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Transfer Matrix Methods for Annular Periodic Multilayered Structures

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Abstract: Analysis of complex one-dimensional photonic systems is a challenge to the scientific community. To solve such a problem in the past computational techniques have been deployed. Transfer matrix or so-called characteristics matrix is one of the widely used methods acceptable for multilayered systems. The convectional transfer matrix method has some drawbacks to solving an annular type multilayered system. This paper presents the transfer matrix method to investigate the electromagnetic wave propagation in the annular multilayered photonic structures.

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1 Introduction

Computational Electromagnetics (CEM) has evolved enormously in the past decades to the point that its methods can examine the performance of the optical structures with extreme accuracy. There are various methods employed such as transfer matrix method (TMM), plane wave expansion (PWE) method, finite difference time domain (FDTD) method, finite element method (FEM), boundary element method (BEM), etc to compute the solution of electromagnetic waves that can be propagated through the optical structure [12]. Though several methods are used for computation purposes; however, the TMM method is frequently used to avoid empirical calculation and provides many accurate solutions to its counterpart and is therefore well accepted in the research community specifically while practising multilayered optical structures. For example, relevant for the design of anti-reflective coatings, dielectric mirrors, omnidirectional band gap materials, and narrow filters [15]. It should be noted that the mentioned applications are using the flat photonic multilayered for such a purpose.

On the other hand, annular periodic multilayers have a tremendous advantage over the flat type multilayered for specific applications such as LEDs design and Bragg reflectors [11, 10]. The optical properties of such an annular structure can be calculated using the transfer matrix method. However, the conventional transfer matrix method in a Cartesian coordinates [3, 19] for flat type multilayered systems is not able to calculate the reflectance and transmittance spectra of the system. Therefore, in this paper, we followed the previous works by Kaliteevski et al. [10] and Hu et al. [9], and present the modified transfer matrix

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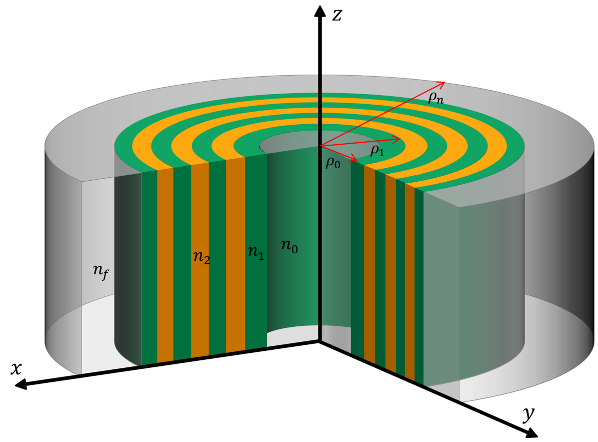

Figure 1: Schematic of an annular photonic crystal structures where the refractive index of the gray inner and outer regions are n_0 and n_f, respectively. The green and orange layers are indicated the materials with the refractive index of n_1 and n_2.

method to calculate the optical properties of annular multilayered systems. This transfer matrix of the annular multilayered system is solved with the help of the cylindrical coordinate system, Bessel function, and the Neumann function.

2 Transfer Matrix Method

The structure of an annular photonic crystal that also named Bragg reflector is shown in Figure 1, in which the inner core gray region has a refractive index of n_0 and a starting radius of ρ_0, the green layer with index n_1 is assumed to be the first material, and the second orange layer having index n_2. In addition, the index of refraction of the outer gray region is denoted by n_f. To calculate the reflectance at the first circular boundary, $\rho = \rho_0$, the transfer matrix method in the cylindrical waves is employed. The cylindrical wave is assumed to be diverging from the axis of symmetry, $\rho = 0$, and then impinges normally on the first circular interface of $\rho = \rho_0$.

Assuming an $e^{(j\omega t)}$ time dependence for the electromagnetic fields, the source-free two curl Maxwell's equations are given by [11, 10, 9]

$$\nabla \times E = -j\omega\mu H, \tag{2.1a}$$

$$\nabla \times H = j\omega\varepsilon E. \tag{2.1b}$$

In cylindrical coordinate, Eq. 2.1:

$$\frac{1}{\rho}\begin{vmatrix} \hat{\rho} & \rho\hat{\phi} & \hat{z} \\ \partial_\rho & \partial_\phi & \partial_z \\ E_\rho & \rho E_\phi & E_z \end{vmatrix} = -j\omega\mu\left(\hat{\rho}H_\rho + \hat{\phi}H_\phi + \hat{z}H_z\right), \tag{2.2}$$

which leads to [10, 9]

$$\frac{1}{\rho}\frac{\partial E_z}{\partial \phi}-\frac{\partial E_\phi}{\partial z}=-j\omega\mu H_\rho, \tag{2.3a}$$

$$\frac{\partial E_\rho}{\partial z}-\frac{\partial E_z}{\partial \rho}=-j\omega\mu H_\phi, \tag{2.3b}$$

$$\frac{1}{\rho}\left[\frac{\partial\left(\rho E_\phi\right)}{\partial \rho}-\frac{\partial E_\rho}{\partial \phi}\right]=-j\omega\mu H_z, \tag{2.3c}$$

and expanded of Eq. 2.1b has the following form:

$$\frac{1}{\rho}\begin{vmatrix} \hat{\rho} & \rho\hat{\phi} & \hat{z} \\ \partial_\rho & \partial_\phi & \partial_z \\ H_\rho & \rho H_\phi & H_z \end{vmatrix}=j\omega\varepsilon\left(\hat{\rho}E_\rho+\hat{\phi}E_\phi+\hat{z}E_z\right), \tag{2.4}$$

which is leading to [10, 9]

$$\frac{1}{\rho}\frac{\partial H_z}{\partial \phi}-\frac{\partial H_\phi}{\partial z}=j\omega\varepsilon E_\rho, \tag{2.5a}$$

$$\frac{\partial H_\rho}{\partial z}-\frac{\partial H_z}{\partial \rho}=j\omega\varepsilon E_\phi, \tag{2.5b}$$

$$\frac{1}{\rho}\left[\frac{\partial\left(\rho H_\phi\right)}{\partial \rho}-\frac{\partial H_\rho}{\partial \phi}\right]=j\omega\varepsilon E_z. \tag{2.5c}$$

To simplify first we are considering the case that the propagation of cylindrical wave diverging from or converging to the axis of symmetry $\rho = 0$ (z axis). Note that the derivatives of the fields with respect to z vanish and Eq. 2.3 can be reduced to [10, 9]

$$\frac{1}{\rho}\frac{\partial E_z}{\partial \phi}=-j\omega\mu H_\rho, \tag{2.6a}$$

$$\frac{\partial E_z}{\partial \rho}=j\omega\mu H_\phi, \tag{2.6b}$$

$$\frac{1}{\rho}\left[\frac{\partial\left(\rho E_\phi\right)}{\partial \rho}-\frac{\partial E_\rho}{\partial \phi}\right]=-j\omega\mu H_z. \tag{2.6c}$$

In the same way the Eq. 2.5 changes to:

$$\frac{1}{\rho}\frac{\partial H_z}{\partial \phi} = j\omega\varepsilon E_\rho, \tag{2.7a}$$

$$\frac{\partial H_z}{\partial \rho} = -j\omega\varepsilon E_\phi, \tag{2.7b}$$

$$\frac{1}{\rho}\left[\frac{\partial\left(\rho H_\phi\right)}{\partial \rho} - \frac{\partial H_\rho}{\partial \phi}\right] = j\omega\varepsilon E_z. \tag{2.7c}$$

In the circular cylindrical coordinates there are two possible modes. One of them is named Transverse Electric (TE) polarized and the other of is called Transverse Magnetic (TM) polarized. For TE case, the nonzero fields, E_z, H_ϕ, and H_ρ in each single layer satisfy the three equations, Eqs. 2.6a-c. Solutions for these equations can be obtain for TM polarized case, which has non-zero components H_z, E_ϕ and E_ρ. In the case of TE-polarized wave, the electromagnetic field (E_z, H_ϕ, H_ρ) obeys the relations [9]:

$$H_\rho = j\frac{1}{\omega\mu}\frac{1}{\rho}\frac{\partial E_z}{\partial \phi}, \tag{2.8}$$

$$H_\phi = -j\frac{1}{\omega\mu}\frac{\partial E_z}{\partial \rho}, \tag{2.9}$$

$$\frac{\partial\left(\rho H_\phi\right)}{\partial \rho} - \frac{\partial H_\rho}{\partial \phi} = j\omega\varepsilon\rho E_z. \tag{2.10}$$

Therefore, Eq. 2.10 becomes

$$\frac{\partial}{\partial \rho}\left(-j\rho\frac{1}{\omega\mu}\frac{\partial E_z}{\partial \rho}\right) - \frac{\partial}{\partial \phi}\left(j\frac{1}{\omega\mu}\frac{\partial E_z}{\partial \rho}\right) = j\omega\varepsilon\rho E_z. \tag{2.11}$$

By using the Eq. 2.2, the governing equation for tangential electric field E_z is written as [10, 9]

$$\rho\frac{\partial}{\partial \rho}\left(\rho\frac{\partial E_z}{\partial \rho}\right) - \rho^2\frac{1}{\mu}\frac{\partial \mu}{\partial \rho}\frac{\partial E_z}{\partial \rho} + \frac{\partial}{\partial \phi}\left(\frac{\partial E_z}{\partial \phi}\right) + \omega^2\mu\varepsilon\rho^2 E_z = 0. \tag{2.12}$$

The solution of Eq. 2.12 can be obtained the method of separation of variables. By substituting $E_z(\rho,\phi) = V(\rho)\Phi(\phi)$ in Eq. 2.6 and using Eq. 2.12 we obtain:

$$\frac{1}{V}\rho\frac{\partial}{\partial \rho}\left(\rho\frac{\partial V}{\partial \rho}\right) - \frac{1}{V}\rho^2\frac{1}{\mu}\frac{\partial \mu}{\partial \rho}\frac{\partial V}{\partial \rho} + \frac{1}{\Phi}\frac{\partial^2 \Phi}{\partial \phi^2} + \omega^2\mu\varepsilon\rho^2 = 0, \tag{2.13}$$

which can be expressed as

$$\frac{1}{V}\rho\frac{\partial}{\partial \rho}\left(\rho\frac{\partial V}{\partial \rho}\right) - \frac{1}{V}\rho^2\frac{1}{\mu}\frac{\partial \mu}{\partial \rho}\frac{\partial V}{\partial \rho} + \omega^2\mu\varepsilon\rho^2 = -\frac{1}{\Phi}\frac{\partial^2 \Phi}{\partial \phi^2} = m^2. \tag{2.14}$$

For the angular part [9]:

$$\frac{\partial^2\Phi}{\partial\phi^2} + m^2\Phi = 0, \tag{2.15}$$

which has a solution $\Phi \sim e^{im\phi}$. In addition, the radial part of the Eq. 2.12 is given by

$$\frac{1}{V}\rho\frac{\partial}{\partial\rho}\left(\rho\frac{\partial V}{\partial\rho}\right) - \frac{1}{V}\rho^2\frac{1}{\mu}\frac{\partial\mu}{\partial\rho}\frac{\partial V}{\partial\rho} + \omega^2\mu\varepsilon\rho^2 = m^2, \tag{2.16}$$

where m which is called the azimuthal number and can be zero or a positive or negative integer. Then, we obtain:

$$E_z(\rho,\phi) = V(\rho)\Phi(\phi), \tag{2.17a}$$

$$H_\rho = -\frac{m}{\omega\mu}\frac{V(\rho)}{\rho}e^{jm\phi}, \tag{2.17b}$$

$$H_\phi = -\frac{1}{j\omega\mu}\frac{\partial V(\rho)}{\partial\rho}e^{jm\phi} = U(\rho)e^{jm\phi}. \tag{2.17c}$$

where the functions $U(\rho)$ and $V(\rho)$ are related by [10]

$$\frac{\partial V}{\partial\rho} = j\omega\mu U(\rho). \tag{2.18}$$

It should be noted that we have

$$\rho\frac{\partial}{\partial\rho}\left(\rho\frac{\partial V}{\partial\rho}\right) + \omega^2\mu\varepsilon\rho^2 V - m^2 V = 0, \tag{2.19}$$

and it can be reduced to

$$\rho\frac{\partial}{\partial\rho}\left(\rho\frac{\partial V}{\partial\rho}\right) + k^2 V - m^2 V = 0, \tag{2.20}$$

which is the standard Bessel's differential equation with the solution expressible as

$$V = AJ_m(k\rho) + BY_m(k\rho). \tag{2.21}$$

Here, A and B are constants, J_m is a Bessel function, Y_m is a Neumann function, and $k = \omega\sqrt{\mu\varepsilon}$ is the wave number in the layer. The function $U(\rho)$ can be found from Eq. 2.18 as [10, 9]

$$U(\rho) = \frac{k}{j\omega\mu}\left(AJ'_m(k\rho) + BY'_m(k\rho)\right), \tag{2.22}$$

$$U(\rho) = -jp\left(AJ'_m(k\rho) + BY'_m(k\rho)\right), \tag{2.23}$$

where $p = \sqrt{\varepsilon/\mu}$ is the intrinsic admittance of the layer, and the primes represent differentiation by the whole argument of the function (not just by ρ).

From Eqs. 2.17b and 2.17c we see that V and U determine the magnetic field components H_ρ and H_ϕ, respectively. Equations 2.7-2.10 enable us to construct a single layer matrix relating the electric and magnetic fields at its two interfaces. For instance, the matrix for the first layer (with refractive index n_1 and interfaces at $\rho = \rho_0$ and ρ_1) is written as [10, 9]

$$\begin{pmatrix} V(\rho_1) \\ U(\rho_1) \end{pmatrix} = \mathbb{M}_1 \begin{pmatrix} V(\rho_0) \\ U(\rho_0) \end{pmatrix} \tag{2.24}$$

The element of transfer matrix can be found by considering the relations Eqs. 2.21-2.24 when the vector $(V(\rho_0), U(\rho_0))$ has the special values $(1, 0)$ and $(0, 1)$. Solving the equation with the help of the identity [10]

$$J_m(x)Y'_m(x) - J'_m(x)Y_m(x) = 2/\pi x \tag{2.25}$$

therefore the single layer matrix

$$\mathbb{M}_1 = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} \tag{2.26}$$

has the following matrix elements

$$\begin{aligned}
M_{11} &= \frac{\pi}{2} k_1 \rho_0 \left[Y'_m(k_1\rho_0) J_m(k_1\rho_1) - J'_m(k_1\rho_0) Y_m(k_1\rho_1) \right] \\
M_{12} &= j\frac{\pi}{2}\frac{k_1}{p_1} \rho_0 \left[J_m(k_1\rho_0) Y_m(k_1\rho_1) - Y_m(k_1\rho_0) J_m(k_1\rho_1) \right] \\
M_{21} &= -j\frac{\pi}{2} k_1 \rho_0 p_1 \left[Y'_m(k_1\rho_0) J'_m(k_1\rho_1) - J'_m(k_1\rho_0) Y'_m(k_1\rho_1) \right] \\
M_{22} &= \frac{\pi}{2} k_1 \rho_0 \left[J_m(k_1\rho_0) Y'_m(k_1\rho_1) - Y_m(k_1\rho_0) J'_m(k_1\rho_1) \right]
\end{aligned}$$

where $p_1 = \sqrt{\varepsilon_1/\mu_1}$. Note that the determinant of the transfer matrix $\mathbb{M}$ in both cases is given by the ratio of the initial and final radii:

$$\det\begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} = |M| = M_{11}M_{22} - M_{12}M_{21} = \frac{\rho_0}{\rho} \tag{2.27}$$

Obviously, the matrix elements are dependent on the radii of the two interfaces. Similarly, for i th layer the matrix can be obtained by some simple replacements, i.e., $\rho_0 \to \rho_{i-1}, \rho_1 \to \rho_i, k_1 \to k_i = \omega\sqrt{\mu_i\varepsilon_i}$, and $p_1 \to p_i = \sqrt{\varepsilon_i/\mu_i}$. In addition, with structure being periodic, one has $\varepsilon_i = \varepsilon_1$ if i = odd, and $\varepsilon_i = \varepsilon_2$ if i = even. For an N-period bilayer periodic reflector we have, in total, $2N$ layers and therefore there should be $2N$ matrices in order to set up the total system matrix $\mathbb{M}$ that relates the first and final interfaces as

$$\begin{pmatrix} V(\rho_f) \\ U(\rho_f) \end{pmatrix} = \mathbb{M} \begin{pmatrix} V(\rho_0) \\ U(\rho_0) \end{pmatrix} \tag{2.28}$$

where

$$\mathbb{M} = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} = \mathbb{M}_{2N} \cdots \mathbb{M}_2\mathbb{M}_1 \tag{2.29}$$

Note that the analytic expressions for the matrix elements of $\mathbb{M}$ for an annular photonic crystal structures cannot be obtained because the elements of each single layer matrix are functions of the radii of the two interfaces. It thus has to be numerically calculated. Consider an outgoing wave incident on the interface between 0 and 1 , which we take to have radius $\rho = \rho_0$, and propagating to the medium f, which extends from $\rho = \rho_f$ to $\rho = \infty$. The amplitudes of the electric field and magnetic fields at ρ_0 and ρ_f can be written in terms of the amplitude reflection and transmission coefficients r_d and t_d and are related by the transfer matrix $\mathbb{M}$ defined in Eq. 2.28 and subsequence discussion:

$$\begin{pmatrix} V(\rho_0) \\ U(\rho_0) \end{pmatrix} = \mathbb{M}^{-1} \begin{pmatrix} V\left(\rho_f\right) \\ U\left(\rho_f\right) \end{pmatrix}$$

which is expressed as [10, 9]

$$\begin{pmatrix} 1 + r_d \\ -jp_0 C_{m0}^{(2)} - jp_0 C_{m0}^{(1)} t_d \end{pmatrix} = \mathbb{M}^{-1} \begin{pmatrix} t_d \\ -jp_f C_{mf}^{(2)} t_d \end{pmatrix} \tag{2.30}$$

where the inverse matrix in the above equation is defined by [9]

$$\mathbb{M}^{-1} = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix}^{-1} = \frac{1}{|\mathbb{M}|} \begin{pmatrix} M_{22} & -M_{21} \\ -M_{12} & M_{11} \end{pmatrix} \equiv \begin{pmatrix} M'_{11} & M'_{12} \\ M'_{21} & M'_{22} \end{pmatrix}. \tag{2.31}$$

Equation 2.30 enables us to calculate the reflection and transmissior coefficients for multilayered structure,

$$\begin{pmatrix} 1 + r_d \\ -jp_0 C_{m0}^{(2)} - jp_0 C_{m=}^{(1)} t_d \end{pmatrix} = \begin{pmatrix} M'_{11} & M'_{12} \\ M'_{21} & M'_{22} \end{pmatrix} \begin{pmatrix} t_d \\ -jp_f C_{mf}^{(2)} t_d \end{pmatrix} \tag{2.32}$$

$$1 + r_d = \left(M'_{11} - jp_f C_{mf}^{(2)} M'_{12}\right) t_d \tag{2.33}$$

$$- ip_0 C_{m0}^{(2)} - jp_0 C_{m0}^{(1)} r_d = \left(M'_{21} - jp_f C_{mf}^{(2)} M'_{22}\right) t_d \tag{2.34}$$

From Eqs. 2.33 and 2.34, we can get

$$\frac{-jp_0 C_{m0}^{(2)} - jp_0 C_{m0}^{(1)} r_d}{1 + r_d} = \frac{M'_{21} - jp_f C_{mf}^{(2)} M'_{22}}{M'_{11} - jp_f C_{mf}^{(2)} M'_{12}} \tag{2.35}$$

Hence, the reflection coefficients is given by

$$r_d = \frac{\left(M'_{21} + jp_0 C_{m0}^{(2)} M'_{11}\right) - jp_f C_{mf}^{(2)} \left(M'_{22} + jp_0 C_{m0}^{(2)} M'_{12}\right)}{\left(-jp_0 C_{m0}^{(1)} M'_{11} - M'_{21}\right) - jp_f C_{mf}^{(2)} \left(-jp_0 C_{m0}^{(1)} M'_{12} - M'_{22}\right)} \tag{2.36}$$

From Eqs 2.33,

$$t_d = \frac{1 + r_d}{M'_{11} - jp_f C_{mf}^{(2)} M'_{12}} \tag{2.37}$$

where the numerator

$$
\begin{aligned}
1 + r_d =& 1 + \frac{\left(M'_{21} + jp_0 C^{(2)}_{m0} M'_{11}\right) - jp_f C^{(2)}_{mf}\left(M'_{22} + jp_0 C^{(2)}_{m0} M'_{12}\right)}{\left(-jp_0 C^{(1)}_{m0} M'_{11} - M'_{21}\right) - jp_f C^{(2)}_{mf}\left(-jp_0 C^{(1)}_{m0} M'_{12} - M'_{22}\right)} \\
=& \frac{\left(-jp_0 C^{(1)}_{m0} + jp_0 C^{(2)}_{m0}\right)\left(M'_{11} - jp_f C^{(2)}_{mf} M'_{12}\right)}{\left(-jp_0 C^{(1)}_{m0} M'_{11} - M'_{21}\right) - jp_f C^{(2)}_{mf}\left(-jp_0 C^{(1)}_{m0} M'_{12} - M'_{22}\right)}
\end{aligned} \tag{2.38}
$$

Therefore the transmission coefficient is given by

$$
t_d = \frac{4\sqrt{\varepsilon_0/\mu_0}}{\pi k \rho_0 H^{(2)}_m(k_0\rho_0) H^{(1)}_m(k_0\rho_0)\left[\left(-jp_0 C^{(1)}_{mi} M'_{11} - M'_{21}\right) - jp_f C^{(2)}_{mf}\left(-jp_0 C^{(1)}_{m0} M'_{12} - M'_{22}\right)\right]} \tag{2.39}
$$

where $M'_{11}, M'_{12}, M'_{21}$ and M'_{22} are the matrix elements of the inverse matrix of $\mathbb{M}$, $k = \omega\sqrt{\mu_0\varepsilon_0}$ is the free-space wave number, and [9]

$$
C^{(1,2)}_{ml} = \frac{H^{(1,2)'}_m(k_l\rho_l)}{H^{(1,2)}_m(k_l\rho_l)}, l = 0, f \tag{2.40}
$$

In the above equations, $H^{(1)}_m$ and $H^{(2)}_m$ are the Hankel function of the first and second kind. Equations 2.36 and 2.39 then leads to the reflectance R and the transmittance T, i.e.,

$$
R = |r_d|^2, \quad T = \frac{n_f}{n_0}|t_d|^2, \tag{2.41}
$$

where n_0 and n_f are respectively the refractive indices of the starting and the final media. The results for TM wave are also obtainable by simply replacing $\varepsilon \leftrightarrow \mu$, and $j \leftrightarrow -j$ in the formulas of TE polarization wave. Finally, before going to the conclusion it should be mentioned that the method that is presented in this paper has used before by different research groups to study the optical properties of different annular photonic crystal structures; There structures composed of different materials such as dielectric [9, 7, 1], semiconductor [4], high-temperature superconductor [16, 5, 2], single- and double-negative metamaterials [8, 6, 17], graphene [14], and plasma [13, 18].

3 Conclusion

In this paper, we provided and full details of the transfer matrix method which can be suitable for annular photonic multilayer analysis. The work of conversion a flat type transfer matrix method to the annular type is based on the principle of conversion of the system from a Cartesian coordinate system to the cylindrical coordinate system. We also used the Bessel function as well as the Neumann function for presenting the transfer matrix in a suitable and understandable form. We expect readers will get a clear view from the article for designing their one-dimentional annular photonic system.

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