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Sensitivity Analysis of a New Model of Stewart Platform

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Abstract: In this article, we introduce a new model of the Stewart Platform robot with rotary servo motors that is very cheap to build. First of all, studied a kinematic of the Stewart Platform, and then, with a simple geometric analysis, we'll find the angles of servo motors for a specified position. Because of using the angular or rotational Servo motors in this model, sensitivity analysis is a crucial point. After sensitivity analysis of the model, we show the sensitivity level of a type of rotary servo in different coordinates x, y, and z.

Keywords: Stewart Platform; Parallel Robot; Sensitivity Analysis; Rotary Servo;
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1 Introduction

With the increasing proliferation of bots in various applications, the need for structures that have no series robots limitations and have special features such as accuracy and high acceleration, and high load-carrying capability, have increasingly felt. The solution is to look back to nature and be inspired by it. As humans use both hands for precise movement or lifting heavy objects, it can be assumed that the use of closed kinematic chains of the multi-arm parallel to each other and being involved in the movement may be an appropriate response to this problem. A parallel robot with a closed kinematic chain is a mechanism in which a mobile platform by several independent kinematic chains is attached to the base. Thus, a parallel robot is a static platform that consists of a connected multi-arm moving to the platform. These arms often include a sliding actuator or rotating spherical joints, which are connected to fixed platforms. Parallel robots offer several advantages to a series robot that are obvious: One of the features of parallel robots having some kinematic closure ring. Given this, a parallel robot has more intentions than a series robot. As a result, could expect higher speed and acceleration and also the larger forces applied to it. On the other hand, in these robots, every branch bears part of the burden-sharing and it causes the system to consume less energy while heavier loads retreat. The Stewart Platform is a platform

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that has six degrees of freedom and can move in three linear directions (x, y, z) and also three angles' directions or combinations of them. Stewart's first flight simulator was built in 1965 [6] that are used widely for this purpose now. Stewart Platform is used in many industries, including automotive, defense, transport, and machine tool technology applications. In this article, we try to find the sensitivity of the Stewart Platform by analyzing geometrics. The photograph of the built Stewart Platform is shown in Figure 1. The photograph also indicates the range of motion of this platform. Mr.Tannous and partners in [7] using a linear method, studied sensitivity to Series mechanisms. Mr. Han and colleagues in [3], have studied the sensitivity of a parallel mechanism with three degrees of freedom. In the following first, check the robot kinematics problem, and then we will do a sensitivity analysis. Mr. Spong in [5] has studied different robot kinematics. The parallel robot model examined in this article in [4] is given. An example of this type of sensitivity analysis torque and power Servo motors of these platforms are in [1].

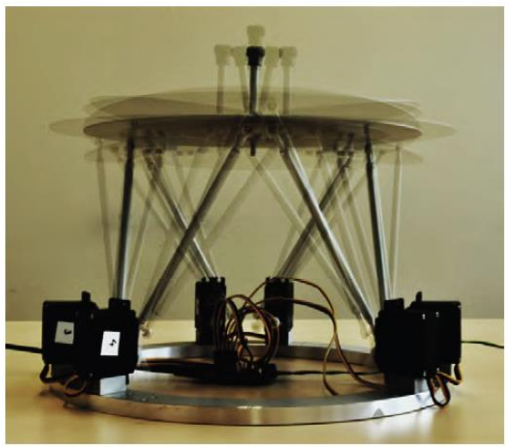

Figure 1: Stewart platform

2 Ease of Use

The Stewart Platform consists of 2 rigid frames connected by 6 variable-length legs. The Base is considered to be the reference framework, with orthogonal axes x, y, and z. The Platform has its orthogonal coordinates .

The Platform has 6 degrees of freedom concerning the Base the origin of the Platform coordinates can be defined by 3 translational displacements concerning the Base, one for each axis. Three angular displacements then define the orientation of the platform concerning the Base. A set of Euler angles are used in the following sequence:

- Rotate an angle around the z-axis
- Rotate an angle around the y-axis

- Rotate an angle around the x-axis

If we consider the first rotation around the z-axis, then we have (Figure2):

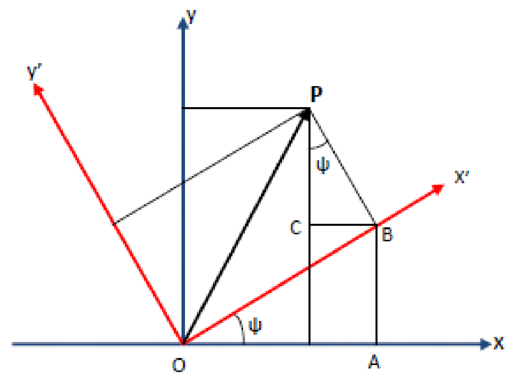

Figure 2: Yaw angle rotation

$$\begin{aligned} &P = i'x' + j'y' + k'z' = ix + jy + kz \\ &x = OA - BC = x'\cos\psi - y'\sin\psi \\ &y = AB + PC = x'\sin\psi - y'\cos\psi \\ &z = z' \end{aligned} \tag{2.1}$$

Now, we define the rotation matrix $R_z(\psi)$ where:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = R_z(\psi) \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix}, \; R_z(\psi) = \begin{pmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{2.3}$$

Finally, the full rotation matrix of the Platform relative to the base is then given by:

$$^{P}R_B = R_z(\psi)\cdot R_y(\theta)\cdot R_x(\phi) = \begin{pmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi & \cos\phi \end{pmatrix} =$$

$$\begin{pmatrix} \cos\psi\cos\theta & -\sin\psi\cos\phi + \cos\psi\sin\theta\sin\phi & \sin\psi\sin\phi + \cos\psi\sin\theta\cos\phi \\ \sin\psi\cos\theta & \cos\psi\cos\phi + \sin\psi\sin\theta\sin\phi & -\cos\psi\sin\phi + \sin\psi\sin\theta\sin\phi \\ -\sin\theta & \cos\theta\sin\phi & \cos\theta\cos\phi \end{pmatrix}$$

Now consider a Stewart Platform, for the ith leg: The coordinates q_i of the anchor point P_i with respect to the base reference framework (Figure3) are given by the equation:

$$q_i = T + {}^{P}R_B . p_i \tag{2.4}$$

Where T is the translation vector, giving the positional linear displacement of the origin of the platform frame (Figure3) concerning the Base reference framework, and p_i is the vector defining the coordinates of the anchor point P_i with respect to the Platform framework. Similarly, the length of the i^{th} leg is given by:

$$l_i = T + {}^{P}R_B . p_i - b_i \tag{2.5}$$

Where b_i is the vector defining the coordinates of the lower anchor point B_i. These 6 equations give the lengths of the 6 legs to achieve the desired position and attitude of the platform. When considering the Forward Kinematics, this expression represents 18 simultaneous nonlinear equations in the 6 unknowns representing the position and attitude of the platform. Much work has been done on finding the solutions to these equations; in the general case, there are 40 possible solutions, although in practice many of these solutions would not be practical.

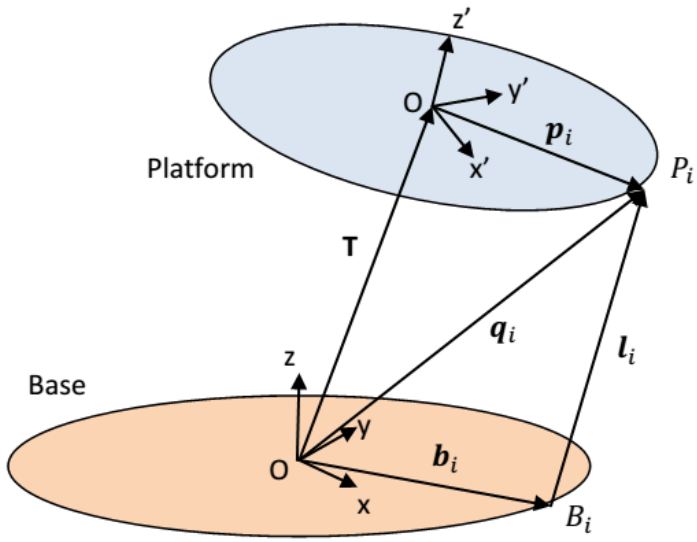

Figure 2: Coordinate system of Stewart platform

2.1 Servo angles

The leg lengths are achieved via rotational servos, rather than linear servos, a further calculation is required to determine the angle of rotation of the servo. Each servo/leg combination can be represented as follows (Figure5):

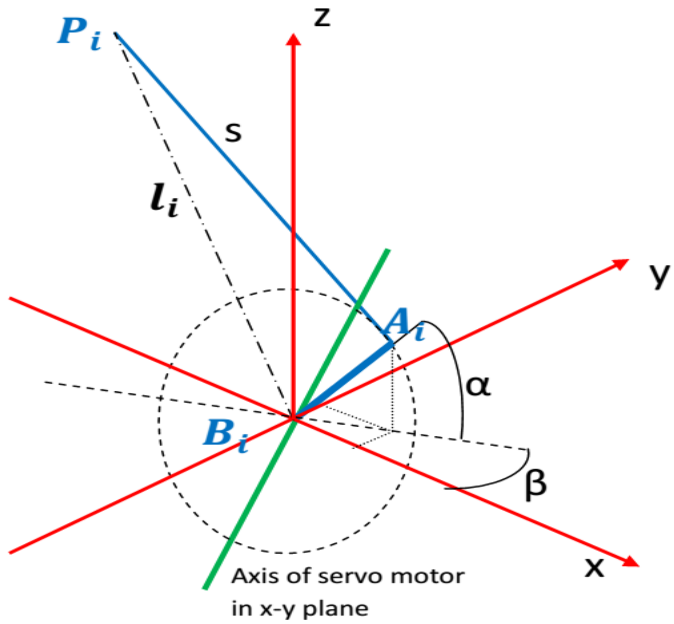

Figure 3: Axis of one of the servo-motors in x-y plane

Where a is length of the servo operating arm and A_i are the points of the arm/leg joint on *ith* the servo with coordinates $a=[x_a \ \ y_a \ \ z_a]^T$ in the base framework. B_i are the points of rotation of the servo arms with the coordinates $b=[x_b \ \ y_b \ \ z_b]^T$ in the base framework. P_i are the points the joints between the operating rods and the platform, with coordinates $p=[x_p \ \ y_p \ \ z_p]^T$ in the platform framework. S is length of operating leg and l_i length of the i^{th} leg as calculated from $l_i = T + {}^{P}R_B . p_i - b_i$ and also α, β are angle of servo operating arm from horizontal and angle of servo arm plane relative to the x-axis (Note that the shaft axis lies in the x-y plane where z = 0).

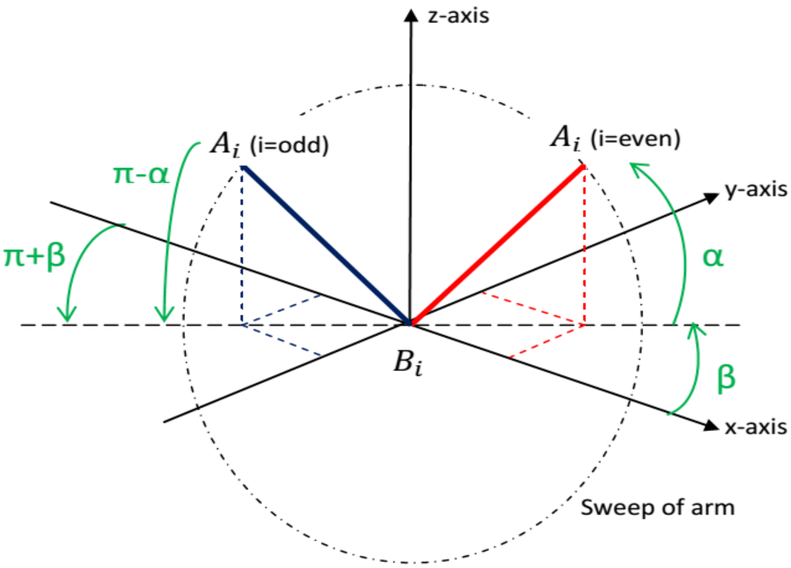

Figure 4: Stewart platform servo angles

Point A is constrained to be on the servo arm, but the arrangement of the servos means that the odd and even arms are a reflection of each other. So, for the even legs we have:

$$x_a = a\cos\alpha\cos\beta + x_b \ , \ y_a = a\cos\alpha\sin\beta + y_b \ , \ z_a = a\sin\alpha + z_b \qquad (2.6)$$

And for the odd legs we have:

$$x_a = a\cos(\pi-\alpha)\cos(\pi+\beta) + x_b \ , \ y_a = a\cos(\pi-\alpha)\ \sin(\pi+\beta) + y_b \ , \ z_a = a\sin(\pi-\alpha) + z_b \ (2.7)$$

But we also have this relationship between odd and even legs:

$$\sin(\pi-\alpha) = \sin\alpha \, , \cos(\pi-\alpha) = -\cos\alpha \, , \sin(\pi+\alpha) = -\sin\alpha \, , \cos(\pi+\alpha) = -\cos\alpha$$

Substituting these values into the equations for the odd legs, we get the same equations as (4) for the even legs.

By Pythagoras we also have:

$$a^2 = (x_a - x_b)^2 + (y_a - y_b)^2 + (z_a - z_b)^2 = (x_a^2 + y_a^2 + z_a^2) + (x_b^2 + y_b^2 + z_b^2) - 2(x_a x_b + y_a y_b + z_a z_b) \ (2.8)$$

$$l^2 = (x_p - x_b)^2 + (y_p - y_b)^2 + (z_p - z_b)^2 = (x_p^2 + y_p^2 + z_p^2) + (x_b^2 + y_b^2 + z_b^2) - 2(x_p x_b + y_p y_b + z_p z_b) \ (2.9)$$

$$s^2 = (x_p - x_a)^2 + (y_p - y_a)^2 + (z_p - z_a)^2 = (x_p^2 + y_p^2 + z_p^2) + (x_a^2 + y_a^2 + z_a^2) - 2(x_p x_a + y_p y_a + z_p z_a) \ (2.10)$$

By substituting equations (2.8) and (2.9) at (2.10), we have:

$$l^2 - (s^2 - a^2) = 2a\ \sin\alpha\,(z_p - z_b) + 2a\ \cos\alpha\cos\beta(x_p - x_b) + 2a\ \cos\alpha\sin\beta\,(y_p - y_b) =$$

$$2a\ \sin\alpha\,(z_p - z_b) + 2a\ \cos\alpha[\cos\beta(x_p - x_b) + \sin\beta\,(y_p - y_b)]$$

Which is an equation of the form $L = M\sin\alpha + N\cos\alpha$, We therefore have another sine function of α with a phase shift δ :

$$L=\sqrt{M^2+N^2}\sin(\alpha+\delta)\ ,\ \delta=\tan^{-1}\frac{N}{M}\Rightarrow \sin(\alpha+\delta)=\frac{L}{\sqrt{M^2+N^2}}\ ,$$
$$\alpha=\sin^{-1}\frac{L}{\sqrt{M^2+N^2}}-\tan^{-1}\frac{N}{M}\Rightarrow L=l^2-(s^2-a^2)\ ,\ M=2a(z_p-z_b)\ ,N=2a[\cos\beta(x_p-x_b)+\sin\beta(y_p-y_b)] \tag{2.11}$$

We now have sufficient information to calculate the lengths of the effective "legs", and the associated angle of the servo arms, for the reverse kinematics for the platform. But to design and implement the hexapod platform we need to define a few constants in order to define the range of movement.

2.2 Home position

We need to define the "home" position of the platform. By definition this will be where the platform is at a height above the base framework, and there being no other translational or rotational movement.

$$l^2=s^2+a^2=(x_p-x_b)^2+(y_p-y_b)^2+(h_0+z_p-0)^2\quad,\quad h_0=\sqrt{s^2+a^2-(x_p-x_b)^2+(y_p-y_b)^2}-z_p \tag{2.12}$$

Similarly, we will define the "home" position where the servo arms and rods are at right angles to each other. since the platform is constructed symmetrically around the z-axis, this equation will give the same result for any leg.

2.3 The initial angle

We can also calculate the angle α_0 of the servo arm at the home position. Using the equation $l_i=T+{}^pR_B.p_i-b_i$ and remembering the symmetrical construction of the platform. The length of the legs in the "home" position is given by: $l^2=(x_p-x_b)^2+(y_p-y_b)^2+(h_0+z_p)^2$ and the angle of the servo arm in the "home" position can be given by equation (2.12). Since we have symmetry, we can look at leg 2 only, where $\beta=0^\circ$:

$$\alpha_0=\sin^{-1}\frac{L_0}{\sqrt{{M_0}^2+{N_0}^2}}-\tan^{-1}\frac{N_0}{M_0}\Rightarrow L_0=l^2-(s^2-a^2)=s^2+a^2-(s^2-a^2)=2a^2$$
$$N_0=2a(h_0+z_p)\quad,\quad M_0=2a[\cos\beta(x_p-x_b)+\sin\beta(y_p-y_b)]=2a(x_p-x_b) \tag{2.13}$$

3 Sensitivity analysis

In this section, we try to geometric analysis platform and find the displacement of a movable plate when a servo moves to a small size. For this purpose, consider Figure 5 :

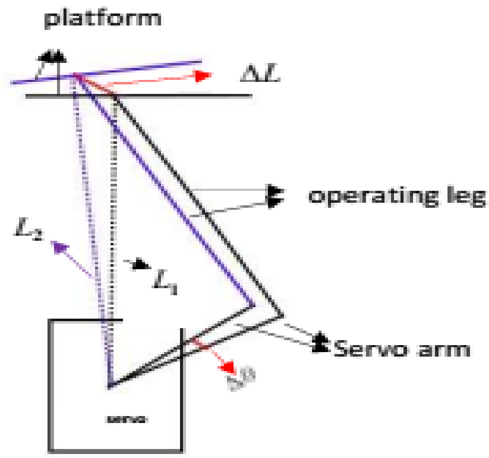

Figure 5: Displacement of Stewart platform movable plate due to servo infinitsemal rotation

In this figure $\Delta\theta$ is amount of variation of servo angles and ΔL is displacement of movable plate. L_1, L_2 are lines corresponding to the servo angles in initial state and the secondary mode (after displacement amount of $\Delta\theta$). Because the amount of displacement of the movable plate is very small, we calculate it with the following statement:

$$\Delta L = \tfrac{1}{2}(L_1 + L_2)\Delta\theta \tag{3.1}$$

For calculate the displacement all the above equation parameters must be known, which means that we must calculate L_2 to find amount of displacement. Note that the $\Delta\theta$ and L_1 are the known parameters and don't need to calculate them. to calculate L_2 use the Trigonometry rule below:

$$\Delta L^2 = L_1^2 + L_2^2 - 2L_1L_2\,Cos(\Delta\theta) \tag{3.2}$$

By substituting equations 13 at 12, We can calculate the amount of L_2, that is equal to:

$$L_2 = \frac{1}{-4+\Delta\theta^2}\left\{\Delta\theta^2 L_1 - 4\cos(\Delta\theta)L_1 \pm 4\left(2\sin^2(\Delta\theta/2)L_1^2 + \Delta\theta^2\sin^2(\Delta\theta/2)L_1^2 - 2\cos(\Delta\theta)\sin^2(\Delta\theta/2)L_1^2\right)^{\frac{1}{2}}\right\} \tag{3.3}$$

There are two answers to L_2, because the robot can be both clockwise and anti-clockwise to move. Since we moved in anti-clockwise direction So, we do all the calculations with the root with negative sign of square root. By substituting from above equations, we can calculate the amount of ΔL, that is equal to:

$$\Delta L = \frac{1}{-8+2\Delta\theta^2}\left\{-8L_1 + 2\Delta\theta^2 L_1 - 4\cos(\Delta\theta)L_1 - 4\left(-2\sin^2(\Delta\theta/2)L_1^2 + \Delta\theta^2\sin^2(\Delta\theta/2)L_1^2 - 2\cos(\Delta\theta)\sin^2(\Delta\theta/2)L_1^2\right)^{\frac{1}{2}}\right\} \tag{3.4}$$

Now we need to get the sensitivity of the motion to change the $\Delta\theta$ angle. As in the second quarter [2] is given, to calculate the sensitivity of the variable α against of variable β, we use the following equation:

$$S_\beta^\alpha \triangleq \frac{\partial\alpha}{\partial\beta}\frac{\beta}{\alpha} \tag{3.5}$$

So, by substituting ΔL instead of α and $\Delta\theta$ instead of β, The platform displacement sensitivity to changes in the servo angle is equal to:

$$S_{\Delta\theta}^{\Delta L}=\frac{\partial \Delta L}{\partial \Delta\theta}\frac{\Delta\theta}{\Delta L}=$$

$$\frac{(-8+2\Delta\theta^2)\left(\left(-\frac{4\Delta\theta*(-8L_1+2\Delta\theta^2L-4Cs(_1\Delta\theta)L_1 \quad -4\sqrt{-2S^2(\Delta\theta/2)L_1^2+\Delta\theta^2S^2(\Delta\theta/2)L_1^2-2C(\Delta\theta)S^2(\Delta\theta/2)L_1^2})}{(-8+2\Delta\theta^2)^2}\right)+\left(\frac{4\Delta\theta L_1+4S(\Delta\theta)L_1-(-4C(\Delta\theta/2)S(\Delta\theta/2)L_1^2+2\Delta\theta C(\frac{\Delta\theta}{2})S(\Delta\theta/2)L_1^2 \quad -4C(\frac{\Delta\theta}{2})C(\Delta\theta)S(\Delta\theta/2)L_1^2+4\Delta\theta S(\Delta\theta/2)L_1^2+4S(\Delta\theta)S^2(\Delta\theta/2)L_1^2}{-8+2\Delta\theta^2}\right)\right)}{\left(\sqrt{-2S^2(\Delta\theta/2)L_1^2+\Delta\theta^2S^2(\Delta\theta/2)L_1^2-2C(\Delta\theta)S^2(\Delta\theta/2)L_1^2}\right)\left(-8L_1+2\Delta\theta^2L_1-4C(_1\Delta\theta)L_1-4\sqrt{-2S^2(\Delta\theta/2)L_1^2+\Delta\theta^2S^2(\Delta\theta/2)L_1^2-2C(\Delta\theta)S^2(\Delta\theta/2)L_1^2}\right)}$$

(3.6)

Where in this relation we use the notation $S\equiv\sin, C\equiv\cos$. Finally, sensitivity can be obtained for small movements.

3.1 Design evaluation

The capabilities of the presented Stewart Platform with fixed rotary actuators strongly depend on the quality of the components used. The photograph of the built Stewart Platform is shown in Figure 1. The photograph also indicates the range of motion of this platform. As there is no sensory feedback from the servos and no external measurements were carried out only a coarse-grained evaluation can be given here. In this context, the platform was commanded to move in *x*, *y,* and *z-direction* as well as to perform positive and negative rotations around these axes as far as possible. The platform did not carry any additional load and the current consumption was monitored to see if the commanded position and orientation were reached - an increased current flow would indicate that one or more servos could not reach the commanded position. Since the employed servos can perform continuous rotation and the angular limits in the joints are large enough the platform was able to reach all the commanded positions/orientations according to the real solutions. However, this pleasing result is to be ascribed to the limited variability in the length of the virtual legs and thus a relatively small work area. The range of motion is approximately ±25, ±28, and ±15 mm for the motion along the *x*, *y,* and *z* axes, respectively. The precision of the platform is limited mainly by the resolution of the employed servos whose angular position can be changed in the smallest increment of 0.1° only. The operating speed of the servos is 0.15sec/60°which corresponds to approximately 40 mm/s for the platform. The overall weight of the low-cost Stewart Platform together with the servo controller and cabling is less than 0.5 kg. The platform can carry payloads of approximately a few kilograms which are much heavier than its weight.

4 Conclusion

Due to the expensive nature Stewart platform, a model in this paper could provide a very cheap price for this type of robot. As we have seen, the platform is enormous sensitivity for the analog servo and without sensor feedback, it is very difficult to control the platform. We can use a Digital servo with a little more cost which benefits from sensor feedback to create a model which is suitable for most industries. Also, a larger workspace can be created by using digital servos.

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