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Investigation of light nuclei in the cluster model by means of relativistic and non-relativistic systems

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Abstract: Calculation of the energy of even-even isotopes using collective models in nuclear physics has its own complication. Therefor different physical models are used to study nuclear isotopes. The cluster model is a new and successful model for investigating the properties of isotopes. Using this model, the interaction between core and cluster can be chosen and static properties, including the eigenvalues energy and wave function, can be calculated. Considering the modified Eckart plus Hulthen potentials and Coulomb repulsive potential for interactions between clusters and with substituting this potential in the Schrödinger equations, by Nikiforov-Uvarov analytical method some of the static properties including the energy levels and wave functions are obtained for ^{14}C, ^{16}O, ^{20}Ne, ^{24}Mg, ^{28}Si and ^{32}S isotopes.

Keywords: Cluster model; Eckart plus Hulthen potentials; Nikiforov-Uvarov method; Energy levels; Structure of Nuclei.
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1 Introduction

The investigation of light nuclei is done through different models in which one of them is using a cluster model [30]. The study of the clustering phenomenon in the nuclei has a long history [7]. The cluster model can provide a good description of the static, electromagnetic properties, and other properties of the nuclei [36]. According to the latest findings of nuclear physicists, cluster arrangement cannot be found typically in the ground state of the nuclei. Usually, clustering arrangement occurs in the ground state of the cores, and in order to make clustering, the system must have enough energy to put down at the threshold of clustering [1,13,39]. The amount of threshold energy for this deformation is demonstrated in the Ikeda diagrams in Figure1 [22]. In this diagram, the required amount of energy for creating a cluster with specific structures in each nucleus are predicted using empirical experiments of light nuclei with N = Z, A= 4K (K = 2, …,7), which have an even, and equal, number of

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protons and neutrons [20]. The main achievement of this diagram is that clustering phenomena are expected to occur mainly within the range of the related energy to the threshold of clustering in each of nuclei [5,18,23].

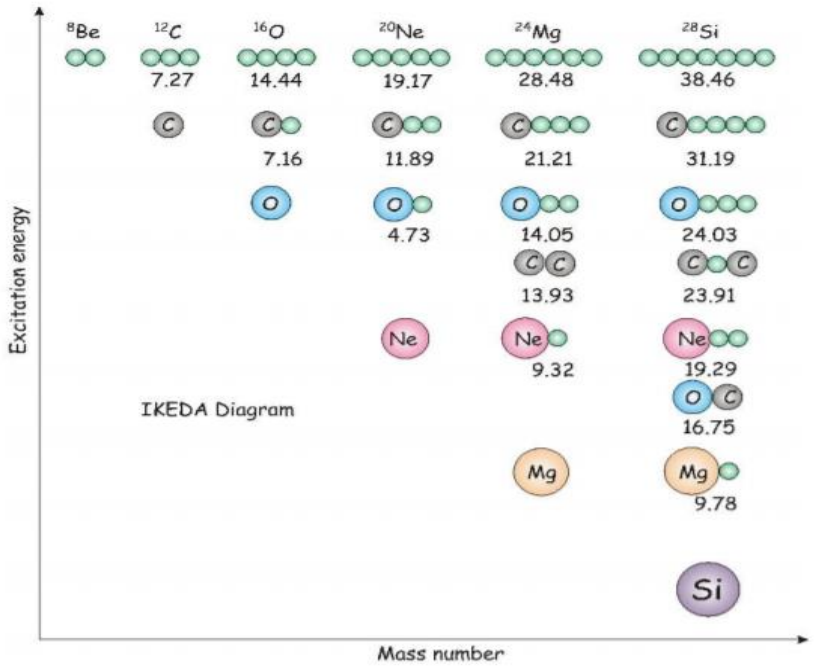

Figure1: The Ikeda threshold diagram for light nuclei with α-clustering. Cluster structures are predicted to appear close to the associated decay thresholds. These energies needed for the decomposition of the normal nucleus into the structures are indicated in MeV, adapted from [24].

The phenomenon of clustering occurs in different forms as:

- nuclei consist of identical clusters: for example, 12C isotope has an α + α+ α cluster structure at the ground state energy [6,19,37].
- nuclei consist of non-identical clusters such as 24Mg→ (20Ne + α) which consist of core and alpha clusters [17,31]. B. Buck and his colleagues have proposed models for studying nuclei, which can be considered as a cluster model. These models include the interaction of a core-cluster [8,10]. An essential consideration in the study of these isotopes is to consider the interactions between the proposed clusters. In this paper, we investigate even-even isotopes by considering core-cluster model for each of isotopes by this configuration of 14C → (10Be + α), 16O → (12C + α), 20Ne → (16O + α), 24Mg→ (20Ne + α), 28Si → (24Mg + α) and 32S → (28Si + α). Some of these nuclei have already been investigated using various models, [9,17] such as 24Mg considered as 12C + 12C and 32S is composed of two 16O clusters (16O + 16O). We can see, the spectrum of energy obtained from the experimental results for some of the mentioned isotopes in Figure 2 and Figure 2 [38].

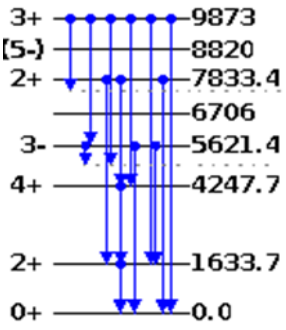

Figure 2: 20Ne Energy spectrum (KeV)

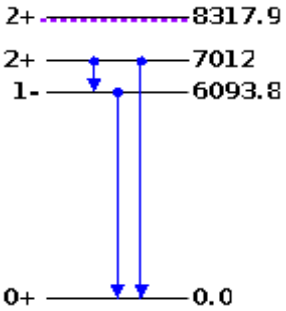

Figure 3: 14C Energy spectrum (KeV)

Each of these isotopes has spin and parity 0^+ [26] because, in these isotopes, the nucleons are placed in pairs at each of the nucleon levels. Using the collective model, these isotopes will be converted into few body systems which is difficult to calculate but by considering the proposed model, they can be converted into a two-body system taking into account the potential of the Eckart [12,14] plus Hulthen [15,35] and Coulomb repulsion potential for interactions of the core-cluster. We consider the following potential to study the interaction between the core-cluster in the proposed model [25].

$$V(r) = V_0 Csch^2(\alpha r) + V_1 \frac{\exp(-2\alpha r)}{1-\exp(-2\alpha r)} + \frac{K}{r} \tag{1-1}$$

The above equation consists of a Coulomb repulsion potential and nuclear potential, where the parameters V_0, V_1 and K are real parameters, these are strength parameters and the parameter α is related to the range of the potential. Based on the proposed model, we used the parametrization Nikiforov-Uvarov method to calculate the energy levels by using potential in the equation (1.1). In this paper, we have calculated the energy levels and charge of the radius using the analytic method of parametrization Nikiforov-Uvarov [28]. Finally, we have compared obtain results with experimental data.

2 Review of parametrization Nikiforov-Uvarov method (PNU):

The PNU method is a direct method for easy access to the NU method without the need to investigate various functions. Now, to calculate the equations of the proposed model which is consisted of core – cluster, first, we introduce the general form of the Schrödinger equation in the following way [29,34]:

$$\left[\frac{d^2}{ds^2} + \frac{c_1 - c_2 s}{s(1-c_3 s)}\frac{d}{ds} + \frac{(-\rho_2 s^2 + \rho_1 s - \rho_0)}{s^2(1-c_3 s)^2}\right]\psi_n(s) = 0 \tag{2-1}$$

For the Schrödinger equation, in the presence of any interaction potential that can be written as the above equation, the equations of eigenvalues energy and wave function are following, respectively:

$$nc_2 - (2n+1)c_5 + (2n+1)(\sqrt{c_9} + c_3\sqrt{c_8}) + n(n-1)c_3 + c_7 + 2c_3c_8 + 2\sqrt{c_8c_9} = 0 \tag{2-2}$$

$$R_{nl(s)} = N_{nl} s^{c_{12}} (1-c_3 s)^{c_{13}} p_n^{(c_{10},c_{11})}(1-2c_3 s) \tag{2-3}$$

Where N is the normalization constant and the functions $P_n^{(\mu,\nu)}(x)$ are the Jacobi polynomials. And the constant coefficients C_i which are used in the above equations are given in the following table.

Table 1: The constant coefficients C_i=(i=1,…,13), $c_1 = c_2 = c_3 = 1$

$c_4 = \frac{1}{2}(1-c_1)$	$c_5 = \frac{1}{2}(c_2 - 2c_3)$	$c_6 = c_5^2 + \rho_2$
$c_7 = 2c_4c_5 - \rho_1$	$c_8 = c_4^2 + \rho_0$	$c_9 = c_3(c_7 + c_3c_8) + c_6$
$c_{10} = c_1 + 2c_4 + 2\sqrt{c_8} - 1\rangle - 1$	$c_{11} = 1 - c_1 - 2c_4 + \frac{2}{c_3}\sqrt{c_9}\rangle - 1, c_3 \neq 0$	
$c_{12} = c_4 + \sqrt{c_8}\rangle 0$	$c_{13} = -c_4 + \frac{1}{c_3}\left(\sqrt{c_9} - c_5\right)\rangle 0, c_3 \neq 0$	

Using the Eq. (2.2) and Eq. (2.3) and the given coefficients in Table 1 , we can obtain the energy equation and the radial wave function

3 Mathematical Calculations in Non-Relativistic Model

To investigate isotopes in a non-relativistic shell model, we use the Schrödinger equation, which is the radial part of the Schrödinger equation in spherical coordinates is given as [11,32]:

$$\left\{\frac{d^2}{dr^2} + \frac{2}{r}\frac{d}{dr} - \frac{l(l+1)}{r^2} - \frac{2\mu}{\hbar^2}\left[V(r) - E_{nl}\right]\right\} R_{nl}(r) = 0 \tag{3-1}$$

Where $R_{nl}(r)$, E_{nl} and l represent the radial part, the energy Eigen-values, and the orbital angular momentum. By considering radial wave function as $U_{nl}(r) = r\,R_{nl}(r)$ and taking central potential into account and putting in the Schrödinger equation, the above equation is given as follows:

$$\frac{d^2U_{nl}(r)}{dr^2} + \frac{2\mu}{\hbar^2}\left[E_{nl} - 4V_0\frac{\exp(-2\alpha r)}{(1-\exp(-2\alpha r))^2} - V_1\frac{\exp(-2\alpha r)}{(1-\exp(-2\alpha r))} - \frac{K}{r} - \frac{\hbar^2}{2\mu}\frac{l(l+1)}{r^2}\right] U_{nl}(r) = 0 \tag{3-2}$$

Eq. (3.2) is exactly solved only for the case of l=0. Hence, we shall use an approximation in order to deal with the centrifugal-like terms. In order to obtain the analytical solutions of Eq. (3.2), we employ the improved Green and Aldrich approximation and replace the spin-orbit coupling term with an expression that is valid for $\alpha \leq 1$ [21,27]. The main characteristic of these solutions lies in the substitution of the centrifugal term by an approximation so that one can obtain an equation, normally geometric, which is solvable [16]:

$$\frac{1}{r^2} = \frac{4\alpha^2\exp(-2\alpha r)}{(1-\exp(-2\alpha r))^2} \tag{3-3}$$

Applying the approximations of Eq. (3.3) and the central potential along with introducing a new variable of the form $s = \exp(-2\alpha r)$, we can write Eq. (3.2) as follows:

$$\frac{d^2U(s)}{ds^2} + \frac{(1-s)}{s(1-s)}\frac{dU(s)}{ds} + \frac{1}{s^2(1-s)^2}(-\rho_2 s^2 + \rho_1 s - \rho_0)U(s) = 0 \tag{3-4}$$

Where the parameters ρ_2, ρ_1 and ρ_0 are considered as follows:

$$\rho_2 = -\frac{2\mu}{4\alpha^2\hbar^2}\left[E + V_1 + 2K\alpha\right]$$

$$\rho_1 = -\frac{2\mu}{4\alpha^2\hbar^2}\left[2E + 4V_0 + V_1 + 2K\alpha\right] + l(l+1) \tag{3-5}$$

$$\rho_0 = -\frac{2\mu}{4\alpha^2\hbar^2}E$$

Applying the PNU method, we obtain the energy equation and the radial function can be written in the form as follows respectively:

$$(2n+1)[\sqrt{\rho_2 - \rho_1 + \rho_0 + \frac{1}{4}} + \sqrt{\rho_0} + \frac{1}{4}(2n+1)] + 2\sqrt{\rho_0(\rho_2 - \rho_1 + \rho_0)} + 2\rho_0 - \rho_1 + \frac{1}{4} = 0 \tag{3-6}$$

$$R_{nl}(s) = \frac{N}{r}\exp(-2\alpha r)^{\sqrt{\rho_0}}(1-\exp(-2\alpha r))^{\sqrt{\rho_2-\rho_1+\rho_0+\frac{1}{4}}+\frac{1}{2}} p_n^{(2\sqrt{\rho_0},2\sqrt{\rho_2-\rho_1+\rho_0+\frac{1}{4}})}(1-2(\exp(-2\alpha r))) \tag{3-7}$$

4 Mathematical Calculations in Relativistic Model

The corresponding Klein-Gordon equation is given by [4,33]:

$$\{-\hbar^2c^2\nabla^2 + (Mc^2 + S(r))^2 - (E_{nl} - V(r))^2\}\psi_{nlm}(r,\theta,\phi) = 0 \tag{4-1}$$

Where E_{nl}, $V(r)$ and $S(r)$ are the relativistic energy of the particle, vector and scalar potentials, respectively. For spherical symmetrical scalar and vector potentials, we can open assume

$$\psi_{nlm}(r,\theta,\phi) = \frac{u_{nl}(r)}{r}Y_{lm}(\theta,\phi) \tag{4-2}$$

Where $Y_{lm}(\theta,\phi)$ is the spherical harmonic function and l is the angular momentum quantum number. We obtain the radial Klein-Gordon equation as:

$$\frac{d^2U_{nl}(r)}{dr^2} + \frac{1}{\hbar^2c^2}\left\{(E_{nl} - V(r))^2 - (Mc^2 + S(r))^2 - \frac{l(l+1)}{r^2}\hbar^2c^2\right\}U_{nl}(r) = 0 \tag{4-3}$$

Where M is the rest mass, E_{nl} is the relativistic energy, c is the speed of light, $\hbar$ is the reduced Planck's constant, $V(r)$ and $S(r)$ are vector and scalar potentials, respectively. In the case that the scalar and vector potential have equal magnitudes, $V(r)=S(r)$, we have: Writing a conclusion for your research paper can be difficult.

$$\frac{d^2U_{nl}(r)}{dr^2}+\frac{1}{\hbar^2c^2}\left\{E_{nl}^2-M^2c^4-V(r)(Mc^2+E_{nl})-\frac{l(l+1)}{r^2}\hbar^2c^2\right\}U_{nl}(r)=0 \tag{4-4}$$

And

$$V(r)=V_0\,Csch^2(\alpha r)+V_1\frac{\exp(-2\alpha r)}{1-\exp(-2\alpha r)}+\frac{K}{r} \tag{4-5}$$

$$\frac{d^2U_{nl}(r)}{dr^2}+\left\{\frac{(E_{nl}^2-M^2c^4)}{\hbar^2c^2}-\frac{(Mc^2+E_{nl})}{\hbar^2c^2}\left(4V_0\frac{\exp(-2\alpha r)}{(1-\exp(-2\alpha r))^2}+V_1\frac{\exp(-2\alpha r)}{(1-\exp(-2\alpha r))}+\frac{K}{r}\right)-\frac{l(l+1)}{r^2}\right\}U_{nl}(r)=0 \tag{4-6}$$

Applying the approximations presented in the previous section for a centrifugal term and hyper central potential with introducing the new variable of the form $s=\exp(-2\alpha r)$, Eq. (4.6) can be reduced to:

$$\begin{aligned}&\frac{d^2U_{nl}(s)}{ds^2}+\frac{(1-s)}{s(1-s)}\frac{dU_{nl}(s)}{ds}+\frac{1}{s^2(1-s)^2}\left\{\frac{(E_{nl}^2-M^2c^4)}{4\alpha^2\hbar^2c^2}(1-s)^2\right.\\&\left.-\frac{(E_{nl}+Mc^2)}{4\alpha^2\hbar^2c^2}\left(8V_0\,s+2V_1s(1-s)\right)+4\alpha K\,s(1-s)-l(l+1)s\right\}U_{nl}(s)=0\end{aligned} \tag{4-7}$$

Eq. (4.7) can be summarized as follows:

$$U_{nl}''(s)+\frac{(1-s)}{s(1-s)}U_{nl}'(s)+\frac{1}{s^2(1-s)^2}(-\kappa_2s^2+\kappa_1s-\kappa_0)U_{nl}(s)=0 \tag{4-8}$$

Where the parameters κ_2,κ_2 and κ_0 are considered as follows:

$$\begin{aligned}&\kappa_2=-\left[\xi+2V_1\delta+4\alpha\,K\delta\right]\\&\kappa_1=-\left[2\xi+8V_0\,\delta+2V_1\delta+4\alpha K\delta+l(l+1)\right]\\&\kappa_0=-\xi\quad,\qquad\delta=\frac{(E_{nl}+Mc^2)}{4\alpha^2\hbar^2c^2}\quad,\qquad\xi=\frac{(E_{nl}^2-M^2c^4)}{4\alpha^2\hbar^2c^2}\end{aligned} \tag{4-9}$$

Applying the PNU method, we obtain the energy equation and the radial function can be written in the form as follows respectively:

$$(2n+1)[\sqrt{\kappa_2-\kappa_1+\kappa_0+\frac{1}{4}}+\sqrt{\kappa_0}+\frac{1}{4}(2n+1)]+2\sqrt{\kappa_0(\kappa_2-\kappa_1+\kappa_0)}+2\kappa_0-\kappa_1+\frac{1}{4}=0 \tag{4-10}$$

$$\psi_{nl}(s)=\frac{N'}{r}\exp(-2\alpha r)^{\sqrt{\kappa_0}}(1-\exp(-2\alpha r))^{\sqrt{\kappa_2-\kappa_1+\kappa_0+\frac{1}{4}}+\frac{1}{2}}p_n^{(2\sqrt{\kappa_0},2\sqrt{\kappa_2-\kappa_1+\kappa_0+\frac{1}{4}})}(1-2(\exp(-2\alpha r))) \tag{4-11}$$

Where N' is the normalization constant and the functions $P_n^{(\mu,\nu)}(x)$ are the Jacobi polynomials.

5 Discussions and Resulting

The ground state and by two excited energies for the even-even isotopes are obtained in non- relativistic and relativistic analytical approach using Eq. (3.6) and Eq. (4.10), which is related to V_0, V_1 and α to variables. Now, using the numerical method, the ground state energy and by excited energies for each of the isotopes referred to calculated according to the following table. The obtained results are shown in Table 2 and Table 3 for a non-relativistic and relativistic analytical approach.

Table 2: The ground state and the excited energies for each of isotopes with their experimental values (in Non-Relativistic Method).

Isotope	parameters of potential	state	E_{Our}(MeV)	E_{Exp}(MeV) [3]
^{14}C	$\begin{pmatrix} \alpha(fm^{-1}) & 0.098 \\ V_0(MeV) & 4.595 \\ V_1(MeV) & -124.452 \\ K(MeV) & 0.012 \end{pmatrix}$	0^+	-105.712	-105.285
		1^-	-97.463	-99.247
		0^+	-93.684	-92.658
^{16}O	$\begin{pmatrix} \alpha(fm^{-1}) & 0.076 \\ V_0(MeV) & 3.622 \\ V_1(MeV) & -115.201 \\ K(MeV) & 0.121 \end{pmatrix}$	0^+	-127.682	-127.619
		0^+	-120.779	-121.569
		3^-	-117.068	-115.439
^{20}Ne	$\begin{pmatrix} \alpha(fm^{-1}) & 0.081 \\ V_0(MeV) & 9.312 \\ V_1(MeV) & -212.561 \\ K(MeV) & 0.215 \end{pmatrix}$	0^+	-160.667	-160.645
		2^+	-157.547	-159.012
		4^+	-154.543	-154.765
^{24}Mg	$\begin{pmatrix} \alpha(fm^{-1}) & 0.101 \\ V_0(MeV) & 22.129 \\ V_1(MeV) & -385.871 \\ K(MeV) & 0.034 \end{pmatrix}$	0^+	-198.244	-198.257
		2^+	-194.589	-196.888
		4^+	-192.794	-192.134
^{28}Si	$\begin{pmatrix} \alpha(fm^{-1}) & 0.114 \\ V_0(MeV) & 37.826 \\ V_1(MeV) & -573.148 \\ K(MeV) & 0.021 \end{pmatrix}$	0^+	-236.842	-236.537
		2^+	-233.399	-234.758
		4^+	-231.700	-230.141
^{32}S	$\begin{pmatrix} \alpha(fm^{-1}) & 0.096 \\ V_0(MeV) & 36.545 \\ V_1(MeV) & -582.482 \\ K(MeV) & 0.051 \end{pmatrix}$	0^+	-271.789	-271.780
		2^+	-269.041	-269.545
		0^+	-267.680	-267.321

Table 3: The ground state and the excited energies for each of isotopes with their experimental values (in Relativistic Method).

Isotope	parameters of potential	state	E_{Our}(MeV)	E_{Exp}(MeV) [3]
^{14}C	$\begin{pmatrix} \alpha(fm^{-1}) & 0.0125 \\ V_0(MeV) & 1.328 \\ V_1(MeV) & -185.028 \\ K(MeV) & 0.285 \end{pmatrix}$	0^+	-105.385	-105.285
		1^-	-100.589	-99.247
		0^+	-91.095	-92.658
^{16}O	$\begin{pmatrix} \alpha(fm^{-1}) & 0.0115 \\ V_0(MeV) & 1.228 \\ V_1(MeV) & -181.969 \\ K(MeV) & 1.294 \end{pmatrix}$	0^+	-127.630	-127.619
		0^+	-123.179	-121.569
		3^-	-114.357	-115.439
^{20}Ne	$\begin{pmatrix} \alpha(fm^{-1}) & 0.0102 \\ V_0(MeV) & 1.115 \\ V_1(MeV) & -178.752 \\ K(MeV) & 0.346 \end{pmatrix}$	0^+	-160.878	-160.645
		2^+	-159.372	-159.012
		4^+	-153.051	-154.765
^{24}Mg	$\begin{pmatrix} \alpha(fm^{-1}) & 0.0120 \\ V_0(MeV) & 1.693 \\ V_1(MeV) & -220.569 \\ K(MeV) & 0.851 \end{pmatrix}$	0^+	-198.710	-198.257
		2^+	-196.658	-196.888
		4^+	-191.324	-192.134
^{28}Si	$\begin{pmatrix} \alpha(fm^{-1}) & 0.0118 \\ V_0(MeV) & 1.840 \\ V_1(MeV) & -240.306 \\ K(MeV) & 1.421 \end{pmatrix}$	0^+	-236.885	-236.537
		2^+	-234.341	-234.758
		4^+	-230.137	-230.141
^{32}S	$\begin{pmatrix} \alpha(fm^{-1}) & 0.0112 \\ V_0(MeV) & 1.969 \\ V_1(MeV) & -251.641 \\ K(MeV) & 1.623 \end{pmatrix}$	0^+	-271.936	-271.780
		2^+	-269.683	-269.545
		0^+	-266.106	-267.321

According to the above table, and to determine the best potential coefficients, we can calculate the wave function for each of the isotopes using the Eq. (3.7). The radial wave function is obtained from Eq. (3.8), so we can easily calculate the charge radius of studied isotopes by calculating $\langle r^2 \rangle^{1/2}$. The obtained results are shown in Table 4 for the non-relativistic and relativistic analytical approach by using Eq. (3.7) and Eq. (4.11), respectively.

$$\langle r^2 \rangle^{\frac{1}{2}} = \left[\frac{\int R^*_{nl}(r) r^2 R_{nl}(r) d^3 r}{\int R^*_{nl}(r) R_{nl}(r) d^3 r} \right]^{\frac{1}{2}} \tag{5-1}$$

Table 4: The charge radius for each of isotopes with their experimental values for ground state energy (in Non-Relativistic and Relativistic Method by using parameters potential in Table 2 and Table 3).

Isotope	$\langle r^2 \rangle^{1/2}_{our\ work}(fm)$		$\langle r^2 \rangle^{1/2}_{\exp}(fm)$ [2]
	In Non-Relativistic	**In Relativistic**	
^{14}C	2.521	2.503	2.502
^{16}O	2.665	2.692	2.699
^{20}Ne	3.003	3.007	3.005
^{24}Mg	3.067	3.054	3.056
^{28}Si	3.103	3.127	3.122
^{32}S	3.265	3.265	3.261

As a comparison between these results and experimental data implies, the suggested model can estimate the charge radius of this isotopes in the ground state.

6 Conclusions

In the present work, we have investigated different isotopes using the cluster model. Our proposed model involves the consideration of the core -alpha cluster for isotopes. For the interaction between core and cluster, we proposed the interaction potential of Eckart and Hulthen plus Coulomb repulsion and have calculated the energy levels and the charge of the radius for, even-even isotopes. Using the PNU method we selected the core-cluster model for observation isotopes which converted into a two-body system. In order to obtain the energy levels and the charge of the radius for the mentioned isotopes, we use the Eq. (3.6), Eq. (4.10) and Eq. (5.1), which the results are indicated in Table 2,Table 3 and Table 4. There is a found agreement between our results and the experimental data. Also, the potential coefficients are obtained by fitting with experimental values. Therefore, the proposed model can be used to examine other similar isotopes and can be compared with experimental data.

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