on the temperature where the temperature is found to freeze with increase in time towards the upper plate due to convective heat loss by the ambient surroundings.

Figure 11 shows the impact of Schmidt number (Sc) on temperature profile and the results shows that temperature increase with an increase in Sc number. More interesting the temperature is observed to decrease to freezing point with an increase in space, though there observed a constant temperature on the low plate from $\eta = 0$ to $\eta = 0.4$ then from a significant change of temperature decrease is observed which can be due to convective heat loss by the ambient surrounding.

Figure 12 shows the impact of nanoparticle volume fraction (ϕ) while other parameters are kept constant. The results show that there is a very slightly decrease of temperature with an increasing nanoparticle

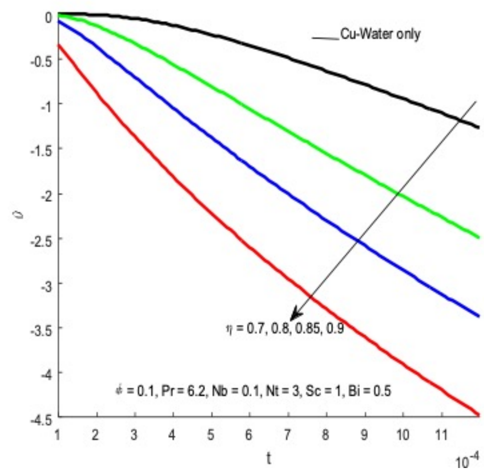

Figure 9: Nanofluids temperature profile with increasing time

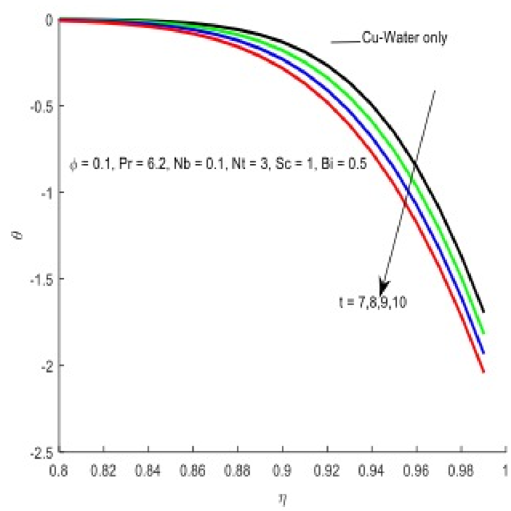

Figure 10: Nanofluids temperature profile with increasing space

volume fraction (ϕ) towards the upper plate. However, there seems a constant temperature with no change from $\eta = 0$ to $\eta = 0.6$ but soon from $\eta = 0.7$ temperature decreases slightly to the freezing points towards the upper plate.

Results from figure 13 show the impact of Brownian motion parameter (Nb) on the temperature profile and it is marked to have a significant increase in temperature with increasing number of Nb. Also, it is observed to have constant temperature on the lower plate from $\eta = 0$ to $\eta = 0.4$ then from $\eta = 0.5$ a significant change of temperature decrease towards the upper plate is observed which can be due to

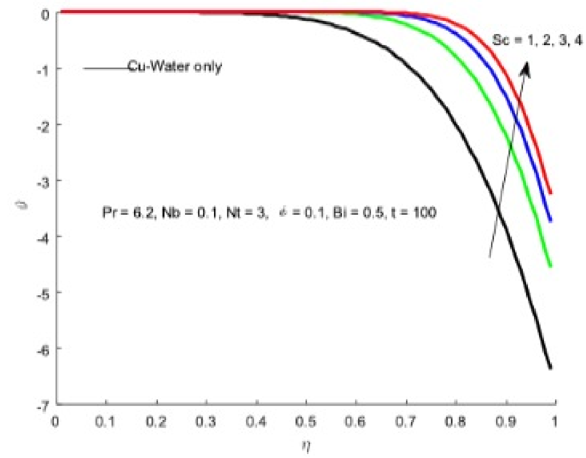

Figure 11: Nanofluids temperature profile with increasing Sc

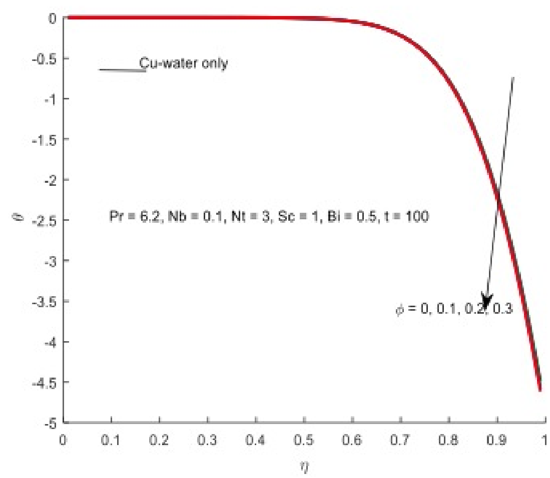

Figure 12: Nanofluids temperature profile with increasing (ϕ)

convective heat loss by the ambient surrounding.

Figure 14 shows the impact of Biot number (Bi) on temperature profile and the results show that temperature increase with an increase in Biot number. More interesting the temperature is observed to decrease slightly to freezing points towards the upper plate with an increase in space, though there observed a constant temperature from $\eta = 0$ to $\eta = 0.4$ then from $\eta = 0.5$ a significant change of temperature decrease is observed which can be due to convective heat loss by the ambient surrounding.

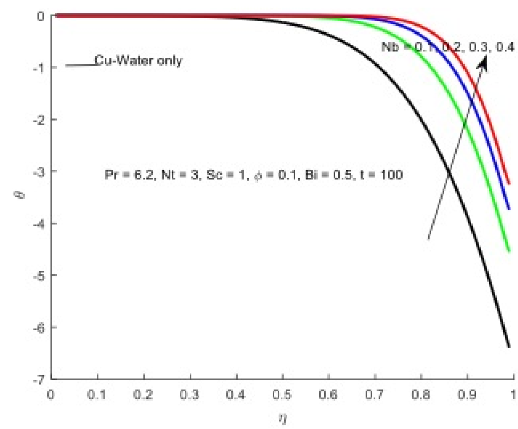

Figure 13: Nanofluids temperature profile with increasing Nb

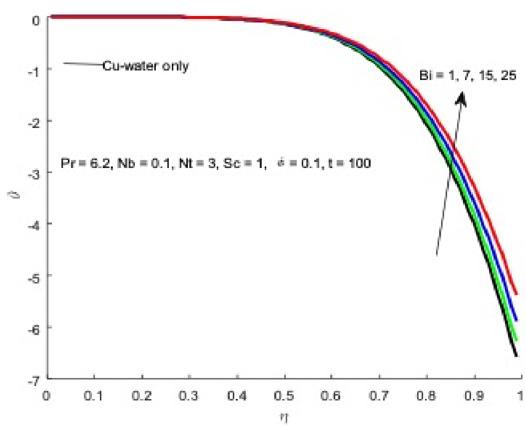

Figure 14: Nanofluids temperature profile with increasing Bi

5 Conclusion

In this work, the impact of nanoparticles in the unsteady channel flow of water based nanofluids was investigated. Using a semi-discretization finite-difference method together with a Runge-Kutta integration scheme implemented on computer using Matlab, the nonlinear governing partial differential equations were transformed into initial value ODEs and then solved numerically. The nanofluid concentration and temperature profiles were discussed for various parameters values governing the flow and from the results and subsequent discussions the following conclusions are deduced.

The Alumina-water nanofluid tends to flow in the same way as Copper-water nanofluid. For both Alumina-water and Copper-water nanofluids their concentration increases with time. The nanofluid concentration increases with with increasing thermophorensis parameter (Nt) and nanoparticle volume fraction and reduced with the increasing of the Biot number (Bi) whereas nanofluid temperature increases with Biot number (Bi), Schmidt number (Sc) and the Brownian motion parameter (Nb) and decrease with an increase nanoparticle volume fraction (ϕ).

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