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Approximate solutions of Schrodinger equation in D Dimensions with the modified Mobius square plus Hulthen potential

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Abstract: The study presents the approximate solutions of Schrodinger equation in D-dimensions with the modified Mobius square plus Hulthen potential. The energy eigenvalues and corresponding wave functions are obtained using the Nikiforov-Uvarov (NU) method. Special cases of this potential are reported. Numerical results are also computed.

Keywords: Schrodinger equation; Modified Mobius square plus Hulthen potential; Nikiorov-Uvarov method
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1 Introduction

In quantum mechanics, one area that has continued to stimulate study interest is the solutions of various wave equations. This is because the solutions to a wave equation serve as useful framework to describe any system under investigation. Recently published articles [2,8,21,23,29], have attested to the significance of the solutions of wave equations. Attempts have been made to probe some wave equations using techniques such as Laplace transform method [3], asymptotic iteration method [4], algebraic method [30], shifted 1/N expansion [11], Nikiforov –Uvarov (NU) method [16,17,20], and others. Out of necessity to extend solutions of wave equations to various dimensional space, reports on D dimensional solutions have emerged [6,13,18,25].

This study is aimed at investigating the solutions of Schrodinger equation in D-dimensions with combination of modified Mobius square (MMS) and Hulthen potentials. Combining the two potentials can be helpful in nuclear physics to understand interaction in nucleus. The Mobius square potential is the general case of some potentials,

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and has many importance [12,28]. Using newly proposed NU - functional analysis approach, Ikot et al. [12], recently solved Schrodinger equation for Mobius square potential. They used their results to study Shanon entropy and Fisher information. By the use of NU method, the authors in Ref [28] solved Schrodinger equation in D dimensions under Mobius square potential. Maghsoodi et al. [15], investigated arbitrary solutions of Dirac equation for Mobius square potential using NU method. Applying Supersymmetry approach, the authors in Ref. [14] obtained solutions of Dirac equation with Mobius square and Mie potentials. In addition, the Hulthen potential is very relevant in particle, atomic, nuclear, chemical and condensed matter physics [26]. By Exact quantization approach, Qiang et al. [23], presented approximate solutions of Schrodinger equation for Hulthen potential. Ikhdair [10] obtained solutions of Schrodinger equation with Hulthen potential for arbitrary state, using NU method to achieve energy eigenvalues and corresponding wavefunctions. Reports on scattering states of Hulthen potential have been published [5]. Adebimpe et al. [1], solved Duffin-Kemmer Petiau equation and Schrodinger equation with Hulthen and Yukawa potentials combined together. Thus, it is important to study the Schrodinger equation with a combination of modified Mobius square and Hulthen potential, as the study will give an understanding of some physical properties such as thermodynamic properties of diatomic molecules.
The modified Mobius square plus Hulthen potential is written as:

$$V(r) = -V_0\left(\frac{A+Be^{-2\alpha r}}{1-e^{-2\alpha r}}\right)^2 - \frac{V_1 e^{-2\alpha r}}{1-e^{-2\alpha r}} \tag{1.1}$$

where V_0, V_1 are the potential depth , A, B , and α, are the range of the potential, molecular bond length, and the screening parameter, respectively.

2 The Nikiforov-Uvarov (NU) method

In applying NU method, a second order linear differential equation is transformed to a hyper-geometric form [10]. Using $s = s(x)$, one can write an equation

$$\psi_n^{//}(s) + \frac{\tilde{\tau}(s)}{\sigma(s)}\psi_n^{/}(s) + \frac{\tilde{\sigma}(s)}{\sigma^2(s)}\psi_n(s) = 0, \tag{2.1}$$

where $\tilde{\sigma}(s)$ and $\sigma(s)$ are polynomials, at most two degree and $\tilde{t}(s)$ is a polynomial of one degree.

In this paper, we consider a parametric approach to NU method following the proposal by Tezcan and Sever [24]. In this case, we use the differential equation

$$\frac{d^2\psi(s)}{ds^2} + \frac{\alpha_1-\alpha_2 s}{s(1-\alpha_3 s)}\frac{d\psi(s)}{ds} + \frac{-\xi_1 s^2+\xi_2 s-\xi_3}{s^2(1-\alpha_3 s)^2}\psi(s) = 0 \tag{2.2}$$

Considering Eq. (2.2), the energy equation can be calculated using the expression

$$(\alpha_2-\alpha_3)n+\alpha_3 n^2-(2n+1)\alpha_5+(2n+1)(\sqrt{\alpha_9}+\alpha_3\sqrt{\alpha_8})$$
$$+\alpha_7+2\alpha_3\alpha_8+2\sqrt{\alpha_8\alpha_9}=0, \tag{2.3}$$

and the corresponding wave function as

$$\psi(s)=N_{nl}s^{\alpha_{12}}(1-\alpha_3 s)^{-\alpha_{12}-(\alpha_{13}/\alpha_3)}P_n^{\left(\alpha_{10}-1,\frac{\alpha_{11}}{\alpha_3}-\alpha_{10}-1\right)}(1-2\alpha_3 s). \tag{2.4}$$

The parameters in Eqs. (2.3) and (2.4), are defined as follows:

$$\psi(s)=N_{nl}s^{\alpha_{12}}(1-\alpha_3 s)^{-\alpha_{12}-(\alpha_{13}/\alpha_3)}P_n^{\left(\alpha_{10}-1,\frac{\alpha_{11}}{\alpha_3}-\alpha_{10}-1\right)}(1-2\alpha_3 s). \tag{2.5}$$

$$\left.\begin{aligned}&\alpha_4=\frac{1}{2}(1-\alpha_1),\alpha_5=\frac{1}{2}(\alpha_2-2\alpha_3),\alpha_6=\alpha_5^2+\xi_1,\alpha_7=2\alpha_4\alpha_5-\xi_2,\\&\alpha_8=\alpha_4^2+\xi_3,\alpha_9=\alpha_3\alpha_7+\alpha_3^2\alpha_8+\alpha_6,\alpha_{10}=\alpha_1+2\alpha_4+2\sqrt{\alpha_8},\\&\alpha_{11}=\alpha_2-2\alpha_5+2\left(\sqrt{\alpha_9}+\alpha_3\sqrt{\alpha_8}\right),\alpha_{12}=\alpha_4+\sqrt{\alpha_8},\alpha_{13}=\alpha_5-(\sqrt{\alpha_9}+\alpha_3\sqrt{\alpha_8})\end{aligned}\right\} \tag{2.6}$$

3 D-dimensional Schrodinger equation

The Schrodinger equation in D-dimension for spherically symmetric potential [9] reads:

$$H\psi_{nlm}(r,\Omega_m)=E_{nl}\psi_{nlm}(r,\Omega_m). \tag{3.1}$$

H is the Hamiltonian given by

$$H=-\frac{\hbar}{2\mu}\left[r^{1-D}\frac{\partial}{\partial r}\left(r^{1-D}\frac{\partial}{\partial r}\right)+\frac{\Lambda_D^2}{r^2}\right]+V(r) \tag{3.2}$$

where $V(r)$ is the potential , μ as reduced mass, $\hbar$ as planck constant, E_{nl} as energy, and Ω_D is the angular form of co-ordinate. A hyperspherical harmonic function describes eigenfunction of an operator $\Lambda_D^2(\Omega_D)$. This gives

$$\psi_{nlm}(r,\Omega_m) = r^{-\frac{(D-1)}{2}} R_{nl}(r) Y_l^m(\Omega_D) \tag{3.3}$$

where $Y_l^m(\Omega_D)$, and $R_{nl}(r)$ are hyperspherical function, and hyper radial wave function, respectively. In addition, a general form of centrifugal barrier in D-dimension, involving (Ω_D) is given by $\frac{\Lambda_D^2(\Omega_D)}{r^2}$

Substituting Eqs. (3.2) and (3.3) into Eq. (3.1) transforms Eq. (3.1) into the Schrodinger equation in D dimension as [9],

$$\frac{d^2R_{nl(r)}}{dr^2} + \frac{2\mu}{\hbar^2}\left[E - V(r)\right]R_{nl}(r) + \frac{1}{r^2}\left[\frac{(D+2l-1)(D+2l-3)}{4}\right]R_{nl}(r) = 0 \tag{3.4}$$

Substituting Eq. (1.1) into Eq. (3.4) gives

$$\begin{aligned} &\frac{d^2R_{nl(r)}}{dr^2} + \frac{2\mu}{\hbar^2}\left[E + V_0\left(\frac{A+Be^{-2\alpha r}}{1-e^{-2\alpha r}}\right)^2 + \frac{V_1e^{-2\alpha r}}{1-e^{-2\alpha r}}\right]R_{nl}(r) \\ &+ \frac{1}{r^2}\left[\frac{(D+2l-1)(D+2l-3)}{4}\right]R_{nl}(r) = 0 \end{aligned} \tag{3.5}$$

For $l \neq 0$, Eq. (3.5) is solvable by use of approximation [7]

$$\frac{1}{r^2} \approx \frac{4\alpha^2 e^{-2\alpha r}}{(1-e^{-2\alpha r})^2} \tag{3.6}$$

Substitution of Eq. (3.6) in Eq. (3.5), and taking $s = e^{-2\alpha r}$, yields

$$\frac{d^2R(r)}{ds^2} + \frac{(1-s)}{s(1-s)}\frac{dR(s)}{ds} + \frac{-\varepsilon^2 s^2 + \theta\, s - M}{s^2(1-s)}R(r) = 0 \tag{3.7}$$

where,

$$\varepsilon^2 = -\frac{\mu(E+V_0B^2 - V_1)}{2\alpha^2\hbar^2} \tag{3.8}$$

$$\theta = \frac{\mu(V_1 - 2E + 2ABV_0)}{2\alpha^2\hbar^2} + \frac{(D-1)(D-3)}{4} + l(l+D-2) \tag{3.9}$$

$$M = -\frac{\mu(E+V_0A^2)}{2\alpha^2\hbar^2} \tag{3.10}$$

Comparing Eq. (2.2) with Eq. (3.7), we obtain values of constants in Eq. (2.5) as:

$$\left.\begin{aligned}&\alpha_1 = \alpha_2 = \alpha_3 = 1,\ \alpha_4 = 0,\ \alpha_5 = -\frac{1}{2},\ \alpha_6 = \frac{1}{4} + \varepsilon^2,\ \alpha_7 = -\theta,\ \alpha_8 = M,\ \alpha_9 = \frac{1}{4} + \varepsilon^2 + M - \theta\\ &,\ \alpha_{10} = 1 + 2\sqrt{M}\ ,\ \alpha_{11} = 2 + 2\left(\sqrt{\frac{1}{4} + \varepsilon^2 + M - \theta} + \sqrt{M}\right),\ \alpha_{12} = \sqrt{M}\ ,\\ &\alpha_{13} = -\frac{1}{2} - \left(\sqrt{\frac{1}{4} + \varepsilon^2 + M - \theta} + \sqrt{M}\right)\end{aligned}\right\} \quad (3.11)$$

Substituting the appropriate parameters obtained in Eq. (2.5) into Eqs. (2.3), we obtain the energy equation as

$$E_{nl} = -V_0 A^2 - \frac{2\alpha^2\hbar^2}{\mu}\left(\frac{n(n+1) + \frac{1}{2} - \Lambda - \frac{\mu(V_1 + 2V_0A^2 + 2ABV_0)}{2\alpha^2\hbar^2} + \left(n + \frac{1}{2}\right)\sqrt{\frac{1}{4} - \frac{\mu V_0 (A+B)^2}{2\alpha^2\hbar^2} - \Lambda}}{2n + 1 + \sqrt{\frac{1}{4} - \frac{\mu V_0 (A+B)^2}{2\alpha^2\hbar^2} - \Lambda}}\right)^2 \quad (3.12)$$

Where

$$\Lambda = \frac{(D + 2l - 1)(D + 2l - 3)}{4} \quad (3.13)$$

In 3D, Eq. (3.12) becomes

$$E_{nl} = -V_0 A^2 - \frac{2\alpha^2\hbar^2}{\mu}\left(\frac{n(n+1) + \frac{1}{2} - l(l+1) - \frac{\mu(V_1 + 2V_0A^2 + 2ABV_0)}{2\alpha^2\hbar^2} + \left(n + \frac{1}{2}\right)\sqrt{2l + 1^{\ 2} - \frac{\mu V_0 (A+B)^2}{2\alpha^2\hbar^2}}}{2n + 1 + \sqrt{2l + 1^{\ 2} - \frac{\mu V_0 (A+B)^2}{2\alpha^2\hbar^2}}}\right)^2 \quad (3.14)$$

Substitution of Eq. (2.5) into Eq. (2.4) gives the corresponding wave function

$$R(s) = NS^{\sqrt{M}}(1 - s)^{\chi} P_n^{2\sqrt{M},\, 2\chi - 1}(1 - 2s) \quad (3.15)$$

where,

$$\chi = \frac{1}{2} + \frac{1}{2}\sqrt{\frac{1}{4} + \varepsilon^2 + M - \theta} \quad (3.16)$$

4 Discussions

Fig.1 examines variation of the potential in Eq. (1.1) against internuclear distance (r) for values of α. It is observed that the potential increases as r increases. Fig. 2 shows the accuracy of the approximation used. Thus, the approximation has validity within small values of screening parameter, α <<1. In fig. 3, the energy of the potential is seen to decrease as V_0 increases but cuts-off just above V_0 = 0.00006 eV for all values of quantum number, n. In fig.4, there is a sharp decrease in energy as V_1 increases for all values of n. In figs.5 and 6, the energy is seen to increase with the potential range, A and molecular bond length, B, respectively. In fig. 7, the energy of the system is observed to increase monotonically as the screening parameter, α increases for all values of n. We computed the energy eigenvalue for different n and l states for $D = 3,4,5,6$ with some values of α as presented in Tables 1-2.The energies obtained are valid for short range potentials. The energy is observed to increase as the quantum numbers, n and l, and spatial dimensionality, D, increase. In table 3, the numerical values of energy of the modified Mobius square potential are presented. To show the accuracy of our result, numerical values of the energy of Hulthen potential as obtained from this study is compared to those found in the literature using the NU method [27], Asymptotic iteration method (AIM) [19], exact quantization rule (EQR) [23] and supersymmetric method (SUSY) [10] in table 4. Our results are observed to be in good agreement with literature.

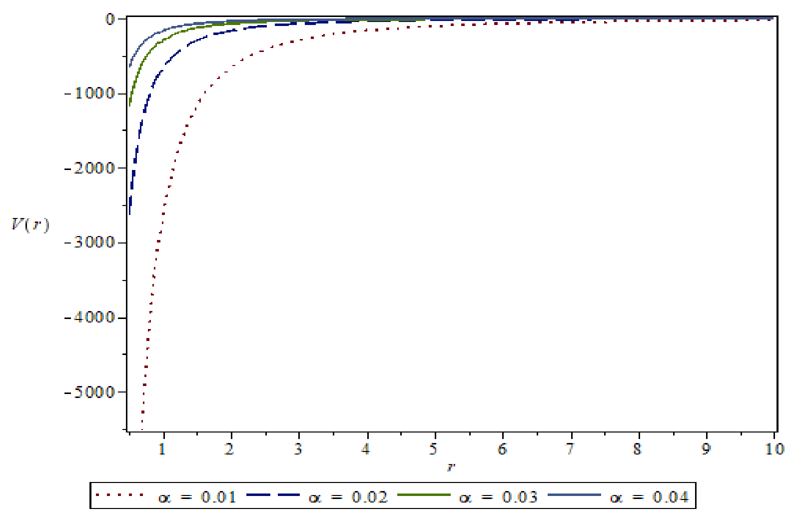

Figure 1: Plot of modified Mobius square plus Hulthen potential against r for different values of α

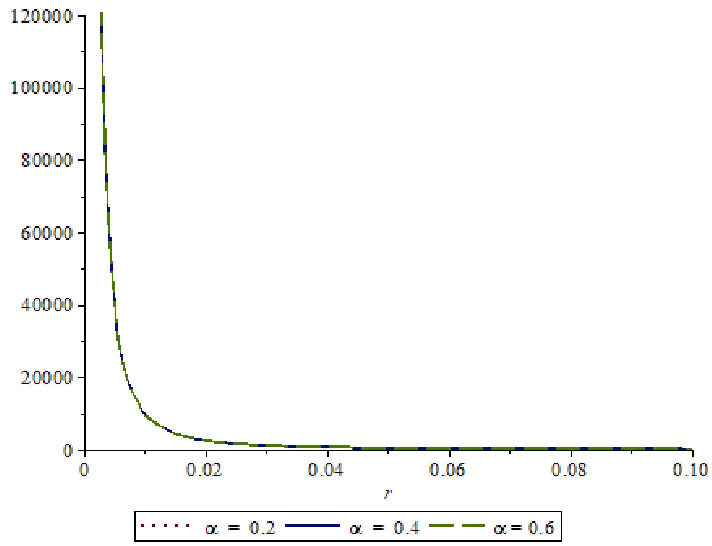

Figure 2: A plot of the variation of the centrifugal term of $\frac{1}{r^2}$ and the approximation as a function of r for different values of $\alpha = 0.2,\ 0.4,$ and $\alpha = 0.6$

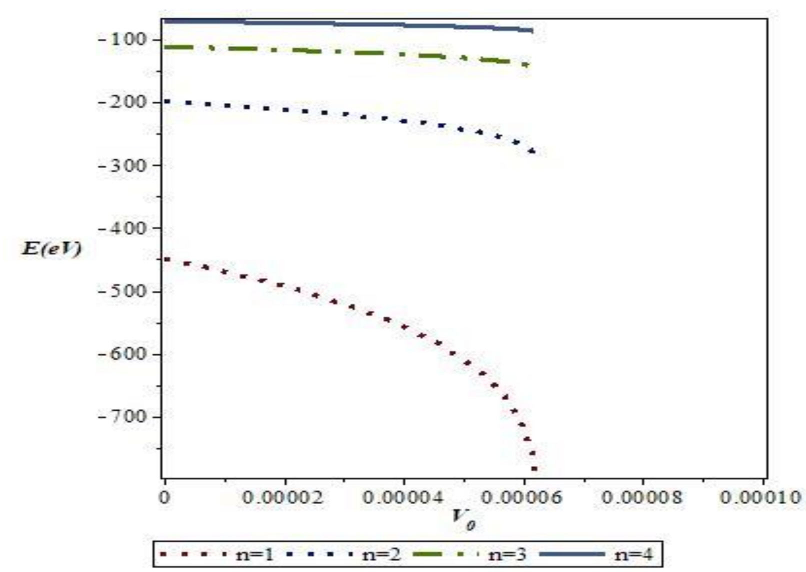

Figure 3: Variation of energy of the potential with the potential depth, V_0, for various values of quantum number, n

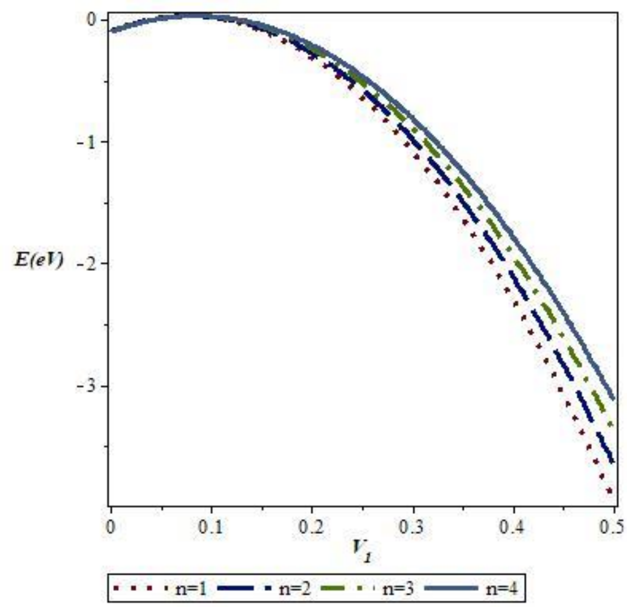

Figure 4: Variation of energy of the potential with the potential depth, V_1, for various values of quantum number, n

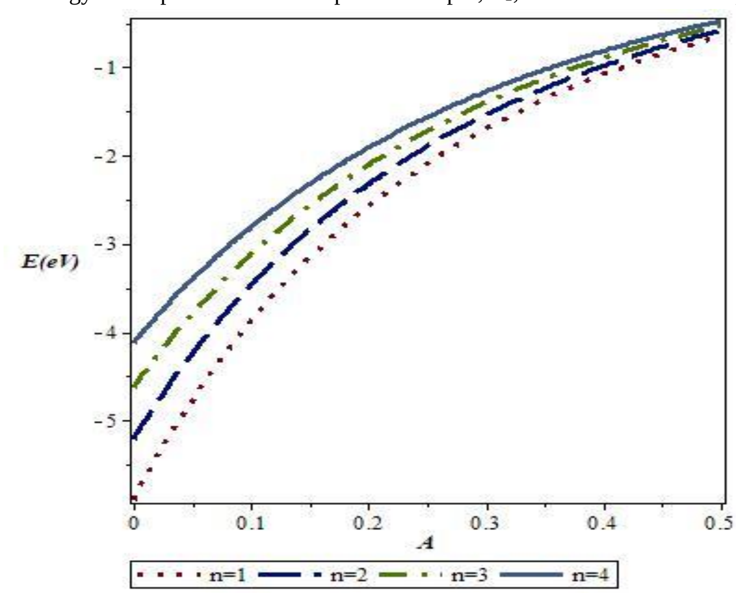

Figure 5: Variation of energy of the potential with the potential range, A, for various values of quantum number, n

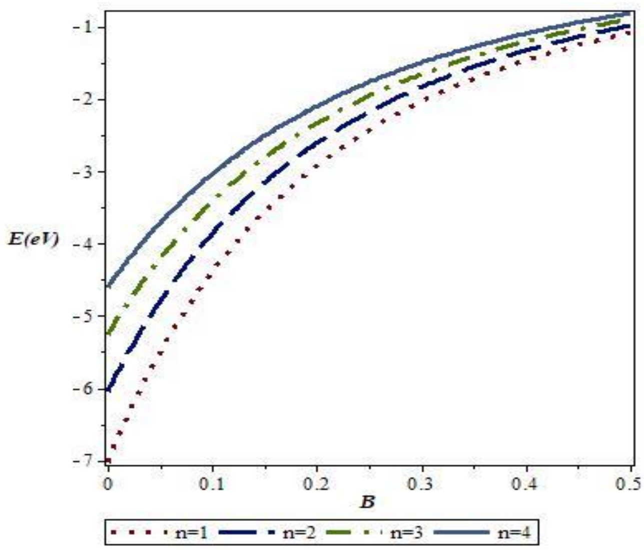

Figure 6: Variation of energy of the potential with the molecular bond length B, for various values of quantum number n

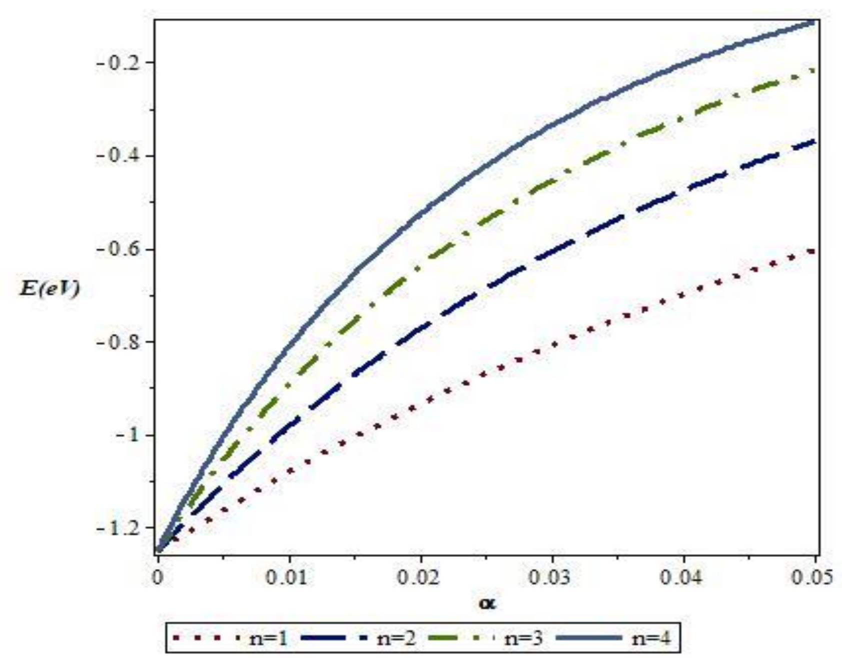

Figure 7: Variation of energy of the potential with the screening parameter α, for various values of quantum number n

Table 1: Energy eigenvalues for $D = 3,4,5$ and 6 with $\alpha = 0.02$ and $V_0 = 0.2, V_1 = 2, B = 0.5, A = 1.5$, $\mu = \hbar = 1$.

n	l	$E_{nl}^{D}(D=3)$	$E_{nl}^{D}(D=4)$	$E_{nl}^{D}(D=5)$	$E_{nl}^{D}(D=6)$
0	0	-3.649000000	-3.647802358	-3.645810774	-0.6472641912
1	0	-3.637839488	-3.636657940	-3.634693098	-0.6421173445
	1	-3.634693098	-3.631951526	-3.628442346	-0.6392176363
2	0	-3.615732002	-3.614582164	-3.612670010	-0.6319175005
	1	-3.612670010	-3.610001876	-3.606586558	-0.6291101192
	2	-3.606586558	-3.602435268	-3.597561530	-0.6252416027
3	0	-3.583095774	-3.581992330	-3.580157274	-0.6168484872
	1	-3.580157274	-3.577596596	-3.574318628	-0.6141762136
	2	-3.574318628	-3.570333980	-3.565655462	-0.6104935556
	3	-3.565655462	-3.560298000	-3.554278530	-0.6058570579
4	0	-3.540536678	-3.539492982	-3.537757216	-0.5971766407
	1	-3.537757216	-3.535334942	-3.532233886	-0.5946784012
	2	-3.532233886	-3.528463902	-3.524036884	-0.5912351992
	3	-3.524036884	-3.518966698	-3.513269084	-0.5868995775
	4	-3.513269084	-3.506961548	-3.500063266	-0.5817366364
5	0	-3.488824120	-3.487851866	-3.486234830	-0.5732402484
	1	-3.486234830	-3.483978060	-3.481088580	-0.5709501545
	2	-3.481088580	-3.477575346	-3.470480368	-0.5677934164
	3	-3.473449178	-3.468722686	-3.463410204	-0.5638178586
	4	-3.463410204	-3.457527680	-3.451092596	-0.5590828148
	5	-3.451092596	-3.444123832	-3.436641584	-0.5536576534

Table 2. Energy eigenvalues for $D = 3,4,5$ and 6 with $\alpha = 0.03$ and $V_0 = 0.2, V_1 = 2, B = 0.5, A = 1.5$ $(\mu = \hbar = 1)$.

n	l	$E_{nl}^{D}(D=3)$	$E_{nl}^{D}(D=4)$	$E_{nl}^{D}(D=5)$	$E_{nl}^{D}(D=6)$
0	0	-0.6488750001	-0.6478678042	-0.6461991732	-0.6438839740
1	0	-0.6372635384	-0.6362922546	-0.6346830771	-0.6324502821
	1	-0.6346830771	-0.6324502821	-0.6296135284	-0.6261975411
2	0	-0.6145142738	-0.6136124449	-0.6121182603	-0.6100448660
	1	-0.6121182603	-0.6100448660	-0.6086621402	-0.6042375758
	2	-0.6074103736	-0.6042375758	-0.6005535976	-0.5963894897
3	0	-0.5815311795	-0.5807278961	-0.5802661357	-0.5775497289
	1	-0.5793968886	-0.5775497289	-0.5752023720	-0.5723749166
	2	-0.5752023720	-0.5723749166	-0.5690913119	-0.5653790137
	3	-0.5690913119	-0.5653790137	-0.5612686010	-0.5567933637
4	0	-0.5395685259	-0.5387778633	-0.5377572787	-0.5361894490
	1	-0.5377572787	-0.5361894490	-0.5341967345	-0.5317959912
	2	-0.5341967345	-0.5317959912	-0.5290073401	-0.5258538995
	3	-0.5290073401	-0.5258538995	-0.5223614826	-0.5185582712
	4	-0.5223614826	-0.5185582712	-0.5144744697	-0.5101419492
5	0	-0.4901260056	-0.4895821217	-0.4886807640	-0.4874295391
	1	-0.4886807640	-0.4874295391	-0.4858389635	-0.4839223343
	2	-0.4858389635	-0.4839223343	-0.4816955687	-0.4791770143
	3	-0.4816955687	-0.4791770143	-0.4763872333	-0.4733487687
	4	-0.4763872333	-0.4733487687	-0.4689938718	-0.4666243462
	5	-0.4700858928	-0.4666243462	-0.4629910688	-0.4592139313

Table 3: Numerical values energy for modified Mobius square potential with A = 0.4, B = 0.5, V0 = 1

n	l	$\alpha=0.01$	$\alpha=0.02$	$\alpha=0.03$
0	0	-0.0000253086	-0.0001012346	-0.0002277778
1	0	-0.0001290090	-0.0005159959	-0.0011608412
2	0	-0.0003363915	-0.0013452292	-0.0030255237
	1	-0.0004351676	-0.0017404594	-0.0039152526
3	0	-0.0006474198	-0.0025883676	-0.0058190673
	1	-0.0007462068	-0.0029837662	-0.0067096054
	2	-0.0009437769	-0.0037745107	-0.0084904476
4	0	-0.0010620399	-0.0042445896	-0.0095376415
	1	-0.0011608413	-0.0046402052	-0.0104291790
	2	-0.0013584402	-0.0054313873	-0.0122120583
	3	-0.0016548302	-0.0066180365	-0.0148858765
5	0	-0.0015801808	-0.0063128513	-0.0141766601
	1	-0.0016790000	-0.0067087264	-0.0150693309
	2	-0.0018766344	-0.0075004321	-0.0168545185
	3	-0.0021730780	-0.0086878778	-0.0195319074
	4	-0.0025683196	-0.0102709250	-0.0231010042

Table 4: Numerical values of energy in eV of the Hulthen potential for various states of various values of the screening parameter with Z = 1 in atomic units ($\hbar = \mu = e = 1$)

State	***A***	**Present**	**NU[27]**	**AIM[19]**	**EQR[23]**	**SUSY[10]**
2p	0.025	-0.11281250000	-0.11281250000	-0.1128125	-0.1128125	-0.1127605
	0.050	-0.10125000000	-0.10125000000	-0.1012500	-0.1012500	-0.1010425
	0.075	-0.09031250005	-0.09031249994	-0.0903125	-0.0903125	-0.0898478
	0.10	-0.08000000000	-0.08000000000	-0.0800000	-0.0800000	-0.0791794
	0.15	-0.06124999995	-0.06124999998	-0.0612500	-0.0612500	-0.0594415
3p	0.025	-0.04375868056	-0.04070312500	-0.0437590	-0.0437590	-0.0437068
	0.050	-0.033368055560	-0.03336810000	-0.0333681	-0.0333681	-0.0331632
	0.075	-0.024383680560	-0.02438370000	-0.0243837	-0.0243837	-0.0239331
	0.10	-0.016805555560	-0.01680560000	-0.0168056	-0.0168056	-0.0160326
	0.15	-0.005868055545	-0.00586810000	-0.0058681	-0.0058681	-0.0043599
3d	0.025	-0.04375868056	-0.04360440000	-0.0437587	-0.0437587	-0.0436030
	0.050	-0.03336805556	-0.03275080000	-0.0333681	-0.0333681	-0.0327532
	0.075	-0.02438368056	-0.02299480000	-0.0243837	-0.0243837	-0.0230306
	0.10	-0.01680555556	-0.01433640000	-0.0162600	-0.0162600	-0.0144832
	0.15	-0.005868055545	-0.00031240000	-0.0058681	-0.0058681	-0.0132820
4p	0.025	-0.02000000000	-0.01994860000	-0.0200000	-0.0200000	-0.0199480
	0.050	-0.01125000000	-0.01104420000	-0.0112500	-0.0112500	-0.0110430
	0.075	-0.005000000000	-0.00453700000	-0.0050000	-0.0050000	-0.0045385
	0.10	-0.001250000000	-0.00042690000	-0.0012500	-0.0012500	-0.0004434

4.1 Special cases

In this section, we examine special cases of Eq. (1.1) by adjusting the parameters

Case I: Hulthen potential

When $V_0 = 0$, Eq. (1.1) transforms to Hulthen potential of the form

$$V_H(r) = -\frac{V_1 e^{-2\alpha r}}{1-e^{-2\alpha r}} \tag{3.17}$$

with energy equation as

$$E_{nl} = \frac{-2\alpha^2\hbar^2}{\mu}\left(\frac{n(n+1) + \frac{1}{2} - \Lambda - \frac{\mu V_1}{2\alpha^2\hbar^2} + (2n+1)\sqrt{\frac{1}{4} - \Lambda}}{2n+1+2\sqrt{\frac{1}{4}-\Lambda}}\right)^2. \tag{3.18}$$

Further, if we take $D = 3$, and substitute $2\alpha = \delta$, the energy equation becomes

$$E = \frac{-\delta^2\hbar^2}{2\mu}\left(\frac{-\frac{2\mu V_1}{\delta^2\hbar^2} + (l+n+1)^2}{2(n+l+1)}\right)^2 \tag{3.19}$$

which is similar to Eq. (61) of Ref. [1].

Case II: modified Mobius square potential

Similarly, when $V_1 = 0$, Eq. (1.1) transforms to modified Mobius square potential

$$V(r) = -V_0\left(\frac{A+Be^{-2\alpha r}}{1-e^{-2\alpha r}}\right)^2. \tag{3.20}$$

Here, Eq. (3.12) becomes

$$E_{nl} = -V_0 A^2 - \frac{2\alpha^2\hbar^2}{\mu}\left(\frac{n(n+1) + \frac{1}{2} - \Lambda - \frac{\mu V_0 A(A+B)}{\alpha^2\hbar^2} + \left(n+\frac{1}{2}\right)\sqrt{\frac{1}{4} - \frac{\mu V_0 (A+B)^2}{2\alpha^2\hbar^2} - \Lambda}}{2n+1+\sqrt{\frac{1}{4} - \frac{\mu V_0(A+B)^2}{2\alpha^2\hbar^2} - \Lambda}}\right)^2 \tag{3.21}$$

With the choice of $D = 3$, Eq. (3.21) transforms to (3.22)

$$E_{nl} = -V_0 A^2 - \frac{2\alpha^2\hbar^2}{\mu}\left(\frac{n(n+1)+\frac{1}{2} - l\ \ l+1\ - \frac{\mu V_0 A(A+B)}{\alpha^2\hbar^2} + \left(n+\frac{1}{2}\right)\sqrt{(1+2l)^2 - \frac{2\mu V_0 (A+B)^2}{\alpha^2\hbar^2}}}{2n+1+\sqrt{(1+2l)^2 - \frac{2\mu V_0 (A+B)^2}{\alpha^2\hbar^2}}}\right)^2$$

5 Conclusion

In summary, we studied the analytical solutions of Schrodinger equation in D-dimensions for modified Mobius Square plus Hulthen potential. To derive energy spectrum and the wave functions, the N-U method is employed and the centrifugal term is approximated suitably with Greene and Aldrich proposed scheme. We note that the potential yields Hulthen and modified Mobius square potential when $V_0 = 0$, and $V_1 = 0$, respectively. Our result is consistent with literature [1]. To show the accuracy of our results obtained in the study, we carried out a further check by providing numerical results of the Hulthen potential and compared to literature results obtained by using various methods. Our results are observed to be in good agreement with literature, which further highlights the Nikiforov Uvarov (NU) method as a reliable method. This study may have some applications in molecular and chemical physics.

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