MATHEMATICS AND COMPUTATIONAL SCIENCES

Vol 3(3), 2022, pp:57-75
DOI:10.30511/mcs.2022.255061
ISSN:2717-2708
http://mcs.qut.ac.ir/

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Modelling and analysis of counter-terrorism: via drone

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Abstract: The September 11, 2001 attacked in Washington District of Colombia (DC) and Pennsylvania in US known as 9/11 attacks remain the most destructive terrorist assaults in recorded history, and the attacks led to far bloodier conflicts as part of the subsequent war on terror. Terrorism has become the defining issue of International politics of the 21st century. Because of the spread of the ideology, the drone together with Information Communication Technology (ICT) contribute to the counter-terrorism mission by providing universal intelligence, surveillance and hostile strike capabilities to control and reduce the recruitment pool of terrorist groups. This paper focuses on Modelling and Analysis of Counter-terrorism using Drone as a control technique with five compartments. The results showed that use of drone to fight against terrorist/insurgency appear to have 80% significant reduction on their strength, kill leaders, foot soldiers and put the criminal elements in a state of confusion. Finally, effective counter-terrorist and counter-insurgent depend on drone deployment across the borders for monitoring.

Keywords: Modelling; Analysis; Counter-terrorism; Drone.
2020 Mathematics Subject Classification: 00A71; 34D20; 93A30
Receive: 23 May 2022, **Accepted:** 29 September 2022

1 Introduction

Terrorism can be seen as an unlawful organization, attempting or threatening to cause death or serious bodily injury to any person and causing damage to private property including public places or government facilities in [20] . The United Nations Security Council (UNSC) in [21] adopted at its 814th and 8496th meetings, stressed that the primary responsibility of member states in countering terrorist acts and reiterating their obligation to prevent and suppress the financing of terrorism as well as the call upon all states to become party to the international counter-terrorism conventions and protocols. In Global terrorism index [4] defined terrorism as a threatened or actual use of illegal force and violence by a non-state actor to attain political, religious, or social goals through fear, oppression or intimidation. This definition indicates that terrorism is not only the physical act of an attack but also the mental blow it has on humanity for many years in [4].

The term drone is an Unmanned Aerial Vehicles (UAV) that can be used for military or commercial purposes. The United States is also using combat drones to strike at terrorist and insurgent groups. These include combat drone strikes that target militants in Pakistan who backing al-Qaeda and insurgents operating in Afghanistan, al-Qaeda in the Arabian Peninsula in Yemen, and the al-Shabaab movement in

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Somalia, Walsh in [23]. The purposes of these combat drone strikes are to control and to prevent insurgent/terrorist groups from gaining momentum. Drone punishes the groups by killing or assassinating leaders and creating fear and doubt among the member of their foot-soldiers. To understand how drone warfare contributes to effective counter-terrorism, it is important to explain counter-terrorism and counter-insurgency. Moreover, Farrow in [3] study showed how the drone aircraft fits into the military approach. The drone contributes to the counter-terrorism mission by providing universal intelligence and hostile strike capabilities. The result of his work showed that the drone is a particularly effective military instrument of counter-terrorism strategy because of its unique capabilities to employ pervasive reconnaissance and aggressive air-strikes simultaneously. The UAV assisted the military operation in a volatile locations worldwide. The new development in the use of UAV led to a new aviation era of self-sufficient aerial vehicles in both the civilian and military fields. Therefore, Yaacoub *et al.,* in [24] examined the use of drone within Information and Communication Technology links to track criminals in a population. Also, Onuoha *et al.,* in [14] examined the Counter-insurgency operations capable of defeating the Boko Haram insurgency. Their results show how military campaigns at the strategic, operational and tactical levels have been challenged by numerous factors, including corruption. A mathematical model of the dynamics behaviour of terrorism was developed by Adamu and Ibrahim in [1]. The model is developed to control the spread of terrorist ideologies in the society and to check the recruitment pool. Furthermore, Santoprete in [19] built a model of countering violent extremism (CVE) for prevention programs, and sought to stop the radicalization process from occurring. Also, Momoh *et al.,* in [13] concluded that in order to reduce or eliminate drug abuse and violence co-menace in the population, government needed to invest in mass campaign against drug abuse, mass campaign against violence, effective policing and rehabilitation of drug abusing and violent individuals. Rivera-Castro *et al.,* in [17] developed a mathematical model, analysis and simulation of the spread of gangs in interacting youth and adult populations. The model used to study the spread of gangs in vulnerable youth and adult sub-populations. Moreover, Ibrahim et al., in [5, 6] examines the impact of correctional intervention measures on the population dynamics of the criminal gangs in Nigeria, following an age-structured paradigm and the principles of optimal control theory applied to a non-linear mathematical model for the population dynamics of criminal gangs with variability in the sub-population.

To examine how warfare contributes to effective counter-terrorism, it is important to clarify a general distinction between counter-terrorism and counterinsurgency. Counter-terrorism and counter insurgency are two different types of warfare; Counter-terrorism focuses on the enemy while counterinsurgency focuses on the population. For example, counter-terrorism often involves disrupting terrorists ability to conduct operations while counter-insurgency often involves building and solidifying domestic institutions and societies (hearts and minds). But these two forms of warfare can be counteracting, producing a variety of potential offsetting effects in [3] . There is no single approach to prevent terrorism rather a combination of techniques to combat the potential threats. The best way to counter terrorism is by using a three approach method, one military intervention approach and two civil intervention approach. The military intervention approaches focuses on the use of armed drones to target leaders and foot-soldiers within terrorist organizations, in order to slow the spread of terrorist cells and their influence. The civil intervention approach focuses on untying financial aid for local population to assist in development projects, programme design for convected criminals in detention facilities and de-radicalized terrorist. This aid would be used to finance social, economic, and physical development projects which will lead to the development of local areas. Thus, the programme focuses on improving access to education and vocational training, Ryan in [18]. Okoye, Collins, and Mbah in [16] developed a mathematical model to examine the dynamics of terrorism and likely counter-terrorism techniques that can moderate terrorism in a given population. Their result forecasts the terrorism dynamics like Boko Haram and some related criminals in Nigeria. Moreover, Mohammad and Roslan in [12] built a Crime Model using Dynamical Approach to examine the spread of the crime system in a population. From their result, bifurcation analysis shows the number of criminals that are imprisoned increases as well as parameter values increases in the model. Similarly, Okrinya and Consul in [15] construct a simple mathematical model on kidnapping by incorporating rehabilitation of kidnappers in a population. The result shows that increasing the rehabilitation rate of kidnappers is affirmed the effective way of ensuring a kidnapping free society.

Therefore, The aim of this paper is to examine Counter-Terrorist activities and deploy UAVs for surveillance across the border to reduce the threats, while the objectives are to; test the model formulated with both military and civil intervention, evaluate the basic reproduction number of the terrorism in a population, test for stability analysis, examine the drone deployment ability across the border for intelligence gathering, surveillance and strike, examine the terrorist behaviour and informant.

2 Formulation of Model

The model focuses on Modelling and Analysis of Counter-terrorism via Drone with five (5) compartments which includes military and rehabilitation programme in a population. Subsequently, the network model diagram of terrorist movement can be deduced in figure 1 as;

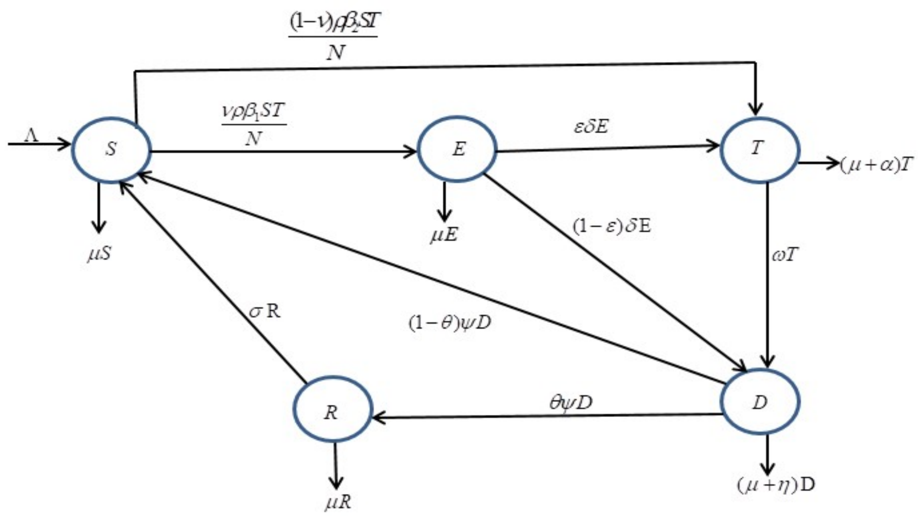

Figure 1: Flow Diagram of Terrorism

Table 1: Description of State Variables of the Model

Variables	**Description**
$S(t)$	Susceptible group t
$E(t)$	Moderate group t
$T(t)$	Terrorist group t
$D(t)$	Detention facility t
$R(t)$	Rehabilitation centre t

Table 2: Description of Parameters of the Model

Parameters	Description
β_1	Contact rate with terrorist individuals
β_2	Contact rate at which terrorists use force recruitment
ε	Is the proportion of individuals that join terrorist from moderate
η	Death due to torture in the detention facility
θ	The rate at which certified terrorists move to susceptible
Λ	Recruitment rate (birth and immigration)
ψ	Certified terrorists that joins the rehabilitation centre
σ	The proportion of successful rehabilitee individuals
δ	The rate at which exposed individuals become terrorist
ω	The rate at which terrorist move to the detention facility due to drone activities
ρ	Is the terrorist transmission probability per contact
ν	Is the proportion of susceptible individuals who becomes terrorists
μ	The death rate of all groups
α	Death due to counter-terrorist activities by drone, suicide

2.1 Description of Model

The total population $N(t)$ is divided into five compartments. The five compartments are:
Susceptible Group $S(t)$ is at the risk of contact with the ideology through interaction. It is assumed that susceptible individuals are recruited by increasing the population byΛ(birth and immigration), σ(proportion of successful rehabilitee individuals who move to join susceptible individuals and these rehabilitee individuals are not permanently de-radicalized, because some may re-join terrorist since the ideology will not be completely remove), $(1-\theta)\psi$ (proportion of not all individuals will move to join rehabilitee individuals from the detention facility after ending jail term together with certified terrorist to join the susceptible group) and decreasing by μ (natural death), $\frac{\nu\rho\beta_1 ST}{N}$ (proportion of susceptible individuals who become terrorist and move to moderate group with transmission probability per contact together with a contact rate of terrorist individuals) and $\frac{(1-\nu)\rho\beta_2 ST}{N}$ (proportion of not all susceptible individuals become terrorist with transmission probability per contact together with contact rate at which terrorist use force recruitment).

$$\frac{dS}{dt} = \Lambda + \sigma R + (1-\theta)\psi D - \left(\frac{\nu\rho\beta_1 T}{N} + \frac{(1-\nu)\rho\beta_2 T}{N} + \mu\right) S \tag{2.1}$$

The Moderate Group $E(t)$ is signifying the number of individuals that are exposed to the terrorist ideology, but not complete terrorist. This group is increasing by $\frac{\nu\rho\beta_1 ST}{N}$ (proportion of susceptible individuals who become terrorist and move to moderate group with transmission probability per contact together with a contact rate of terrorist individuals) and decreasing by $\mu + \delta$ (natural death and progression rate of the terrorist group).

$$\frac{dE}{dt} = \frac{\nu\rho\beta_1 ST}{N} - (\mu + \delta)E \tag{2.2}$$

The Terrorist Group $T(t)$ is representing the terrorist group. This compartment is increasing by $\frac{(1-\nu)\rho\beta_2 ST}{N}$ (proportion of not all susceptible individuals become terrorists with transmission probability per contact together with contact rate at which terrorist use force recruitment), $\varepsilon\delta$(ε progressed at a rate of δ to the terrorist group and the remaining proportion will move to the detention facilities) and decreasing by $\mu + \alpha + \omega$ (natural death, death due to suicide or UAV attack and the rate at which captured terrorists move to detention facilities with the use of UAV).

$$\frac{dT}{dt} = \varepsilon\delta E + \left(\frac{(1-\nu)\rho\beta_2 S}{N} - (\mu + \alpha + \omega)\right)T \tag{2.3}$$

The Detention Facility $D(t)$ is representing the number of suspected individuals who are in the detention facility. This group is increasing by $(1 - \varepsilon)\delta$ (not all individuals captured and move to detention are terrorist, some will progress at a rate of δ proportion to the detention facilities), ω (the rate at which captured terrorist move to detention facilities with the use of UAV) and decreasing by $\mu + \eta + \psi$ (natural death, death due to torture in detention facilities and individuals that move to join rehabilitates from the detention facilities after ending jail term or certified by law agency and move at a rate of ψ to join the susceptible group).

$$\frac{dD}{dt} = (1 - \varepsilon)\delta E + \omega T - (\psi + \mu + \eta)D \tag{2.4}$$

The Rehabilitation Centre $R(t)$ is signifying the number of terrorist individuals that are certified by the counter-terrorist operation. This group of individuals can increase by $\theta\psi D$(proportion of individuals that move to join the rehabilitated from the detention facilities after ending jail term or certified by law agencies with a rate of ψ to join the susceptible group) and decreasing by $\mu + \sigma$ (natural death rate and proportion of successful rehabilitated individuals who move to join sub-population).

$$\frac{dR}{dt} = \theta\psi D - (\mu + \sigma)R \tag{2.5}$$

It is assumed that the terrorist individuals that are certified by NGOS and law enforcement agencies, move to a rehabilitation centre and stay for some time to acquire a skill programme designed for them is not permanent de-radicalised since some of them may return to terrorist/radical groups. In this study, the natural death rate μ is for all humans and the total population in this research work is given by;

$$\frac{dN}{dt} = S(t) + E(t) + T(t) + D(t) + R(t) \tag{2.6}$$

The equation corresponding to the model diagram in figure 3.2 is given by non-linear differential equation;

$$\left.\begin{aligned}
\frac{dS}{dt} &= \Lambda + \sigma R + (1-\theta)\psi D - \left(\frac{\nu\rho\beta_1 T}{N} + \frac{(1-\nu)\rho\beta_2 T}{N} + \mu\right)S \\
\frac{dE}{dt} &= \frac{\nu\rho\beta_1 ST}{N} - (\mu + \delta)E \\
\frac{dT}{dt} &= \varepsilon\delta E + \left(\frac{(1-\nu)\rho\beta_2 S}{N} - (\mu + \alpha + \omega)\right)T \\
\frac{dD}{dt} &= (1 - \varepsilon)\delta E + \omega T - (\psi + \mu + \eta)D \\
\frac{dR}{dt} &= \theta\psi D - (\mu + \sigma)R \\
\frac{dN}{dt} &= S + E + T + D + R
\end{aligned}\right\} \tag{2.7}$$

with the initial condition;

$$S(0) = S_0 \geq 0, E(0) = E_0 \geq 0, T(0) = T_0 \geq 0, D(0) = D_0 \geq 0, R(0) = R_0 \geq 0.$$

2.2 Basic Properties of the Terrorism Model

2.3 Invariant Region

Let determine $N = (S, E, T, D, R) = S(t) + E(t) + T(t) + D(t) + R(t)$. Then, N respect to time obtain;

$$\frac{dN}{dt} = \frac{dS}{dt} + \frac{dE}{dt} + \frac{dT}{dt} + \frac{dD}{dt} + \frac{dR}{dt} \tag{2.8}$$

If there is no terrorism, the total population can be obtained as;

$$\frac{dN}{dt} = \Lambda - \mu N \tag{2.9}$$

Since; $N = S + E + T + D + R$, then equation (2.9) resolved to the linear differential equation of the form;

$$\frac{dN}{dt} + \mu N - \Lambda \tag{2.10}$$

solving equation (2.10) by using the method of Integrating factor and evaluating it for $t \to \infty$, obtain; $\Omega = \left\{(S, E, T, D, R) \in \Re^5_+ : N(t) \leq \frac{\Lambda}{\mu}\right\}$, which is the feasible solution set for the model system (2.7) and all the solution set is bounded by Ω.

2.4 Positivity of Solution

Theorem 2.1. *Let $S(0) > 0, E(0) > 0, T(0) > 0, D(0) > 0, R(0) > 0$ are positive in the feasible set Ω, then the solution set $S(t), E(t), T(t), D(t), R(t)$ of the system (2.7) is positive for all $t \geq 0$.*

Proof. From the first equation of the system (2.7) □

$\frac{dS}{dt} = \Lambda + \sigma R + (1-\theta)\psi D - \frac{\nu\rho\beta_1 ST}{N} - \mu S + \frac{(1-\nu)\rho\beta_2 ST}{N}$, which can be obtained as;

$$\frac{dS}{dt} \geq \mu S \tag{2.11}$$

By solving and evaluating, the equation (2.11) gives;

$$S \geq S(0)\exp(-\mu t) \tag{2.12}$$

Similarly, this follows as;

$$\left.\begin{array}{l} E \geq E(0)\exp -(\mu + \delta)t \\ T \geq T(0)\exp -(\mu + \alpha + \omega)t \\ D \geq D(0)\exp -(\mu + \psi + \mu)t \\ R \geq R(0)\exp -(\mu + \sigma)t \end{array}\right\} \tag{2.13}$$

in which all the solutions sets are positive for all $t \geq 0$.

2.5 Terrorist-Free Equilibrium Point (TFEP)

In a situation, where there is no terrorist the points are at the steady-state solutions, this means $E = 0$. The TFEP is occurred and obtained by taking the right side of the system (2.7) to zero, used Maple17 software to solved the equations and obtained;

$$E_0\left(\frac{\Lambda}{\mu}, 0, 0, 0, 0\right) \tag{2.14}$$

2.6 Calculation of Basic Reproduction Number R_0

The basic reproduction number is the expected number of new terrorist coming to meet an already infected individual with the terrorist ideology. In this study, the next-generation matrix method described by Van den Driessche in [22] employed to solve the basic reproduction number. Therefore, the model equations can be rewritten by starting with newly terrorist individuals in the compartment.
Thus; Let$X = (S, E, T, D, R)^T$, then the system (2.7) can be written as$X^{'} = F(X) - V(X)$. The basic reproduction numberR_0, denoted by$\Gamma(FV^{-1})$, where Γ is a spectral radius and at terrorist-free equilibrium gives;

$$F = \begin{bmatrix} 0 & \nu\rho\beta_1 & 0 & 0 \\ 0 & (1-\nu)\rho\beta_1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$V = \begin{bmatrix} \mu+\delta & \nu\rho\beta_1 & 0 & 0 \\ -\varepsilon\delta & \mu+\alpha+\omega & 0 & 0 \\ -(1-\varepsilon)\delta & -\omega & \mu+\eta+\psi & 0 \\ 0 & 0 & -\theta\psi & \mu+\sigma \end{bmatrix}$$

The inverse of V is given by;

$$V^{-1} = \begin{bmatrix} \frac{1}{\mu+\delta} & 0 & 0 & 0 \\ K_1 & \frac{1}{\mu+\alpha+\omega} & 0 & 0 \\ K_2 & K_3 & \frac{1}{\mu+\eta+\omega} & 0 \\ K_4 & K_5 & K_6 & \frac{1}{(\mu+\sigma)} \end{bmatrix}$$

where;

$$\begin{aligned} K_1 &= \frac{\varepsilon\delta}{(\mu+\delta)(\mu+\alpha+\omega)} \\ K_2 &= \frac{-\delta(-\mu-\alpha-\omega+\varepsilon\mu+\varepsilon\alpha)}{(\mu+\alpha+\omega)(\mu+\delta)(\mu+\eta+\psi)} \\ K_3 &= \frac{\omega}{(\mu+\alpha+\omega)(\mu+\eta+\omega)} \\ K_4 &= \frac{-\theta\psi\delta(-\mu-\alpha-\omega+\varepsilon\mu+\varepsilon\alpha)}{(\mu+\eta+\omega)(\mu+\alpha+\omega)(\mu+\delta)(\mu+\sigma)} \\ K_5 &= \frac{\theta\psi\omega}{(\mu+\eta+\psi)(\mu+\alpha+\omega)(\mu+\sigma)} \\ K_6 &= \frac{\theta\psi}{(\mu+\eta+\psi)(\mu+\sigma)} \end{aligned}$$

and

$$F \times V^{-1} = \begin{bmatrix} \frac{\nu\rho\beta_1\varepsilon\delta}{(\mu+\delta)(\mu+\alpha+\omega)} & \frac{\nu\rho\beta_1}{\mu+\alpha+\omega} & 0 & 0 \\ \frac{(-1+\nu)\rho\beta_2\varepsilon\delta}{(\mu+\delta)(\mu+\alpha+\omega)} & -\frac{(-1+\nu)\rho\beta_2}{\mu+\alpha+\omega} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Therefore, the basic reproduction number is the spectral radius of the next generation matrix $\Gamma(FV^{-1})$ and obtain;

$$\Re_0 = \frac{\rho((-1+\nu)(\mu+\delta)\beta_2 - \nu\beta_1\varepsilon\delta)}{(\mu+\delta)(\mu+\alpha+\omega)} \tag{2.15}$$

Scientifically, the reproduction number shows the character of a threshold rate for the dynamics of the system and the terrorist threats. If $\Re_0 > 1$, the terrorist threat remains in the population indefinitely. In this case, the terrorists become rampant in the population and if $\Re_0 < 1$, the number of terrorists progressively drops to zero and the terrorist threat reduces in a population. This implies that the terrorists cannot recruit one person in a population.

2.7 Local Stability of Terrorist-Free Equilibrium Point (LSTFEP)

The local stability is determined by the eigenvalues of the Jacobian calculated at that terrorist-free equilibrium point. Using the basic reproduction number obtained from the system (2.7), the result of local stability analysis of the equilibrium point follows.

Theorem 2.2. *The terrorist-free state,*E_0 *, is locally asymptotically stable if* $\Re_0 < 1$*and unstable if* $\Re_0 > 1$.

Proof. Let the Jacobian matrix of system (2.7) evaluated at the disease-free equilibrium pointE_0 □

By using Maple17 software obtain;

$$J(E_0) = \begin{bmatrix} -\mu & 0 & -(\nu\rho\beta_1 + (1-\nu)\rho\beta_2) & (1-\theta)\psi & \sigma \\ 0 & -(\mu+\delta) & \nu\rho\beta_1 & 0 & 0 \\ 0 & \varepsilon\delta & -((1-\nu)\rho\beta_1 + (\mu+\alpha+\omega)) & 0 & 0 \\ 0 & (1-\varepsilon)\delta & \omega & -(\psi+\mu+\eta) & 0 \\ 0 & 0 & 0 & \theta\psi & -(\mu+\sigma) \end{bmatrix}$$

Thus;

$$J(E_0) = \begin{bmatrix} -\mu & 0 & -K_{11} & (1-\theta)\psi & \sigma \\ 0 & -(\mu+\delta) & K_{12} & 0 & 0 \\ 0 & \varepsilon\delta & -K_{13} & 0 & 0 \\ 0 & (1-\varepsilon)\delta & \omega & -K_{14} & 0 \\ 0 & 0 & 0 & \theta\psi & -(\mu+\sigma) \end{bmatrix}$$

where;

$$\begin{aligned} K_{11} &= (\nu\rho\beta_1 + (1-\nu)\rho\beta_2) \\ K_{12} &= \nu\rho\beta_1 \\ K_{13} &= (1-\nu)\rho\beta_1 + (\mu+\alpha+\omega) \\ K_{14} &= (\psi+\mu+\eta) \end{aligned}$$

The eigenvalues of the matrix $J(E_0)$ are the roots of the characteristic equation and are given by;

$$(K_{14}+\lambda)(\mu+\sigma+\lambda)(\mu+\lambda)\left(\lambda^2 + (\mu+K_{13}-\delta)\lambda + K_{13}\mu - \delta(K_{13}+\varepsilon K_{12})\right) = 0 \tag{2.16}$$

Thus; From (2.16), the expression can be rewriting as;

$$(K_{14}+\lambda)(\mu+\sigma+\lambda)(\mu+\lambda)\left[\lambda^2 + b_1\lambda + b_2\right] = 0 \tag{2.17}$$

where;

$$\begin{aligned} b_1 &= \mu + K_{13} - \delta \\ b_2 &= K_{13}\mu - \delta K_{13} - \delta\varepsilon K_{12} \end{aligned} \tag{2.18}$$

From (2.17) by solving for λ

$$\begin{aligned} &(K_{14}+\lambda) \Rightarrow \lambda_1 = -K_{14} < 0, \\ &(\mu+\lambda) \Rightarrow \lambda_2 = -\mu < 0, \\ &(\lambda+\mu+\sigma) \Rightarrow \lambda_3 = -\mu-\sigma < 0 \end{aligned} . \tag{2.19}$$

The last expression of (2.17) resolved to the quadratic equation as;

$$\lambda^2 + b_1\lambda + b_2 = 0 \tag{2.20}$$

2.8 The Routh-Hurwitz Criteria

The Routh-Hurwitz criteria are used to establish asymptotic stability of equilibrium for non-linear system of differential equations. The Routh-Hurwitz criteria provide the necessary and sufficient condition for all roots of the characteristic polynomial to contain negative parts, therefore entails asymptotic stability. According to Routh-Hurwitz Criteria in [10] stated that suppose the polynomial;
$P(\lambda) = \lambda^n + a_1\lambda^{n-1} + ... > a_{n-1}\lambda + a_n$
where;
$a_0 \neq 0$ and $a_n > 0$, for $i = 1, ...n$ (defined the n Hurwitz matrices using the coefficients a_j of the characteristic polynomial)
$H_1 = (a_1)$,

$$H_2 = \begin{pmatrix} a_1 & 1 \\ a_2 & a_3 \end{pmatrix}, H_3 = \begin{pmatrix} a_1 & 1 & 0 \\ a_3 & a_2 & a_1 \\ a_5 & a_4 & a_3 \end{pmatrix},$$

$$H_4 = \begin{pmatrix} a_1 & 1 & 0 & 0 \\ a_3 & a_2 & a_1 & 0 \\ a_5 & a_4 & a_3 & a_2 \\ a_7 & a_6 & a_5 & a_4 \end{pmatrix}, H_5 = \begin{pmatrix} a_1 & 1 & 0 & 0 & 0 \\ a_3 & a_2 & a_1 & 0 & 0 \\ a_5 & a_4 & a_3 & a_2 & 0 \\ a_7 & a_6 & a_4 & a_4 & a_3 \\ a_9 & a_8 & a_7 & a_6 & a_5 \end{pmatrix},$$

$$H_n = \begin{pmatrix} a_1 & 1 & 0 & 0 & ... & 0 \\ a_3 & a_2 & a_1 & 1 & ... & 0 \\ a_5 & a_4 & a_3 & a_2 & ... & 0 \\ . & . & . & . & ... & . \\ . & . & . & . & ... & . \\ a_{2n-1} & a_{2n-2} & a_{2n-3} & a_{2n-4} & ... & a_n \end{pmatrix},$$

where $a_j = 0, j > n$.
All of the roots of polynomial $P(\lambda)$ are negative real part iff the determinants of all Hurwitz matrices are positive:
$\det H_j > 0, j = 1.2...n$. When $n = 2$, the Routh-Hurwitz criteria simplify to $\det H_1 = a_1 > 0$ and $\det H_2 = \det \begin{pmatrix} a_1 & 0 \\ 0 & a_2 \end{pmatrix} = a_1 a_2 > 0$, therefore the polynomials of degree $n = 1, 2...5$ are summarized and the characteristic equation has the form
$n = 1$,
$a_0\lambda + a_1 = 0$
$a_0 > 0$.

$n = 2 : a_1 > 0, a_2 > o$,
$a_0\lambda^2 + a_1\lambda + a_2 = 0$
$a_1 > 0, a_2 > 0$
$n = 3 : a_1 > 0, a_3 > o, a_1a_2 > a_3$,
$a_0\lambda^3 + a_1\lambda^2 + a_2\lambda + a_3 = 0$
$\Delta_3 = a_3\Delta_2 > 0$.
$n = 4 : a_1 > 0, a_3 > o, a_4 > 0, a_1a_2a_3 > a_3^2 + a_1^2 a_4$,
$a_0\lambda^4 + a_1\lambda^3 + a_2\lambda^2 + a_3\lambda + a_4 = 0$
$\Delta_4 = a_4\Delta_3 > 0$.
$n = 5 : a_j > 0, i = 1, ..., 5$
$a_1a_2a_3 > a_3^2 + a_1^2 a_4$ and

$(a_1a_4 - a_0a_5)(a_1a_2a_3 - a_3^2 - a_0a_1^2a_4) > a_5(a_1a_2 - a_3)^2 + a_0a_1a_5^2 > 0$
$a_0\lambda^5 + a_1\lambda^4 + a_2\lambda^3 + a_3\lambda^3 + a_4\lambda^2a_5\lambda + a_0 = 0$ The asymptotic stability condition
$\Delta_5 = a_5\Delta_4 > 0.$
holds and satisfies the above inequalities. The local stability of the equilibrium may be determined from the jacobian matrix.
This shows that TFEP is locally asymptotically stable, since $\Re_0 < 1$ and unstable if $\Re_0 > 1$. Going by Routh-Hurwitz criteria in [10] the criteria was employed and equation (2.20) has strictly negative real root iff $b_1 > 0$ and $b_2 > 0$. It can be deduced from the expression in (2.20) that$b_1 > 0$, because it is the sum of positive parameters and

$$\left.\begin{array}{l} b_2 = K_{13}\mu - \delta K_{13} - \delta\varepsilon K_{12} \\ b_2 = (\mu+\delta)(1-\Re_0) > 0 \end{array}\right\} \tag{2.21}$$

This result shows that terrorist threats can be reduced if the initial size of the terrorist individuals is in the basin of attraction of the TFEP. Separately, to ensure the terrorist threats reduce, it is essential to show that the TFEP is globally stable.

2.9 Global Stability of Terrorist-Free Equilibrium Point (GSTFEP)

Equilibrium is called globally stable iif is stable for almost all initial conditions, not just those that are close to it. Global stability of equilibrium cannot always be proved. A locally stable equilibrium may be globally stable if there are no other locally stable equilibria coexisting with it. Global stability of the terrorist-free equilibrium can be proven for many models for which the terrorist-free equilibrium is the only equilibrium when $\Re_0 < 1$ in [9]. Building global stability in mathematical modelling is possible through different methods. The method presented in this study work well for many models.

Theorem 2.3. *Suppose $\Re_0 < 1$. Then the terrorist-free equilibrium is globally asymptotically stable.*

Proof. Using the idea of Alemneh and Alemu in [2] considers the following Lyapunov function, this need to satisfy the system $E = T = 0$ to be asymptotically stable. Following the terrorist group of a system (2.7) a function. □

$$V = m_1E + m_2T \tag{2.22}$$

Differentiating V with respect to t obtaining;

$$\frac{dV}{dt} = m_1\frac{dE}{dt} + m_2\frac{dT}{dt} \tag{2.23}$$

Substituting for m_1 and m_2 upon simplification, obtain;

$$\frac{dV}{dt} = m_1\left[\left(\frac{\nu\rho\beta_1ST}{N}\right) - (\mu+\delta)E\right] + m_2\left[\varepsilon\delta E + \frac{(1-\nu)\rho\beta_2ST}{N} - (\mu+\alpha+\omega)T\right] \tag{2.24}$$

by rearranging (2.24) obtain;

$$\frac{dV}{dt} = m_1\left(\frac{\nu\rho\beta_1ST}{N}\right) + m_2\frac{(1-\nu)\rho\beta_2ST}{N} - m_2(\mu+\alpha+\omega)T - m_1(\mu+\delta)E + m_2\varepsilon\delta E \tag{2.25}$$

Taking$m_1 = \left(\frac{\varepsilon\delta}{\mu+\delta}\right)m_2$, this gives;

$$\frac{dV}{dt} = \frac{\varepsilon\delta}{\mu+\delta}m_2\nu\rho\beta_1T + m_2(1-\nu)\rho\beta_2T - m_2(\mu+\alpha+\omega)T \tag{2.26}$$

$$\leq \left[\frac{\varepsilon\delta}{\mu+\delta}\nu\rho\beta_1 + (1-\nu)\rho\beta_2 - (\mu+\alpha+\omega)\right] m_2 T \tag{2.27}$$

Taking$m_2 = 1$

$$\leq \left[\frac{\rho\left((-1+\nu)(\mu+\delta)\beta_2 - \nu\beta_1\varepsilon\delta\right)}{\mu+\delta} - (\mu+\alpha+\omega)\right] T \tag{2.28}$$

Substituting $\Re_0$in the equation (2.28) obtain;

$$\leq \left(\frac{\rho\left((-1+\nu)(\mu+\delta)\beta_2 - \nu\beta_1\varepsilon\delta\right)}{\mu+\delta(\mu+\alpha+\omega)} - (\mu+\alpha+\omega)\right) T \tag{2.29}$$

By further simplification, the equation (2.29) can be reduced and obtain;

$$\leq (\mu+\alpha+\omega)\left(\frac{\rho\left((-1+\nu)(\mu+\delta)\beta_2 - \nu\beta_1\varepsilon\delta\right)}{\mu+\delta(\mu+\alpha+\omega)} - 1\right) T$$

then,

$$\frac{dV}{dt} \leq (\mu+\alpha+\omega)(\Re_0 - 1)T \tag{2.30}$$

This shows that for $N \cong S$ which implies; $S < S_0 = \frac{\Lambda}{\mu}$ and $\frac{dV}{dt} \leq 0 \forall \Re_0 < 1$ $\frac{dV}{dt} = 0$ iff $T = 0$. This indicates $\frac{dV}{dt} \leq 0$ is E_0. Therefore, Lassalles's invariance principle E_0 is globally asymptotically stable Ω. This completes the proof of the theorem by La Salle [7].

2.10 The Endemic Stability of Terrorist Equilibrium Point (ESTEP)

The endemic equilibrium point is a positive steady-state solution where the terrorist persists in the population and is denoted by $(S^*, E^*, T^*, D^*, R^*)$. Similarly, it can be acquired by equating (2.7) to zero gives;

$$\frac{dS}{dt} = \frac{dE}{dt} = \frac{dT}{dt} = \frac{dD}{dt} = \frac{dR}{dt} = 0.$$

Thus,
The terrorist model two has a unique equilibrium iff $\Re_0 > 1$ and endemic equilibrium exists with a positive solution which, gives;

$$\left.\begin{array}{l} S^* = \frac{N(\mu+\delta)(\mu+\alpha+\omega)}{\rho(1-\nu)(\mu+\delta)(\beta_2-\varepsilon\delta\nu\beta_1)} \\ E^* = \frac{\nu\rho\beta_1 S^* T^*}{N(\mu+\delta)} \\ T^* = \frac{\varepsilon\delta E^*}{\left(\frac{(1-\nu)\rho\beta_2 S^*}{N} - (\mu+\alpha+\omega)\right)} \\ D^* = \frac{(1-\varepsilon)\delta E^* + \omega T^*}{(\psi+\mu+\eta)} \\ R^* = \frac{\theta\psi D^*}{(\mu+\sigma)} \end{array}\right\} \tag{2.31}$$

2.11 Local Stability of Terrorist Endemic Equilibrium (LSTEE)

In theorem 2.4 of Alemneh and Alemu in [2] the proof of endemic equilibrium E_e^* is locally asymptotically stable in Ω if $\Re_0 > 1$, else unstable. Separately, the authors applied Ruth-Hurwitz criterion in [11] and stated that for endemic equilibrium to be locally asymptotically steady, all root of determinants polynomial must have negative real parts iff $a_1 > 0, a_2 > 0, a_3 > 0, a_4 > 0, a_5 > 0, a_1a_2a_3 > a_3^2 + a_1^2a_4$ and $(a_1a_4 - a_5)(a_1a_2a_3 - a_3^2 + a_1^2a_4) > a_5(a_1a_2 - a_3)^2 + a_1a_5^2$ for $\Re_0 > 1$ which satisfied the above assertion, then endemic equilibrium exists.

Table 3: Parameter, Value and Source for Numerical Simulation

Parameter	**Value**	**Source**
β_2	0.00076	[1]
β_1	0.00000000056	[1]
ρ	0.036	[8]
μ	0.0137	Estimated
σ	0.7	Assumed
ε	0.00035	Assumed
α	0.750	[16]
δ	0.9	Varies
Λ	600	[19]
ν	0.1666	Assumed
ω	0.6	Varies
η	0.0025	[1]
ψ	0.33	Assumed
θ	0.00111	Assumed

3 Numerical Simulations of Model

The numerical methods are used to approximate the exact solutions of an ordinary differential equations. The Numerical Simulation was carry out using Maple17 software, subsequently to investigate the effects of parameters.

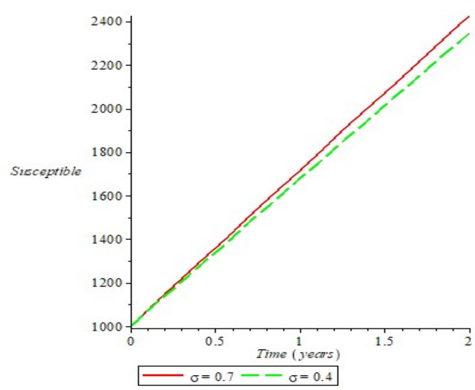

Figure 2: Effect of different rates of σ on Susceptible individuals in time.

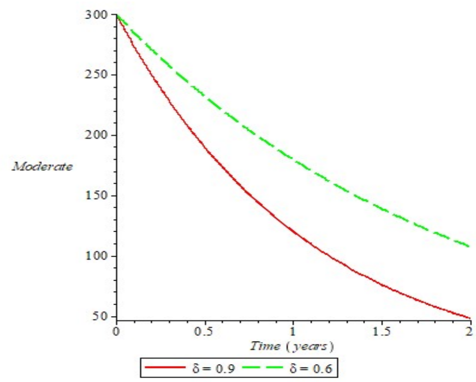

Figure 3: Effect of different rates of δ on Moderate individuals in time.

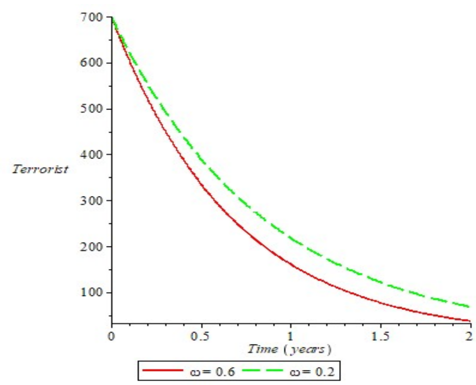

Figure 4: Effect of different rates of ω on Terrorist individuals in time.

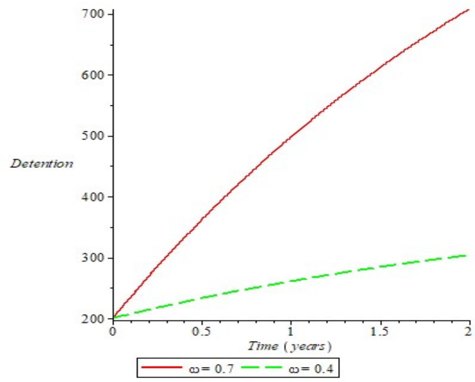

Figure 5: Effect of different rates of ω on Detention individuals in time.

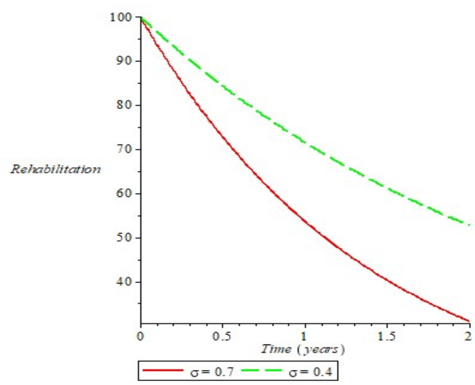

Figure 6: Effect of different rates of σ on Rehabilitation individuals in time.

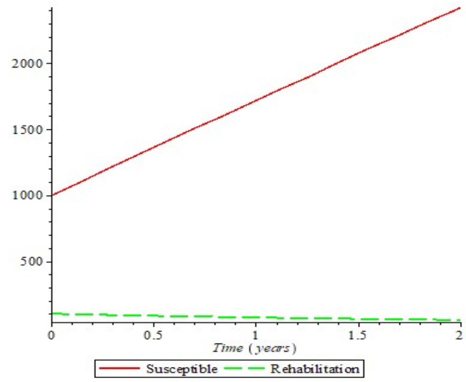

Figure 7: Effect of Civil Intervention on Susceptible and Rehabilitation individuals in time.

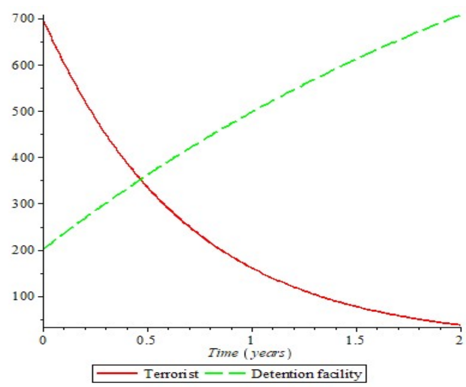

Figure 8: Effect of military intervention on terrorist and detention individuals in time.

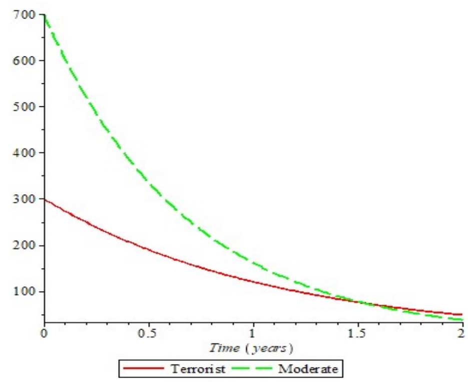

Figure 9: Effect of Military Intervention on Moderate and Terrorist individuals in time.

4 Results and Discussion

The results of the numerical simulation of stability analysis of terrorism dynamics model are presented in graphs. Thus, from figure 4.4: the Effect of σ on the susceptible Population for a period of time shows the proportion of successful rehabilitation individuals are increasing. The recruitment pool of terrorists from the susceptible population with contact rate β increased the population of moderate individuals. This shows that with time, the moderate terrorist will fully convert to terrorist after being indoctrinated and control needed in decreasing the terrorist recruitment in a population. This can be observed from the graph in figure 4.5 to 4.6: were the plots shows the effects of δ and ω on both moderate, terrorist individuals respectively with counter-terrorist identification on the removal of terrorists decreasing the terrorist foot-soldiers. The parameters ω is the rate at which terrorist move to the detention facility due counter-terrorist operation. The effect in figure 4.7: shows that at 40% the counter-terrorist operation is at minimum rate, when is 70% the terrorist number in detention centre increasing and force them to appoint weak leaders. However, figure 4.8: shows the effect of civil intervention on rehabilitation centre where large number of terrorist in rehabilitation facility are moved to susceptible individuals after been certify or end jail term. Furthermore, in Figure 4.9, 4.10 and 4.11 shows the successful in countering the foot-soldiers and leaders of terrorist organizations such as Al-Qaeda, Boko-Haram, ISWAP, ISIS and other insurgency groups under counter-terrorist program. Finally, in addition to Figure 4.7: the effect shows in a period of two (2)years, detention facilities will be close due to effective intervention program.

5 Conclusion

This paper examined the Modelling and Analysis of counter-terrorism in a population: Via Drone. The basic reproduction number R_0 was computed, and the stability of equilibrium points was investigated. Through Lyapunov theory, the terrorist-free equilibrium point is globally asymptotically stable whenever $R_0 > 1$ was proven. The results of the numerical simulations which presented on graphs shows that the integrated control strategy using drone or CUAV should be taken to fight against terrorist/insurgent individuals. Finally, optimal control strategy and other issues are plans to address in future study.

Acknowledgement(s) : I am profoundly grateful to my supervisor, an erudite scholar, Professor M.O. Ibrahim, for his intellectual stimulation, valuable suggestions and availability at all times despite his tight schedules at University of Ilorin, Nigeria.

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