Vol 3(4), 2022, pp: 55-63
DOI:10.30511/mcs.2022.1974430.10926
ISSN:2717-2708
http://mcs.qut.ac.ir/

Reconstructed variational iteration algorithm via third-kind shifted Chebyshev polynomials for the numerical solution of seventh-order boundary value problems

Christie Yemisi Ishola[1], Musiliu Tayo Raji [2], Mutiu Lawal Olaosebikan [3], Mfon Okon Etuk [4], Asimiyu Olalekan Oladapo [5], Aduragbemi Adekunle Victor [6,*]

Abstract: The variational iteration algorithm using shifted Chebyshev polynomials of the third kind was used to obtain the numerical solution of seventh order Boundary Value Problems (PVBs) in this paper. The proposed method is made by constructing the shifted Chebyshev polynomials of the third kind for the given boundary value problems and used as a basis functions for the approximation. Numerical examples where also given to show the efficiency and reliability of the proposed method. Calculations were performed using maple 18 software.

Keywords: Variational iteration algorithm, Boundary value problems, Shifted Chebyshev polynomials of the third kind, Approximate solutions.
2020 Mathematics Subject Classification: 43A12, 34A04
Receive: 26 November 2022, **Accepted:** 25 December 2022

1 Introduction

Consider the seventh order boundary value problem of the form:

$$v^7(\varphi) = G(\varphi, v(\varphi)), \qquad \alpha \le \varphi \le \beta \qquad (1)$$

With boundary conditions

$$v(\alpha) = \emptyset_0,\ v'(\alpha) = \emptyset_1,\ v''(\alpha) = \emptyset_2,\ v'''(\alpha) = \emptyset_3 \qquad (2)$$
$$v(\beta) = \emptyset_4,\ v'(\beta) = \emptyset_5, v''(\beta) = \emptyset_6$$

A few of these problems are important in the mathematical modeling of actual circumstances, including viscoelastic flow, heat transfer, and other engineering sciences. Numerous numerical methods have been created over time for dealing with issues of this nature. Solution of Seventh Order Boundary Value Problem Using Canonical Polynomials was carried out by [1] using a modified

[1] Department of Mathematics, National Open University Jabi Abuja, Nigeria
[2] Department of Mathematics, Federal University of Agriculture, Abeokuta, Ogun State, Nigeria
[3] Department of Mathematics, Osun State University, Osogbo, Osun State, Nigeria
[4] Department of Mathematics and Statistics, Federal Polytechnic Bida, Niger State, Nigeria
[5] Department of Mathematics, Osun State University, Osogbo, Osun State, Nigeria
[6] Corresponding author: Department of Mathematics, University of Ilorin, Kwara State, Nigeria, Email: adekeyebisi5@gmail.com

variational iteration method. He's polynomials were employed in the Variational Iteration Method (VIM) by [11] to solve a fifth order BVPS. [10] used the Modified Variational Iteration Method (MVIM) to tackle tenth- and ninth-order BVPs.

A novel method for resolving fifth order BVPs was created by and [4]. Quintic B-splines were utilized by [5] to solve ninth order BVPs. For an eight order BVPs, [6] used the Variational Iteration Decomposition (VIDM) approach. In order to solve a seventh order BVPs utilizing He's polynomials, [7] used the VIM. For the numerical solutions of generalized nth Order BVPs, [8] devised the power series approximation method. In addition, [9] used Tau-collocation approximation approach for first and second order ordinary differential equations. Fifth-order and other higher-order BVPs can be resolved using the Adomian decomposition method, [10–11] developed, as well as the VIM. The variation of parameters method was utilized by [12] to resolve seventh-order BVPs. Recently, [13] introduced the power series approximation approach as a generalized solution to this problem. Additionally, [14] overused the tau approach and tau-collocation approximation method to solve the first and second ODEs. The fifth order BVPs with sixth degree B-splines is another problem that [15] want to numerically solve. The seventh order BVPs were solved numerically using the octal spline method by [16]. Seventh order BVPs were solved using the Petrov-Galerkin method by [17] using quintic B-splines as the basis function and septic B-splines as the weight function. This study applies the variational iteration approach to solve BVPs of the seventh order using shifted Chebyshev polynomials of the third kind. Other related topic on the above technique can be found in [18-24]. The correction functional is adjusted for the BVP in the suggested techniques, and the Lagrange multiplier is derived using variational theory in the best possible way. The suggested technique is effective, and the preliminary findings are quite positive and trustworthy. The solution is then provided in an infinite series that typically leads to exact solution.

2. The standard variational iteration algorithm

We take the following generic differential equation into consideration in order to demonstrate the algorithm's fundamental principle.

$$Lv + Nv - g(\varphi) = 0, \qquad (3)$$

L is a linear operator, N is a nonlinear operator, and the inhomogeneous term is $g(\varphi)$.

We can create a correction functional as follows using the variational iteration approach.

$$v_{m+1} = v_m(\varphi) + \int_0^{\varphi} \lambda(t)\big(Lv_m(t) + N\widetilde{v_m(t)} - g(t)\big)dt \qquad (4)$$

$\widetilde{v_m}$ is regarded as a restricted variation. i.e., $\widetilde{v_m}$= 0A correction functional is the name given to the relation (4). Due to the precise identification of the Lagrange multiplier, both linear and nonlinear problems can be solved in a single iteration step. The successive approximation of solution v will be easily acquired utilizing the langrange multiplier and our $v_0$in this manner since we must determine the langrange multiplier $\lambda(t)$ideally. The solution is given by

$$\lim_{m-\infty} v_m = v$$

The Lagrange Multiplier also play an important role in determining the solution of the problem, and can be defined as follows:

$$(-1)^m \frac{1}{(m-1)!}(t-\varphi)^{m-1}$$

3. Chebyshev polynomials of the third kind

The third class of Chebyshev polynomials are orthogonal polynomials with respect to the weight function $(\varphi) = \left(\frac{1+\varphi}{1-\varphi}\right)^{\frac{1}{2}}$ $\forall\ \varphi \in [-1,1]$.

The third kind of Chebyshev polynomials are defined by $U_m(\varphi) = \frac{\cos[\left(m+\frac{1}{2}\right)cos^{-1}\varphi]}{cos\left(\frac{1}{2}cos^{-1}\varphi\right)}$; $m = 0,1,2,\dots$ with $U_0(\varphi) = 1$ and $U_1(\varphi) = 2\varphi - 1$.
Hence, the first few Chebyshev Polynomials of the third kind is given below:

$U_0(\varphi) = 1,$
$U_1(x) = 2\varphi - 1,$ (5)
$U_2(x) = 4\varphi^2 - 2\varphi - 1,$
.
.
.

4. Shifted Chebyshev polynomials of the third kind

The Shifted Chebyshev Polynomials of the third kind are orthogonal polynomials with respect to the weight function $w^*(\varphi) = \sqrt{\frac{\varphi}{1-\varphi}}$ $\forall\ \varphi \in [0,1]$ with starting values ${U^*}_0(\varphi) = 1$ and ${U^*}_1(\varphi) = 4\varphi - 3$.
This is defined by the relation ${U_{m+1}}^*(\varphi) = 2(2\varphi - 1){U_m}^*(\varphi) - {U_{m-1}}^*(\varphi) : m = 1,2,\dots$
Hence, the first few Shifted Chebyshev Polynomials of the third third is given below:
${U^*}_0(\varphi) = 1,$
${U^*}_1(\varphi) = 4\varphi - 3,$ (6)
${U^*}_2(x) = 16\varphi^2 - 20\varphi + 5,$
.
.

5. Reconstructed variational iteration algorithm using shifted Chebyshev polynomials of the third kind (RVIASCP)

Using (3) and (4), we assume an approximate solution of the form

$$v_{m,N-1}(\varphi)=\sum_{m=0}^{N-1}a_{m,N-1}U^*{}_{m,N-1}(\varphi) \qquad (7)$$

Where $a_{m,N-1}$ are constants that need to be found, m is the number of degrees of approximation, and $U^*{}_{m,N-1}(\varphi)$ are shifted Chebyshev polynomials of the third order.

As a result, we arrive to the following iterative technique.

$$v_{m+1,N-1}(\varphi)=\sum_{m=0}^{N-1}a_{m,N-1}U^*{}_{m,N-1}(\varphi)+\int_0^{\varphi}\lambda(t)\big(L\sum_{m=0}^{N-1}a_{m,N-1}U^*{}_{m,N-1}(t)+N_l\sum_{m=0}^{N-1}a_{m,N-1}U^*{}_{m,N-1}(t)\big)dt \qquad (8)$$

6. Numerical applications

In this section we applied the proposed method to solve four examples. Numerical results also show the accuracy and efficiency of the proposed scheme.

Example 6.1 [1, 7, 12, 20]: Reference considers the following seventh order linear boundary value problem

$$v^{(7)}+v=-e^{\varphi}(35+12\varphi+2\varphi^2), \qquad 0\le\varphi\le 1 \qquad (9)$$

With boundary conditions

$$v(0)=0,\ v'(0)=1,\ v''(0)=0,\ v'''(0)=-3 \qquad (10)$$
$$v(1)=0,\ v'(1)=-e,\ v''(1)=-4e$$

The exact solution for the problem is $v(\varphi)=\varphi(1-\varphi)e^{\varphi}$.

The correction functional for the boundary value problem (9) and (10) is given as

$$v_{m+1}=v_m(\varphi)+\int_0^{\varphi}\lambda(t)\left(v^{(7)}-v+e^t(35+12t+2t^2)\right)dt$$

Where $\lambda(t)=\frac{(-1)^7(t-\varphi)^6}{6!}$ is the Lagrange multiplier.

Applying the variational iteration algorithm using the shifted Chebyshev polynomials of the third kind, we assume an approximate solution of the form

$$v_{m,6}(\varphi)=\sum_{m=0}^{6}a_{m,6}U^*{}_{m,6}(\varphi)$$

Hence, we get the following iterative formula:

$$v_{m+1,N-1}(\varphi)=\sum_{m=0}^{6}a_{m,6}U^*{}_{m,6}(\varphi)+\int_0^{\varphi}\frac{(-1)^7(t-\varphi)^6}{6!}\left(\frac{d^7}{dt^7}\big(\sum_{m=0}^{6}a_{m,6}U^*{}_{m,6}(t)\big)-\big(\sum_{m=0}^{6}a_{m,6}U^*{}_{m,6}(t)\big)+e^t(35+12t+2t^2)\right)dt$$

$$v_{m+1,N-1}(\varphi)=a_{0,6}U^*{}_{0,6}(\varphi)+a_{1,6}U^*{}_{1,6}(\varphi)+a_{2,6}U^*{}_{2,6}(\varphi)+a_{3,6}U^*{}_{3,6}(\varphi)+a_{4,6}U^*{}_{4,6}(\varphi)+a_{5,6}U^*{}_{5,6}(\varphi)+a_{6,6}U^*{}_{6,6}(\varphi)+\int_0^{\varphi}\frac{(-1)^7(t-\varphi)^6}{6!}\left(\frac{d^7}{dt^7}\big(\sum_{m=0}^{6}a_{m,6}U^*{}_{m,6}(t)\big)-\big(\sum_{m=0}^{6}a_{m,6}U^*{}_{m,6}(t)\big)+e^t(35+12t+2t^2)\right)dt$$

Hence: using (4.1), iterating and applying the boundary conditions (10) the values of the unknown constants can be determined as follows

$a_{0,6} = 0.2421386688, \quad a_{1,6} = -0.07419433722, \quad a_{2,6} = -0.1275227869,$ Type equation here.
$a_{3,6} = -0.02857259190, \quad a_{4,6} = -0.003279623214, , \quad a_{5,6} = -0.000227864957$
$a_{6,6} = -0.000008138083$
Type equation here.
Consequently, the series solution is given as

$$v(\varphi) = 1.820000000\ 10^{-10} + 1.000000001\varphi + 4.300000000\ 10^{-9}\varphi^2 - 0.5000000092\varphi^3 -0.333333250\varphi^4 - 0.1249995551\varphi^5 - 0.03333358797\varphi^6 - 0.006944444444\varphi^7 -0.001190476191\varphi^8 - 0.0001736111111\varphi^9 - 0.00002204585537\varphi^{10} + \mathrm{O}(\varphi^{11})$$

Table 1: Comparison of numerical outcomes for Example 6.1

x	The proposed method's absolute error	Absolute Error by (Kasi and Reddy, 2015) [20]
0.1	5.000000e-11	1.415610e-07
0.2	1.000000e-10	6.407499e-07
0.3	5.000000e-10	2.920628e-06
0.4	1.200000e-09	4.410744e-06
0.5	2.000000e-09	6.735325e-06
0.6	2.100000e-09	6.407499e-06
0.7	1.700000e-09	3.665686e-06
0.8	3.000000e-10	3.278255e-07
0.9	8.000000e-10	1.430511e-06

Example 6.2 [20]: Reference considers the following seventh order linear boundary value problem

$$v^{(7)} = \varphi v + g(\varphi), \qquad 0 \le \varphi \le 1 \qquad (11)$$

With boundary conditions

$$v(0) = 1,\ v'(0) = 0,\ v''(0) = -1,\ v'''(0) = -2 \qquad (12)$$
$$v(1) = 0,\ v'(1) = -e,\ v''(1) = -2e$$

Where $g(\varphi) = e^{\varphi}(\varphi^2 - 2\varphi - 6)$
The exact solution for the problem is $v(\varphi) = (1 - \varphi)e^{\varphi}$.
The correction functional for the boundary value problem (11) and (12) is given as

$$v_{m+1} = v_m(\varphi) + \int_0^{\varphi} \lambda(t)\left(v^{(7)} - tv - e^t(t^2 - 2t - 6)\right)dt$$

Where $\lambda(t) = \frac{(-1)^7(t-\varphi)^6}{6!}$ is the Lagrange multiplier.

Applying the variational iteration algorithm using the shifted Chebyshev polynomials of the third kind, we assume an approximate solution of the form

$$v_{m,6}(\varphi) = \sum_{m=0}^{6} a_{m,6} U^*_{m,6}(\varphi)$$

Hence, we get the following iterative formula:

$$v_{m+1,N-1}(\varphi) = \sum_{m=0}^{6} a_{m,6} U^*_{m,6}(\varphi) + \int_0^{\varphi} \frac{(-1)^7(t-\varphi)^6}{6!} \left(\frac{d^7}{dt^7}\left(\sum_{m=0}^{6} a_{m,6} U^*_{m,6}(t)\right) - \left(\sum_{m=0}^{6} a_{m,6} U^*_{m,6}(t)\right)t - e^t(t^2 - 2t - 6)\right) dt$$

$$v_{m+1,N-1}(\varphi) = a_{0,6}U^*{}_{0,6}(\varphi) + a_{1,6}U^*{}_{1,6}(\varphi) + a_{2,6}U^*{}_{2,6}(\varphi) + a_{3,6}U^*{}_{3,6}(\varphi) + a_{4,6}U^*{}_{4,6}(\varphi) + a_{5,6}U^*{}_{5,6}(\varphi) + a_{6,6}U^*{}_{6,6}(\varphi) + \int_0^{\varphi} \frac{(-1)^7(t-\varphi)^6}{6!} \left(\frac{d^7}{dt^7}\left(\sum_{m=0}^{6} a_{m,6}U^*{}_{m,6}(t)\right) - \left(\sum_{m=0}^{6} a_{m,6}U^*{}_{m,6}(t)\right)t - e^t(t^2-2t-6)\right) dt$$

Hence: Using (4.1), iterating and applying the boundary conditions (12) the values of the unknown constants can be determined as follows

$a_{0,6} = 0.4257364903,\ a_{1,6} = -0.3195648196,\ a_{2,6} = -0.09186977815,\ a_{3,6} = -0.01187811969$

$a_{4,6} = -0.000978597151,\ a_{5,6} = -0.000054592625,\ a_{6,6} = -0.000001695432$

Consequently, the series solution is given as

$$v(\varphi) = 0.9999999994 + 2.48\ 10^{-10}\varphi - 0.4999999982\varphi^2 - 0.3333333401\varphi^3 - 0.1250000272\varphi^4 - 0.03333325722\varphi^5 - 0.006944489472\varphi^6 - \frac{1}{840}\varphi^7 - 0.0001736111111\varphi^8 - 0.00002204585538\varphi^9 - 0.000002480158726\varphi^{10} + O(\varphi^{11})$$

Table 2: Comparison of numerical outcomes for Example 6.2

x	The proposed method's absolute error	Absolute Error by (Kasi and Reddy, 2015) [20]
0.1	5.000000e-10	6.556511e-07
0.2	4.000000e-10	9.596348e-06
0.3	9.000000e-10	2.580881e-05
0.4	1.000000e-09	4.059076e-05
0.5	1.100000e-09	4.905462e-05
0.6	9.000000e-10	4.571676e-05
0.7	8.000000e-10	3.087521e-05
0.8	5.000000e-10	1.317263e-05
0.9	3.000000e-10	1.326203e-06

Example 6.3 [20]: Reference considers the following seventh order nonlinear boundary value problem

$$v^{(7)} = vv' + T(\varphi), \qquad 0 \le \varphi \le 1 \qquad (13)$$

With boundary conditions

$$v(0) = 1,\ v'(0) = -2,\ v''(0) = 3,\ v'''(0) = -4 \qquad (14)$$

$$v(1) = 0,\ v'(1) = -e^{-1},\ v''(1) = 2e^{-1}$$

Where $T(\varphi) = e^{-2\varphi}(2 + e^{\varphi}(\varphi - 8) - 3\varphi + \varphi^2)$

The exact solution for the problem is $v(\varphi) = (1-\varphi)e^{-\varphi}$.

The correction functional for the boundary value problem (13) and (14) is given as

$$v_{m+1} = v_m(\varphi) + \int_0^{\varphi} \lambda(t)\left(v^{(7)} - vv' - e^{-2t}(2 + e^t(t-8) - 3t + t^2)\right)dt$$

Where $\lambda(t) = \frac{(-1)^7(t-\varphi)^6}{6!}$ is the Lagrange multiplier.

Applying the variational iteration algorithm using the shifted chebyshev polynomials of the third kind, we assume an approximate solution of the form

$$v_{m,6}(\varphi) = \sum_{m=0}^{6} a_{m,6}U^*{}_{m,6}(\varphi)$$

Hence, we get the following iterative formula:

$$v_{m+1,N-1}(\varphi) = \sum_{m=0}^{6} a_{m,6}U^{*}{}_{m,6}(\varphi) + \int_0^{\varphi} \frac{(-1)^7(t-\varphi)^6}{6!} \left(\frac{d^7}{dt^7}\left(\sum_{m=0}^{6} a_{m,6}U^{*}{}_{m,6}(t)\right) - \frac{d}{dt}\left(\sum_{m=0}^{6} a_{m,6}U^{*}{}_{m,6}(t)\right)\left(\sum_{m=0}^{6} a_{m,6}U^{*}{}_{m,6}(t)\right) - e^{-2t}(2+e^{t}(t-8)-3t+t^2) \right) dt$$

$$v_{m+1,N-1}(\varphi) = a_{0,6}U^{*}{}_{0,6}(\varphi) + a_{1,6}U^{*}{}_{1,6}(\varphi) + a_{2,6}U^{*}{}_{2,6}(\varphi) + a_{3,6}U^{*}{}_{3,6}(\varphi) + a_{4,6}U^{*}{}_{4,6}(\varphi) + a_{5,6}U^{*}{}_{5,6}(\varphi) + a_{6,6}U^{*}{}_{6,6}(\varphi) + \int_0^{\varphi} \frac{(-1)^7(t-\varphi)^6}{6!} \left(\frac{d^7}{dt^7}\left(\sum_{m=0}^{6} a_{m,6}U^{*}{}_{m,6}(t)\right) - \frac{d}{dt}\left(\sum_{m=0}^{6} a_{m,6}U^{*}{}_{m,6}(t)\right)\left(\sum_{m=0}^{6} a_{m,6}U^{*}{}_{m,6}(t)\right) - e^{-2t}(2+e^{t}(t-8)-3t+t^2) \right) dt$$

Hence: using (4.1), iterating and applying the boundary conditions (6.3.2) the values of the unknown constants can be determined as follows

$a_{0,6} = 0.156970166951409$, $a_{1,6} = -0.194699116134633$, $a_{2,6} = 0.0437706786251294$
$a_{3,6} = -0.00509916008646768$, $a_{4,6} = 0.000461819389745091$, $a_{5,6} = -0.0000179775282118455$
$a_{6,6} = 0.00000237256959227080$

Consequently, the series solution is given as

$$v(\varphi) = 0.9999999998 - 2.000000000\varphi + 1.500000002\varphi^2 - 0.6666666682\varphi^3 + 0.2083300412\varphi^4 - 0.04999263530\varphi^5 + 0.009718045049\varphi^6 - 0.001587301587\varphi^7 + 0.0002232142858\varphi^8 - 0.00002755731930\varphi^9 + 0.000003031283378\varphi^{10} + O(\varphi^{11})$$

Table 3: Comparison of numerical outcomes for Example 6.3

x	The proposed method's absolute error	Absolute Error by (Kasi and Reddy, 2015) [20]
0.1	4.000000e-10	8.344650e-07
0.2	3.300000e-09	6.377697e-06
0.3	1.190000e-08	1.281500e-05
0.4	2.590000e-08	1.686811e-05
0.5	4.070000e-08	1.746416e-05
0.6	4.850000e-08	1.436472e-05
0.7	4.290000e-08	8.881092e-06
0.8	2.452000e-08	3.449619e-06
0.9	4.770000e-09	3.911555e-07

7. Conclusion

The variational iteration algorithm using shifted third-kind Chebyshev polynomials has been studied in this study and successfully used to produce numerical solutions for situations involving boundary values of the seventh order. The approach of solution involves the third kind shifted Chebyshev polynomials mixed with variational iteration algorithm. The technique provides quickly convergent series solutions that appear in physical problems. From Tables 1, 2 and 3, it is noted that the suggested approach yields a better result when compared with methods in literature. Finally; the numerical findings demonstrated that the current technique is an effective mathematical method for solving the class of problems put into consideration. The method considered in this work can be used to solve various problems arising in science and engineering.

8. References

[1] G. C. Abanum, O. K. Oke, Modified Variational Iteration Method for Solution of Seventh Order Boundary Value Problem Using Canonical Polynomials, International Journal of Modern Mathematical Sciences, 17(1) 2019, 57-67

[2] G. Akram, S.S. Siddiqi, Solution of seventh order boundary value problems by variation of parameters method, Research Journal of Applied Sciences, Engineering and Technology, 5(1) 2013, 176-179.

[3] A.F. Adebisi, T.A. Ojurongbe, K.A. Okunlola, O.J. Peter, Application of Chebyshev polynomial basis function on the solution of volterra integro-differential equations using Galerkin method, Mathematics and Computational Sciences, 2(1) 2021, 41-51.

[4] A.F. Adebisi, K.A. Okunola, M.T. Raji, J.A. Adedeji, O.J. Peter, Galerkin and perturbed collocation methods for solving a class of linear fractional integro-differential equations, The Aligarh Bulletin of Mathematics, 40(2) 2021, 45-57.

[5] L. Bougoffa, R.C. Rach, A, Mennouni, An approximate method for solving a class of weakly-singular Volterra integro-differential equations, Applied Mathematics and Computation, 217(22) 2011, 8907-8913.

[6] H.N. Caglar, S.H. Caglar, E.H. Twizell, The numerical solution of fifth-order value problems with sixth degree B-spline function, Applied Mathematics Letters, 12(5) 1999, 25-30.

[7] M. Din, A. Yildirim, Solutions of Tenth and Ninth-order Boundary Value Problems by Modified Variational Iteration Method, Application and applied mathematics, 5(1) 2010, 11- 25.

[8] E.J. Mamadu, I.N. Njoseh, Tau-Collocation Approximation Approach for Solving First and Second Order Ordinary Differential Equations. Journal of Applied Mathematics and Physics, 4(2) 2016, 383-390.

[9] C.Y. Ishola, O.A. Taiwo. A.F, Adebisi, O.J. Peter, Numerical solution of two-dimensional Fredholm integro-differential equations by Chebyshev integral operational matrix method, Journal of Applied Mathematics and Computational Mechanics, 21(1) 2022, 29-40.

[10] M.A. Noor, S.T. Mohyud-Din, A new approach for solving fifth order boundary value problem, International Journal of Nonlinear Science, 9(4) 2010, 387-393.

[11] M.A. Noor, S.T. Mohyud-Din, Variational iteration decomposition method for eight order boundary value problem, Differential Equations and Nonlinear Mechanics, 2007.

[12] M. Noor, S.T. Mohyud-Din. Variational iteration method for fifth order boundary value problem using He's polynomials, Mathematical Problems in Engineering, 2008.

[13] S.S. Shahid, M. Iftikhar, Variational iteration method for solution of seventh order boundary value problem using He's polynomials, Journal of the Association of Arab Universities for Basic and Applied Sciences, 18 2015, 60-65.

[14] I.N. Njoseh, E.J. Mamadu. Numerical solutions of a generalized Nth Order Boundary Value Problems Using Power Series Approximation Method. Applied Mathematics, 7 2016, 1215-1224.

[15] O.J. Peter, F.A. Oguntolu, M.M. Ojo, A.O. Oyeniyi, R. Jan, I. Khan, Fractional Order Mathematical Model of Monkeypox Transmission Dynamics, Physica Scripta, 97(8) 2022, 1-26.

[16] O.J. Peter, M.O. Ibrahim, Application of Variational Iteration Method in Solving Typhoid Fever Model, Knowledge and Control Systems Engineering (BdKCSE), Sofia, Bulgaria. 1(1) 2019, 1-5.

[17] O.J. Peter, O.A. Afolabi, F.A. Oguntolu, C.Y. Ishola, A. A. Victor, Solution of a Deterministic Mathematical Model of Typhoid Fever by Variational Iteration Method. Science World Journal, 13(2) 2018, 64-68.

[18] S.M. Reddy, Numerical Solution of Ninth Order Boundary Value Problems by Quintic B-splines International Journal of Engineering Inventions 5(7) 2016, 38-47.

[19] S.S. Siddiqi, M. Iftikhar, Solution of Seventh Order Boundary Value Problems by Variation of Parameters Method. Res. J. Appl. Sci., Engin. Tech., 5(1) 2013, 176–179.

[20] K.N.S. Kasi Viswanadham, S.M. Reddy, Numerical Solution of Seventh Order Boundary Value Problems by Petrov-Galerkin Method with Quintic B-splines as Basis Functions and Septic B-splines as Weight Functions. International Journal of Computer Applications, 12(5) 2015, 0975 – 8887.

[21] O.A. Uwaheren, A.F. Adebisi, C.Y. Ishola, M.T. Raji, A.O. Yekeem, O.J. Peter, Numerical Solution of Volterra integro-differential Equations by Akbari-Ganji's Method, BAREKENG: J. Math. & App., 16(3) 2022, 1123-1130.

[22] O.A. Uwaheren, A.F. Adebisi, O.T. Olotu, M.O. Etuk, O.J. Peter, Legendre Galerkin Method for Solving Fractional Integro-Differential Equations of Fredholm Type, The Aligarh Bulletin of Mathematics, 40(1) 2021, 1-13.

[23] T. Oyedepo, O.A. Uwaheren, E.P. Okperhie, O. J. Peter. Solution of Fractional Integro-Differential Equation Using Modified Homotopy Perturbation Technique and Constructed Orthogonal Polynomials as Basis Functions, Journal of Science Technology and Education, 7(3) 2019, 157-164.

[24] O.A. Uwaheren, A.F. Adebisi, C.Y. Ishola, M.T. Raji, A.O. Yekeem, O.J. Peter, Numerical Solution of Volterra integro-differential Equations by Akbari-Ganji's Method, BAREKENG: J. Math. & App., 16(3) 2022, 1123-1130.