Hypercyclicity of adjoint of convex weighted shift and multiplication operators on Hilbert spaces

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Abstract: A bounded linear operator T on a Hilbert space $\mathfrak{H}$ is convex, if

$$\|\mathfrak{T}^2 v\|^2 - 2\|\mathfrak{T}v\|^2 + \|v\|^2 \geq 0.$$

In this paper, sufficient conditions to hypercyclicity of adjoint of unilateral (bilateral) forward (backward) weighted shift operator is given. Also, we present some example of convex operators such that it's adjoint is hypercyclic. Finally, the spectrum of convex multiplication operators is obtained and an example of convex, multiplication operators is given such that it's adjoint is hypercyclic.

1   Introduction and preliminaries

Let $\mathfrak{H}$ be a separable infinite dimensional Hilbert space, and $\mathfrak{B}(\mathfrak{H})$ be the space of all bounded linear operators on $\mathfrak{H}$. The operator $\mathfrak{T} \in \mathfrak{B}(\mathfrak{H})$ is said to be *convex*, if

$$\|\mathfrak{T}^2 v\|^2 - 2\|\mathfrak{T}v\|^2 + \|v\|^2 \geq 0, \tag{1.1}$$

for all $v \in \mathfrak{H}$. We know that the sequence $(\alpha_n)_n$ of real numbers is convex, if

$$\alpha_{n+1} \leq \frac{1}{2}(\alpha_n + \alpha_{n+2}), \quad n \in \mathbb{N}. \tag{1.2}$$

Hence, if $\mathfrak{T}$ is a convex operator then the sequence $(\|\mathfrak{T}^n v\|^2)_{n \in \mathbb{N}}$ is a convex sequence for each $v \in \mathfrak{H}$. Letting $\Delta_{\mathfrak{T}} = \mathfrak{T}^* \mathfrak{T} - I$, the similar definition of convexity of $\mathfrak{T}$ is the following form.

$$\mathfrak{T}^* \Delta_{\mathfrak{T}} \mathfrak{T} \geq \Delta_{\mathfrak{T}}.$$

In [13] the authors have studied the convexity of composition and multiplication operators, and their adjoints on a weighted Hardy space. Note that if for every $x \in \mathfrak{H}$, equality holds in 1.1, then the operator $\mathfrak{T}$ is called a 2-isometric operator. Moreover, if the inequality in 1.1 is reversed then the operator $\mathfrak{T}$ is called a concave or 2-hyperexpansive operator. For some information on such operators one can see [17] .

The operator $\mathfrak{T} \in B(\mathfrak{H})$ is *hypercyclic* if there exists a vector $v \in \mathfrak{H}$ such that its orbit, that is the set $orb(\mathfrak{T}, v) = \{v, \mathfrak{T}v, \mathfrak{T}^2 v, T^3 v, ...\}$, is a dense set in $\mathfrak{H}$ and such as v is called hypercyclic vector for $\mathfrak{T}$. Similarly the operator $\mathfrak{T}$ is called supercyclic if there exists a vector $v \in \mathfrak{H}$ such that $\{\alpha \mathfrak{T}^n v : \alpha \in \mathbb{C}. n \in \mathbb{N}\}$ is dense in $\mathfrak{H}$. In this case, the vector v is called supercyclic vector for $\mathfrak{T}$.

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The operator $\mathfrak{T} \in \mathfrak{B}(\mathfrak{H})$ is called weakly hypercyclic if there exists a vector $v \in \mathfrak{H}$ such that its orbit is weakly dense. Similarly, the operator $\mathfrak{T} \in \mathfrak{B}(\mathfrak{H})$ is weakly supercyclic if there exists a vector v such that $\{\alpha \mathfrak{T}^n v : \alpha \in \mathcal{C}, n \in \mathcal{N}_0\}$, is weakly dense. The concept weak dense means dense with respect to the weak topology.

Hilden and Wallen [16] in 1974 showed that any (unilateral) backward weighted shift operator is always supercyclic, regardless of the weight sequence w. Hypercyclicity of unilateral backward weighted shift operators is characterized by Salas [19] in 1995.

Note that if the equality holds in 1.1 for every $v \in \mathfrak{H}$, then the operator $\mathfrak{T}$ is called a 2-isometric operator. It is known that 2-isometric operators are neither supercyclic nor weakly hypercyclic [2]. Ansari and Bourdon [3] extended this to the class of all isometries on a Banach space. Later, supercyclicity of 2-isometric operators and more generally m-isometric operators have been investigated in [1, 2, 4, 7]. In addition, supercyclicity of concave and co-concave operators have been discussed in [18]. In recent years, the supercyclicity of operators has received substantial attention. Some good sources on this topic of operators include [5] and [12]. We mention in particular [6, 10, 20].

In this paper, we discuss on hypercyclicity and supercyclicity of adjoint of some convex operators. Section 2 is devoted to some elementary properties and examples of convex weighted shift operators such that their adjoints are hypercyclic or supercyclic. In Section 3 we give some properties of multiplication operators on weighted Hardy space and we give some necessary conditions for hypercyclicity adjoint of such convex operators and present an example of adjoint hypercyclic convex operators. Finally, the spectrum of such operators is obtained.

2 Cyclicity of adjoint of convex weighted shift operators

An operator $\mathfrak{T}$ in $\mathfrak{B}(\mathfrak{H})$ is called a forward unilateral (bilateral) weighted shift if there is an orthonormal basis $\{\mathfrak{e}_n : n \geq 0\}$ ($\{\mathfrak{e}_n : n \in \mathfrak{Z}\}$) and a sequence of bounded complex numbers $\{\omega_n : n \geq 0\}$ ($\{\omega_n : n \in \mathfrak{Z}\}$) such that $\mathfrak{T}\mathfrak{e}_n = \omega_n \mathfrak{e}_{n+1}$ for all $n \geq 0 (n \in \mathfrak{Z})$. It is known that a weighted shift operator $\mathfrak{T}$ is unitarily equivalent to a weighted shift operator with a non-negative weight sequence [5]. So we can assume that $\omega_n \geq 0$ for every n (see Page 52 of [21]). We know that $\mathfrak{T}$ is an isometry if and only if $\omega_n = 1$ for all n.

Lemma 2.1. *Let $\mathfrak{T}$ be a convex bilateral weighted shift operator.*

(a) *If $\Delta_{\mathfrak{T}} \geq 0$ then $\Delta_{\mathfrak{T}^*} \geq 0$*

(b) *If $\Delta_{\mathfrak{T}} \leq 0$ then $\Delta_{\mathfrak{T}^*} \leq 0$*

Theorem 2.2. *Suppose that $\mathfrak{T}$ is a convex unilateral forward weighted shift on $\ell^2(\mathbb{N})$, with $\Delta_{\mathfrak{T}} > 0$, then T^* is hypercyclic.*

Proof. Since $\Delta_{\mathfrak{T}} > 0$ thus $w_n > 1$, for all $n \in \mathbb{N}$ thus

$$\limsup_{n \to \infty}(\omega_1.\omega_2...\omega_n) = \infty.$$

Hence $\mathfrak{T}^*$ is hypercyclic by [5]. $\square$

Example 2.3. *Suppose that $\mathfrak{T}$ is a convex unilateral weighted shift with $\Delta_{\mathfrak{T}} \geq 0$. Adjoint of $\mathfrak{T}$ is defined by*

$$\mathfrak{T}^* \mathfrak{e}_n = \begin{cases} \omega_{n-1}\mathfrak{e}_{n-1}, & n \geq 1; \\ 0, & n = 0. \end{cases}$$

It's clearly $\mathfrak{T}^*$ is always supercyclic, regardless of the weight sequence $\{w_n\}_n$.
On the other hand $\mathfrak{T}^*$ is hypercyclic if and only if

$$\limsup_{n \to \infty}(\omega_1.\omega_2...\omega_n) = \infty.$$

thus if we define $\omega_n = \sqrt{\frac{n+3}{n+1}}$ for all $n \geq 0$ then it's clearly $\mathfrak{T}$ is convex and $\Delta_{\mathfrak{T}} \geq 0$.

$$\limsup_{n\to\infty}(\omega_1.\omega_2...\omega_n) = \limsup_n(\sqrt{\frac{4}{2} \times \frac{5}{3} \times \frac{6}{4} \times \frac{7}{5} \times ... \times \frac{n+1}{n-1} \times \frac{n+2}{n} \times \frac{n+3}{n+1}})$$

$$= \limsup_n \sqrt{\frac{(n+2)(n+3)}{6}} = \infty.$$

Thus $\mathfrak{T}^*$ is hypercyclic.

Theorem 2.4. *Suppose that $\mathfrak{T}$ is a convex supercyclic bilateral weighted shift, then $\mathfrak{T}^*$ is not hypercyclic*

Proof. Since $\mathfrak{T}$ is supercyclic thus $\Delta_{\mathfrak{T}} \geq 0$ or $\Delta_{\mathfrak{T}} \leq 0$, [19] and then Lemma 2.1 implies that $\Delta_{\mathfrak{T}^*} \geq 0$ or $\Delta_{\mathfrak{T}^*} \leq 0$ respectively. Thus $\mathfrak{T}^*$ is not hypercyclic. $\qquad\square$

Theorem 2.5. *If $\mathfrak{T}$ is a convex, bilateral weighted shift with $\Delta_{\mathfrak{T}} \geq 0$ then $\mathfrak{T}^*$ is not supercyclic.*

Proof. Since $\Delta_{\mathfrak{T}^*} \geq 0$, thus $\omega_n \geq 1$ for all n and then $\mathfrak{T}$ is invertible thus by Theorem 3.4 of [9] there exist $\{n_k\}_k$ such that

$$\lim_n \prod_{j=1}^{n_k}(\frac{\omega_j}{\omega_{-j}}) = 0$$

and then

$$\lim_n \prod_{j=1}^{n_k}(\frac{\omega_{-j}}{\omega_j}) \neq 0$$

thus $\mathfrak{T}^*$ is not supercyclic. $\qquad\square$

Corollary 2.6. *If $\mathfrak{T}$ is a convex, bilateral weighted shift with $c < w_n \leq 1$ for some positive real number c, then $\mathfrak{T}^*$ is not supercyclic.*

Now we present an example of bilateral weighted shift operator $\mathfrak{T}$ such that $\mathfrak{T}$ and $\mathfrak{T}^*$ is not supercyclic.

Example 2.7. *Consider the backward bilateral weighted shift operator $\mathfrak{T}$ defined by $\mathfrak{T}\mathfrak{e}_n = \omega_n\mathfrak{e}_{n-1}$ which*

$$\omega_n = \begin{cases} \frac{1}{10} + \frac{1}{n^4}, & n \leq -2; \\ \frac{1}{10}, & n = -1, 0, 1 \\ \frac{1}{10} + \frac{1}{n^2}, & n \geq 2. \end{cases}$$

$\mathfrak{T}$ is convex $\Delta_{\mathfrak{T}} \leq 0$. Since $\{\omega_n\}_n$ is bounded below, $\mathfrak{T}$ is invertible and

$$\mathfrak{T}^{-1}\mathfrak{e}_n = \frac{1}{\omega_{n+1}}e_{n+1}; \quad n \in \mathbb{Z}.$$

On the other hand $\mathfrak{T}^*$ is invertible

$$(\mathfrak{T}^*)^{-1}(\mathfrak{e}_n) = (\mathfrak{T}^{-1})^*(\mathfrak{e}_n) = \frac{1}{\omega_n}\mathfrak{e}_{n-1}; \quad n \in \mathbb{Z}$$

and

$$\lim_n \prod_{j=1}^{n} \frac{w_j}{\omega_{-j}} = \lim_n \prod_{j=2}^{n} \frac{\frac{1}{10} + 1j^2}{\frac{1}{10} + \frac{1}{j^4}} > 0$$

because

$$\sum_{j=2}^{n} 1 - \frac{\frac{1}{10} + 1j^2}{\frac{1}{10} + \frac{1}{j^4}} = 10\sum_{j=2}^{\infty} \frac{10 - j^2}{j^4 + 10} < \infty$$

Thus $\mathfrak{T}^*$ is not supercyclic.

Example 2.8. *Let $\mathfrak{T}$ be a bilateral forward weighted shift operator $\mathfrak{T}\mathfrak{e}_n = \omega_n \mathfrak{e}_{n+1}$ with $\omega_n = \sqrt{\frac{n}{n+1}}$ for $n \geq 1$ and $\omega_n = \frac{1}{2}$ for $n \leq 1$.*

It is clearly $\mathfrak{T}$ is convex and $\Delta_{\mathfrak{T}} \leq 0$ and $\mathfrak{T}^* \mathfrak{e}_n = \omega_{n-1} \mathfrak{e}_{n-1}$ is backward bilateral weighted shift operator. Since for each $q \in N$

$$\liminf_n \frac{1}{\omega_1 ... \omega_{n+q}} = \liminf_n \sqrt{\frac{1}{\frac{1}{2} \cdot \frac{2}{3} ... \frac{n+q}{n+q+1}}} = \liminf_n \sqrt{(n+q+1)} = \infty$$

but

$$\liminf_n \frac{\omega_0 . \omega_{-1} ... \omega_{-n+q+1}}{\omega_1 . \omega_2 ... \omega_{n+q}} = \liminf_n \sqrt{\frac{(\frac{1}{2})^{n-q}}{\frac{1}{n+q+1}}} = 0.$$

Thus $\mathfrak{T}^*$ is supercyclic by 1.39 of [5].

Theorem 2.9. *Suppose that $\mathfrak{T}$ is a convex, bilateral, forward weighted shift operator on the $\ell^2(\mathbb{Z})$ with weight sequence $\{\omega_n\}_n$. If $v \in \ell^2(\mathbb{Z})$ and $\alpha_n = \|\mathfrak{T}^n v\|$ then, $\{\alpha_n\}_n$ is decreasing or eventually strictly increasing. Moreover, if there exist c such that $1 < c \leq w_n$ for all $n \in \mathbb{Z}$ then,*

$$\lim_n \prod_{i=0}^{k} \omega_{n-i} = \infty.$$

Proof. By Proposition 2.1 of [14], $\{\alpha_n\}_n$ is decreasing or eventually increasing. If the sequence $(\alpha_n)_n$ is decreasing thus $\omega_n \leq 1$ for all $n \in \mathbb{Z}$, and then we conclude that $\|\mathfrak{T}^{*^n} v\|$ is eventually increasing.
On the other hand convexity of $\mathfrak{T}$ implies that

$$\omega_n^2 . \omega_{n+1}^2 \geq 2\omega_n^2 - 1$$

thus

$$\omega_{n-1}^2 \omega_n^2 \omega_{n+1}^2 \geq 2\omega_{n-1}^2 \omega_n^2 - \omega_{n-1}^2 \geq 3\omega_{n-1}^2 - 2$$

by using mathematical induction we conclude that

$$\omega_{n-k}^2 \omega_{n-k+1}^2 ... \omega_n^2 \omega_{n+1}^2 \geq (k+2)\omega_{n-k}^2 - (k+1) = (k+2)[\omega_{n-k}^2 - 1] + 1.$$

Now if $k \to \infty$ then we obtain

$$\lim_{k \to \infty} \prod_{j=0}^{k} \omega_{n-i} \quad \geq \quad \lim_{k \to \infty} (k+1)[\omega_{n-k}^2 - 1] + 1$$

$$\geq \quad (c-1) \lim_{k \to \infty} (k+2) = \infty.$$

$\square$

Example 2.10. *Consider bilateral forward weighted shift operator $\mathfrak{T}$ on $\ell^2(\mathbb{Z})$ with weight $\omega_n = 1$ for $n \leq 0$ and $\omega_n = 2$ for $n \geq 1$.*

It's clearly $\mathfrak{T}$ is convex and $\mathfrak{T}^*$ is a bilateral backward weighted shift operator which is defined by

$$\mathfrak{T}^* \mathfrak{e}_n = w_{n-1} e_{n-1}.$$

and $\mathfrak{T}^*$ is weakly hypercyclic which is not hypercyclic.

Note that in the last example if $\omega_n = a < 1$ for all $n \leq 0$ then $\mathfrak{T}^*$ is hypercyclic too.

In [15] Herrero had a question about direct sum of hypercyclic operators. His question was: Does $\mathfrak{T}$ hypercyclic imply that $\mathfrak{T} \oplus \mathfrak{T}$ is also hypercyclic?. Similarity, we extend this question to adjoint of convex operators. That means, if $\mathfrak{T}$ is convex and $\mathfrak{T}^*$ is hypercyclic. Is $\mathfrak{T}^* \oplus \mathfrak{T}^*$ hypercyclic?. For weighted shift operators, the answer is positive. See Corollary 2.10 of [19]. In generality, for an arbitrary convex operator, it's must be check out.

3   Properties of Multiplication operators

Theorem 3.1. *Suppose that ψ is a non-constant function and multiplier of $\mathfrak{H}$ and $\mathfrak{M}_\psi$ is multiplication operator on $\mathfrak{H}$. Then $\mathfrak{M}_\psi^*$ is hypercyclic if $\psi(\mathbb{D})$ meets the unit circle $\mathbb{D}$, [11].*

At first we present an example of convex multiplication operator $\mathfrak{M}_\psi$ with hypercyclic adjoint.

Example 3.2. *Consider $\mathfrak{H}^2(\beta)$, the weighted Hardy space so it's weight sequence $\{\beta(n)\}_n$ is defined by $\beta(n) = n+1$ and map ψ on $\mathbb{D}$ by $\psi(z) = 2v$. It's clearly, $\mathfrak{M}_\psi$ is bounded and for every nonnegative integer k,*

$$
\begin{aligned}
\|\mathfrak{M}_\psi^2 v^k\|^2 - 2\|\mathfrak{M}_\psi v^k\|^2 + \|v^k\|^2 &= 16\|v^{k+2}\|^2 - 8\|v^{k+1}\|^2 + \|v^k\|^2 \\
&= 16(k+3)^2 - 8(k+2)^2 + (k+1)^2 > 0
\end{aligned}
$$

and

$$
\|\mathfrak{M}_\psi v^k\|^2 - \|v^k\|^2 = 4\|v^{k+1}\|^2 - \|v^k\|^2 = 4(k+2)^2 - (k+1)^2 > 0.
$$

Consequently, $\mathfrak{M}_\psi$ is convex but not an isometry and $\Delta_\psi \geq 0$. Since $\psi(\mathbb{D})$ intersects unit circle then last Theorem implies that $\mathfrak{M}_\psi^*$ is hypercyclic.

Corollary 3.3. *Let $\mathfrak{H}^\infty = \mathfrak{M}(\mathfrak{H}^2(\beta))$. Consider the function $\psi \in \mathfrak{H}^\infty$ with $\|\psi\|_\infty \leq 1$. If $\mathfrak{M}_\psi^* \Delta_\psi M_\psi \geq \Delta_\psi \geq 0$, then $\mathfrak{M}_\psi^*$ is not hypercyclic.*

Proof. By Theorem 3.2 of [13] and statement $\|\mathfrak{M}_\psi\| = \|\mathfrak{M}_\psi^*\| = 1$ we conclude $\mathfrak{M}_\psi^*$ is isometry and consequently is not hypercyclic. $\square$

Finally, we obtain the spectrum of Convex Multiplication operators on Hardy, Bergman and Dirichlet spaces.

Theorem 3.4. *If $\mathfrak{M}_\psi$ is a convex, multiplication operator on $\mathfrak{H}^2(\mathbb{D})$ and $\Delta_{\mathfrak{M}_\psi} \geq 0$ then*

$$
sp(\mathfrak{M}_\psi) = \overline{\psi(\mathbb{D})}.
$$

Proof. Let $\mathfrak{M}_\psi$ be a convex multiplication operator on $\mathfrak{H}^2(\mathbb{D})$ such that $\Delta_{\mathfrak{M}_\psi} \geq 0$ and $\zeta \in \psi(\mathbb{D})$ thus $\psi(v_0) = \zeta$ for some $z_0 \in \mathbb{D}$. Consider $(\mathfrak{M}_\psi - \zeta I)f = \psi f - \zeta f = (\psi - \zeta)f$ thus $(\mathfrak{M}_\psi - \zeta I)(f)(v_0) = (\psi(v_0) - \zeta)f(v_0) = 0$ for each $f \in H^2(\mathbb{D})$ but $1 \in H^2(\mathbb{D})$ thus $\mathfrak{M}_\psi - \zeta I = 0$ and hence $\mathfrak{M}_\psi - \zeta I$ is not surjective and consequently is not invertible, hence $\zeta \in sp(\mathfrak{M}_\psi)$. Compacness of spectrum implies that $\overline{\psi(\mathbb{D})} \subseteq sp(\mathfrak{M}_\psi)$.
On the other hand, let $\zeta \notin \overline{\psi(\mathbb{D})}$ thus $\frac{1}{\psi - \zeta} \in \mathfrak{H}^\infty(\mathbb{D})$. Now we show that $\mathfrak{M}_\psi - \zeta I$ is bijective.
Let

$$
\begin{aligned}
f \in ker(\mathfrak{M}_\psi - \zeta I) \Rightarrow (\mathfrak{M}_\psi - \zeta I)f = 0 &\Rightarrow (\psi(v) - \zeta)f(v) = 0 \quad \forall v \in \mathbb{D} \\
&\Rightarrow f \equiv 0.
\end{aligned}
$$

Thus $\mathfrak{M}_\psi - \zeta I$ injective. Now if $g \in \mathfrak{H}^2(\mathbb{D})$ since $\frac{1}{\psi - \zeta} \in \mathfrak{H}^\infty(D) \subseteq \mathfrak{H}^2(\mathbb{D})$ thus $\frac{1}{\psi - \zeta}g \in \mathfrak{H}^2(\mathbb{D})$ and $(\mathfrak{M}_\psi - \zeta I)\frac{1}{\psi - \zeta}g = g$ hence $\mathfrak{M}_\psi - \zeta I$ is surjective and consequently invertible thus $\zeta \notin sp(M_\psi)$ and then

$$
sp(\mathfrak{M}_\psi) = \overline{\psi(\mathbb{D})}.
$$

$\square$

Note that Theorem 2.1 of [14] implies that if $\mathfrak{M}_\psi$ is non-invertible then $\mathbb{D} \subseteq \psi(\mathbb{D})$, and if $\mathfrak{M}_\psi$ is invertible then $\psi(\mathbb{D}) \subseteq \{z : |z| \geq 1\}$ i.e. $|\psi(z)| \geq 1$ for every $z \in \mathbb{D}$.

In the second case since $\mathfrak{M}_\psi$ is not hypercyclic operator on Hilbert space $\mathfrak{H}$, spectrum $sp(\mathfrak{M}_\psi)$ does not intersect the unit circle thus $|\psi(v)| > 1$ for each $v \in \mathbb{D}$.

Similarly same result will obtain for Bergman space $\mathcal{A}^2(\mathbb{D})$ and Dirichlet space $\mathcal{D}$ on unit disk $\mathbb{D}$. Now, consider Hilbert space $\mathfrak{H}$. If each bounded analytic map ψ on region Ω is a multiplier of $\mathfrak{H}$ and $\|\mathfrak{M}_\psi\| = \|\psi\|_\infty$. Multiplication operator $\mathfrak{M}_\psi^*$ is hypercyclic if and only if $\psi(\Omega) \cap \partial\mathbb{D} \neq \emptyset$. Hence, for each convex, multiplication operator $\mathfrak{M}_\psi$ we have

Corollary 3.5. *If $\mathfrak{M}_\psi$ satisties in condition $\mathfrak{M}_\psi^* \Delta_\psi \mathfrak{M}_\psi \geq \Delta_\psi \geq 0$, on $\mathfrak{H}^2(\mathbb{D})$ or $\mathcal{A}^2(\mathbb{D})$, then $\mathfrak{M}_\psi^*$ is not hypercyclic.*

Of course, this is not valid for all Hilbert space $\mathfrak{H}$. Indeed condition $\|\mathfrak{M}_\psi\| = \|\psi\|_\infty$ is necessary. To show validity of this condition assume that $\mathfrak{H}$ is a Hilbert space and ψ is a multiplier of $\mathfrak{H}$ then, the inequqlity $\|\mathfrak{M}_\psi\| \geq \|\psi\|_\infty$, hold. But if consider the Dirichlet space $\mathcal{D}$ on $\mathbb{D}$, consisting of all analytic function $f : \mathbb{D} \to \mathbb{C}$ with

$$\|f\|_\mathbb{D}^2 = |f(0)|^2 + \int_\mathbb{D} |f'(v)|^2 \frac{dA(v)}{\pi} < \infty.$$

we show that $\mathcal{M}(\mathcal{D}) \subset H^\infty(\mathbb{D})$.

$$\mathcal{D} = \{f(v) = \sum_{j=0}^\infty a_j v^j : \quad \sum_{j=0}^\infty |a_j|^2 (j+1) < \infty\}$$

thus

$$\|v^n\|_\mathcal{D}^2 = \frac{1}{\pi} \int_\mathbb{D} |nv^{n-1}|^2 dA(v) = \frac{n^2}{\pi} \int_0^1 \int_0^{2\pi} r^{2n-1} |e^{i(n-1)\Delta}|^2 d\theta dr = n.$$

Let $e_n(v) = \frac{v^n}{\|v^n\|} = \frac{v^n}{\sqrt{n+1}}$

$$\|\mathfrak{M}_v\| = \sup_{\|f\|=1} \|\mathfrak{M}_v f\| \geq \|\mathfrak{M}_v e_1(v)\|$$

$$= \|\frac{v^2}{\|v\|}\| = \|v\| = \sqrt{2}$$

but $\|\psi(v) = v\|_\infty = \sup_{v \in \mathbb{D}} |v| = 1$ thus $\|\mathfrak{M}_v\| > \|v\|_\infty$.

In continuity, we present a convex, multiplication operator $\mathfrak{M}_\psi$ on Dirichlet space $\mathcal{D}$ such that $\mathfrak{M}_\psi^*$ is hypercyclic but $\psi(\mathbb{D}) \cap \partial\mathbb{D} = \emptyset$.

Example 3.6. *Consider the multiplication operator $\mathfrak{M}_\psi$ on Dirichlet space $\mathcal{D}$. Define the map $\psi(v) = z$ on $\mathbb{D}$. The multiplication operator $\mathfrak{M}_\psi$ is convex and $\Delta_\psi \geq 0$.*

$$\|\mathfrak{M}_\psi^2 v^k\|^2 - 2\|\mathfrak{M}_\psi v^k\|^2 + \|v^k\|^2 = \|z^{k+2}\|^2 - 2\|v^{k+1}\|^2 + \|v^k\|^2$$
$$= (k+2) - 2(k+1) + k = 0.$$

and

$$\|\mathfrak{M}_\psi v^k\|^2 - \|v^k\|^2 = \|v^{k+1}\|^2 - \|v^k\|^2 = (k+1) - k = 1 > 0.$$

Thus $\mathfrak{M}_\psi$ is convex and $\Delta_\psi \geq 0$ and M_ψ^* is hypercyclic on $\mathcal{D}$ by [8], but $\psi(\mathbb{D}) \cap \partial\mathbb{D} = \emptyset$.

4   Conclusion

The concept of convex operator on an infinite dimensional Hilbert space $\mathfrak{H}$ is introduced and sufficient conditions to hypercyclicity of adjoint of unilateral (bilateral) forward (backward) weighted shift operator is given. Also, some example of convex operators such that it's adjoint is hypercyclic is presented. Finally, the spectrum of convex multiplication operators is obtained and an example of convex, multiplication operators is given such that it's adjoint is hypercyclic. For further research, it is best to look at all convex operators and their direct sum.

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