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Improvement of Periodic Rates in the Average Internal Rate of Return Method

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Abstract: One of the most important decisions in project appraisal and enterprise economic policy under constrained and inconsistent circumstances is to select an option among several others. If all variables can be measured by a measure called money, methods such as Internal Rate of Return (IRR) and Average Internal Rate of Return (AIRR) can be used. The AIRR method is a mode developed by the IRR method. However, this method (AIRR) may occasionally result in unrealistic (huge) periodic rates. This article adopted a simple technique to address this problem. Finally, the technique is further explained by solving several numerical examples. According to the results, the proposed method led to distribution of large periodic rates over other periodic rates producing slightly unrealistic results. The results of research indicated that the proposed method causes distribution of large size periodic rates between the rates of other periods so that the new values do not go far beyond reality.

Keywords: Engineering Economics (EE), Internal Rate of Return (IRR), Present Value (PV), Average Internal Rate of Return (AIRR), Equivalent Annual Value (EAV)
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1 Introduction

Mathematical-based economic assessment methods are among important tools for deciding how to select projects [16]. Mathematics has provided various sciences with appropriate facilities for accurate analyzing

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and describing relations between phenomena. The quantitative nature of most economic variables along with factors such as the need for planning caused the development of application of mathematics in the economy. Simplification, abbreviation, and in particular the precision of mathematics in providing economic theories have led economists to use this tool to present their theories[2]. Development of mathematical methods in evaluation of economic issues has led to the formation of a branch of economics called engineering economics. Engineering Economics is a collection of mathematical techniques to simplify economic comparison of industrial projects. More simply, engineering economics is a decision-making tool for selecting the most economical projects[12]. Today, it is obvious that the value of money is constantly changing over time. This has led to the emergence of balance methods and more commonly, the method of internal rate of return, in monetary and financial assessments. In this method, a project is accepted or rejected based on the internal rate of return[15].

The IRR method facilitates decision-making through a simple comparison between IRR and the minimum absorption rate (the expected rate of decision-maker). However, IRR is associated with several drawbacks including:

- The difficulty of calculating rates of return [14].
- The emergence of several rates simultaneously[6].
- Imaginary rates of return[13].
- Evaluation of projects with uncertainties (probable) by IRR [4].

In recent years, much scientific effort has been made to overcome these drawbacks. Below, some of the most important studies are reviewed.

1.1 Literature Review

Russell and Richard (1982) proved that multi-rate projects can be recognized by considering cash flows (costs and revenues). To enjoy of a unique rate, the relevant cash flow of economic plans should meet specific conditions in terms of costs and revenues[14].

Neff and Reif (1996) found that increasing period size is associated with an increase in problems related to the calculation of rates of return. In this regard, they presented an efficient algorithm for performing the relevant calculations[13].

Magni(2010) introduced a different approach and showed that IRR changes in all periods. Therefore, all drawbacks of the IRR method can be resolved by calculating the simple arithmetic average of rate of return for each period, leading to results fully consistent with the PV method[10].

Johnston et al (2013) the present value (PV) method is a holistic approach compared to IRR. Methods derived from the present value method give same results and only the IRR method involves a contradiction in the evaluation of projects[9].

Balyeatet al. (2013) studied the issue of multi-rate projects. They proposed the use of modified internal rate of return (MIRR). The MIRR method transfers negative cash flows to the start period while transferring positive cash flows to the end year, thus making it possible to obtain a single rate of return and prevent the project from becoming multi-rate[1].

Jafari and Sheykhan (2015) reviewed multi-rate projects and presented a new method, which can properly evaluate projects by considering each of the rates and an auxiliary cash flow[6].

The research of Ivanovic et al (2015), like the research of Balyeat et al(2013), emphasizes the use of the MIRR method.The study was conducted on the assessment of agricultural projects. To prevent the multi-rate issue, they used the MIRR method[5].

Jafari (2020) simplified the AIRR method to calculate periodic rates of return without direct formation of an auxiliary cash flow[8].

Maravas and Pantouvakis (2018) studied the internal rate of return (IRR) method. According to them, the fuzzy present value (FPV) method has been widely studied, whereas fuzzy internal rate of return (FIRR) has received less attention. Therefore, the authors proposed three-dimensional diagrams to estimate FIRR. In general, they concluded that the new approach is very promising for molding uncertainty in project assessment for both project managers and stakeholders[11].

1.2 Summary

Although the new method proposed by Magni, called the Average Internal Rate of Return (AIRR), is capable of resolving all problems of the rate of return, periodic rates may be unbelievable in some cases. This phenomenon may often occur in the final year of projects of different lifetimes. Simply, this problem arises from the method used for equalizing lifetime of projects. To resolve this problem, AIRR is first fully discussed. Then, a new method is proposed for development of Magni's method.

2 Internal Rate of Return

A cash flow stream is a finite or infinite sequence $X = (x_0, x_1, \dots)$ of monetary values. The monetary amount received initially is x_0, and the amount received after period t is x_t. For a finite stream $X = (x_0, x_1, \dots, x_n)$, we assume the horizon n is chosen so that $x_n \neq 0$. The net present value $PV(X|r)$ of a cash flow stream X at interest rate r is given by[7]:

$$PV(X|r) = \sum_{t=0}^{n} \frac{x_t}{(1+r)^t} \qquad (1)$$

defined for proper interest rates $r > -1$. For a cash flow stream X, let $IRR(X)$ be the set of all interest rates r which make $PV(X|r) = 0$. (Note that $IRR(X)$ cannot contain -1 because $PV(X|r = -1)$ is undefined) For finite streams $X = (x_0, x_1, \dots, x_n)$, the present value function $PV(X|r)$ is a degree-n polynomial in $(1 + r)^{-1}$, so $IRR(X)$ can contain anywhere from 0 to n distinct values. If $k \in IRR(X)$, then we will call k an internal rate of return for X.

As is well known, for conventional cash flows X that are negative for the first few periods but positive thereafter, the internal rate of return exists and is unique. Moreover, the internal rate of return is the largest interest rate at which the cash flow shows a discounted net profit. So if $IRR(X)$ exceeds the available market rate of interest r, then $PV(X|r) > 0$ and the investment which generates the cash flow X is worthwhile. Conversely, if the internal rate of return is smaller than the market rate r, then one is better off investing at the market rate r. This is the fundamental justification for the use of internal rate of return[3].

3 Average Internal Rate of Return (AIRR)

With this view that rates of return do not remain constant over different periods, Magni presented a solution called the average internal rate of return to overcome IRR problems. To easily calculate the rates of return for each period, Magni provided conditions using a virtual flow with a lifetime of one unit less than the lifetime of the project examined. In this method, it is only necessary to construct the auxiliary flow C by equation (2) and obtain the rate of return of the period t from the formula $k_t = \frac{x_t + c_t}{c_{t-1}} - 1$. After identifying all periodic return rates, their arithmetic mean is calculated. This was called the average internal rate of return (AIRR) by Magni. Equation (3) is used for calculating this index[10].

$$C = (c_0 = -x_0, c_1 = -x_0(1+r), c_2 = -x_0(1+r)^2, \dots, c_{T-1} = -x_0(1+r)^{T-1}) \quad (2)$$

$$AIRR(X) = \bar{k} = \frac{1}{T}\sum_{t=1}^{T} k_t \quad (3)$$

3.1 Determining the economic value of a project through AIRR

First, AIRR is calculated and then the following five cases are examined [8]:
If the cash flow is negative in the zero period (x_0) and AIRR is greater than the cost of capital (r), it is better to invest.
If the cash flow is positive in the zero period (x_0) and AIRR is greater than the cost of capital (r), it is better not to invest.
If the cash flow is negative in the zero period (x_0) and AIRR is lower than the cost of capital (r), it is better not to invest.
If the cash flow is positive in the zero period (x_0) and AIRR is lower than the cost of capital (r), it is better to invest.
If AIRR is equal to the cost of capital (r), it does not matter whether the investment is done or not.

3.2 Ranking projects by AIRR

To select the best project from several ones, AIRR is first calculated for each project and then the following two modes are examined:
If the auxiliary cash flow is positive in the zero period (c_0), then the project with a higher AIRR is better.
If the auxiliary cash flow is negative in the zero period (c_0), then the project with lower AIRR is better.

3.3 Magni's method for equalizing lifetime of projects

Assume that the value of one or more projects of the projects $X^i = \left(x_0^i, x_1^i, \dots, x_t^i, \dots, x_{T_i-1}^i, x_{T_i}^i\right); i = 1,2,\dots,m$ is not equal to the rest of the projects $(\exists i, j: x_0^i \neq x_0^j)$ in the zero period. We define the variable τ as one of the cash flow valuesin the zero period, namely $\tau = x_0^i \neq 0$. The auxiliary cash flow is definedas $Z^i = \left(z_0^i = \tau - x_0^i, z_1^i = 0, z_2^i = 0, \dots, z_{T-1}^i = 0, z_T^i = (x_0^i - \tau).(1+r)^T\right); T = \max_i T_i$. Projects as like $X_{new}^i = X^i + Z^i$; $i = 1,2,\dots,m$ are replacing the initial projects (X^i). Clearly, all new cash flows (X_{new}^i) have the same lifetime (T) [10].
Note that $PV\left(X^i \middle| r\right) = PV\left(X_{new}^i | r\right)$.

4 The proposed method for equalizing lifetime of projects

Assume that the value of one or more projects of the projects $X^i = \left(x_0^i, x_1^i, \dots, x_t^i, \dots, x_{T_i-1}^i, x_{T_i}^i\right); i = 1,2,\dots,m$ is not equal to the rest of the projects in the zero period $(\exists i, j: x_0^i \neq x_0^j)$. We define the variable τ as one of the cash flow values in the zero period, namely $\tau = x_0^i \neq 0$. The auxiliary cash flow is defined as $Z^i = \left(z_0^i = \tau - x_0^i,\ z_1^i = \varphi_i\ ,\ z_2^i = \varphi_i\ ,\ \dots\ ,\ z_{T-1}^i = \varphi_i\ ,\ z_T^i = \varphi_i\right)\ ;\ T = \max_i T_i$. Projects like $X_{new}^i = X^i + Z^i\ ; i = 1,2,\dots,m$ are replacing the initial projects (X^i). Clearly, all new cash flows (X_{new}^i) have the same lifetime (T). The parameter φ_i is calculated as follows:

$$\begin{cases} U^i = \left(u_0^i = x_0^i - \tau, u_1^i = 0, u_2^i = 0, \dots, u_{T-1}^i = 0, u_T^i = 0\right) \\ \varphi_i = EAV\left(U^i|r\right) = u_0^i\left(\frac{r(1+r)^T}{(1+r)^T-1}\right) \end{cases} \qquad (4)$$

In equation (4) , $EAV\left(U^i|r\right)$ is the same equivalent annual value U^i([15] for further details).
Note that $PV\left(X^i|r\right) = PV\left(X_{new}^i|r\right)$.

Numerical Example 1

Consider three projects with the following cash flows:
$X^1 = (-100, 40, 0, 80, 0)$
$X^2 = (-100, 60, 10, 10, 20)$
$X^3 = (-10, 30, -25, 0, 0)$

If the cost of capital is assumed to be 5% $(r = 0.05)$, then:
$PV\ (X^1|5\%)\ =\ 7.2$
$PV\ (X^2|5\%)\ =\ -8.69$
$PV\ (X^3|5\%)\ =\ -4.1$

In Magni's method:
$Z^1 =\ (0, 0, 0, 0,0)$
$Z^2 =\ (0, 0, 0, 0,0)$
$Z^3 =\ (-90, 0, 0, 0, 109.4)$

We also have:
$X_{new}^1 = X^1 + Z^1 = (-100, 40, 0, 80, 0)$
$X_{new}^2 = X^2 + Z^2 = (-100, 60, 10, 10, 20)$
$X_{new}^3 = X^3 + Z^3 = (-100, 30, -25, 0, 109.4)$

We also have:
$C^1, C^2, C^3 = (100, 105, 110.25, 115.76)$
Table 1 shows the rates of return for each period and AIRR calculated by Magni's method.

Table 1: The rates of return for each period and AIRR calculated by Magni's method

Project	Periodic Rates				AIRR	Rank
	k_1	k_2	k_3	k_4		
X_{new}^1	45%	5%	77.56%	-100%	6.89%	1
X_{new}^2	65%	14.5%	14.1%	-82.7%	2.72%	3
X_{new}^3	118%	14.5%	5%	-100%	3.92%.	2

The maximum and minimum rates in the project $X^3$are 118% and -100%, respectively.
Below, the rates are calculated using the method proposed in this paper. Therefore, we can say:

$$\begin{cases} \mathrm{U}^1 = (\mathrm{u}_0^1 = -100 - (-100) = 0, \mathrm{u}_1^1 = 0, \mathrm{u}_2^1 = 0, \mathrm{u}_3^1 = 0, \mathrm{u}_4^1 = 0) \\ \varphi_1 = EAV(U^1|0.05) = 0 \times \left(\dfrac{0.05(1+0.05)^4}{(1+0.05)^4 - 1}\right) = 0 \end{cases}$$

$$\begin{cases} \mathrm{U}^2 = (\mathrm{u}_0^2 = -100 - (-100) = 0, \mathrm{u}_1^2 = 0, \mathrm{u}_2^2 = 0, \mathrm{u}_3^2 = 0, \mathrm{u}_4^2 = 0) \\ \varphi_2 = EAV(U^2|0.05) = 0 \times \left(\dfrac{0.05(1+0.05)^4}{(1+0.05)^4 - 1}\right) = 0 \end{cases}$$

$$\begin{cases} \mathrm{U}^3 = (\mathrm{u}_0^3 = -10 - (-100) = 90, \mathrm{u}_1^3 = 0, \mathrm{u}_2^3 = 0, \mathrm{u}_3^3 = 0, \mathrm{u}_4^3 = 0) \\ \varphi_3 = EAV(U^3|0.05) = 90 \times \left(\dfrac{0.05(1+0.05)^4}{(1+0.05)^4 - 1}\right) = 25.3811 \end{cases}$$

We also have:
$\mathrm{Z}^1 = (\mathrm{z}_0^1 = 0, \mathrm{z}_1^1 = 0, \mathrm{z}_2^1 = 0, \mathrm{z}_3^1 = 0, \mathrm{z}_4^1 = 0)$
$\mathrm{Z}^2 = (\mathrm{z}_0^2 = 0, \mathrm{z}_1^2 = 0, \mathrm{z}_2^2 = 0, \mathrm{z}_3^2 = 0, \mathrm{z}_4^2 = 0)$
$\mathrm{Z}^3 = (\mathrm{z}_0^3 = \tau - x_0^3 = -100 - (-10) = -90, \mathrm{z}_1^3 = 25.3811, \mathrm{z}_2^3 = 25.3811, \mathrm{z}_3^3 = 25.3811, \mathrm{z}_4^3 = 25.3811)$

We also have:
$X_{new}^1 = X^1 + Z^1 = (-100, 40, 0, 80, 0)$
$X_{new}^2 = X^2 + Z^2 = (-100, 60, 10, 10, 20)$
$X_{new}^3 = X^3 + Z^3 = (-10 + (-90), 30 + 25.3811, -25 + 25.3811, 0 + 25.3811, 0 + 25.3811)$

The expressions are summarized as follows after simplifying:
$X_{new}^1 = X^1 + Z^1 = (-100, 40, 0, 80, 0)$
$X_{new}^2 = X^2 + Z^2 = (-100, 60, 10, 10, 20)$
$X_{new}^3 = X^3 + Z^3 = (-100, 55.3811, 0.3811, 25.3811, 25.3811)$

We also have:

$C^1, C^2, C^3 = (100, 105, 110.25, 115.76)$
Table 2 shows the rates of return for each period and AIRR calculated by the new method.

Table 2: The rates of return for each period and AIRR calculated by the new method

Project	Periodic Rates				AIRR	Rank
	k_1	k_2	k_3	k_4		
X^1_{new}	45%	5%	77.56%	-100%	6.89%	1
X^2_{new}	65%	14.5%	14.1%	-82.7%	2.72%	3
X^3_{new}	60.38%	5.36%	28.02%	-78.07%	3.92%	2

The maximum and minimum rates in the project $X^3$are 60.38% and -78.07%, respectively. It can be concluded that the production of large size periodic rates is less evident and they are closer to reality in the new method.

Numerical Example 2

Consider three projects with the following cash flows:
$X^1 = (-100, 40, 0, 80, 0,0)$
$X^2 = (-100, 60, 10, 10, 20,15)$
$X^3 = (-10, 30, -25, 0, 0,0)$

If the cost of capital is assumed to be 5% ($r = 0.05$), we can say:
$Z^1 = (0, 0,0,0, 0)$
$Z^2 = (0, 0,0,0, 0)$
$Z^3 = (-90, 25.3811,25.3811,25.3811, 25.3811,25.3811)$

We also have:
$X^1_{new} = X^1 + Z^1 = (-100, 40, 0, 80, 0)$
$X^2_{new} = X^2 + Z^2 = (-100, 60, 10, 10, 20)$
$X^3_{new} = X^3 + Z^3 = (-100, 55.3811,0.3811, 25.3811, 25.3811,25.3811)$

We also have:
$C^1 = C^2 = C^3 = (100, 105, 110.25, 115.76,121.55)$
As in the previous example, the rest of calculations are given in Table3.

Table 3: The rates of return for each period and AIRR calculated by the new method

Project	Periodic Rates					AIRR	Rank
	k_1	k_2	k_3	k_4	k_5		
X^1_{new}	45%	5%	77.56%	5%	-100%	6.51%	1
X^2_{new}	65%	14.5%	14.1%	22.3%	-87.7%	5.64%	2
X^3_{new}	35%	-18.8%	5%	5%	-100%	-14.76%	3

Figure 1shows the periodic rate of return.

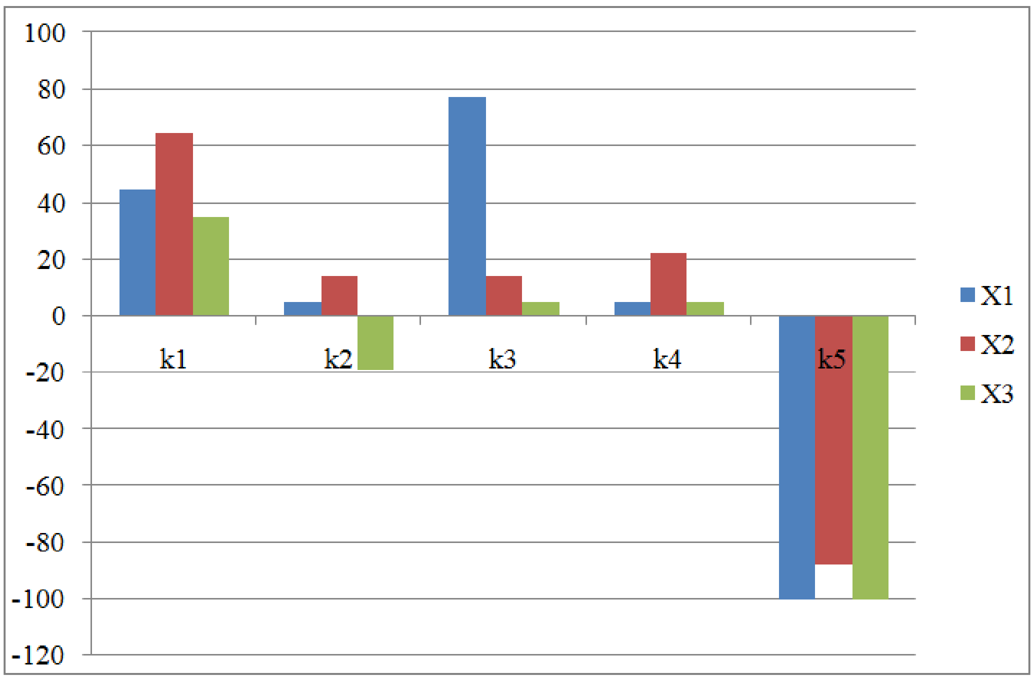

Figure 1: periodic rate of return.

Numerical Example 3

Consider a project with the following cash flow:

$$X = (-460, +48, +48, +48, +48, +48, +48, +48, +48, +48, +48, +48, +48)$$

The cash flow diagram of this project is shown in Figure 2.

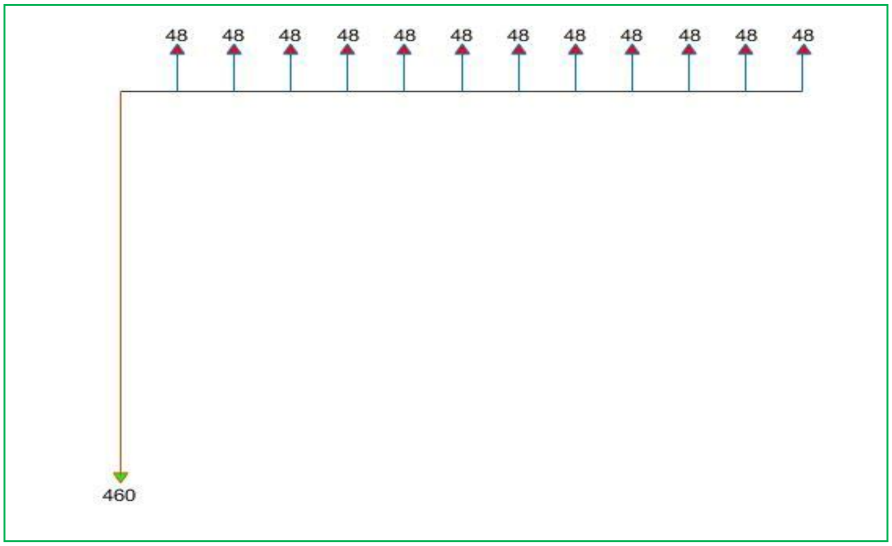

Figure 2: Diagram related to the financial flow of the printer purchase project

This is the cash flow for the project of buying a printer for a private company.
The cost of buying this printer is $460. It is predicted that the income gained from this printer would be $48 at the end of each period (every month). Now, the question is, given the said conditions, whether buying this printer be profitable for the company (Based on the market research, the cost of capital for every period is estimated to be 1.5%).
To answer this question, first we need to calculate AIRR using the recommended approach and then, compare it with the cost of capital. The results of this calculation are presented in Table 4.

Table 4: Returns for Every Period along with AIRR Calculated with the New Method

Periodic Rates												*New-AIRR*
k_1	k_2	k_3	k_4	k_5	k_6	k_7	k_8	k_9	k_{10}	k_{11}	k_{12}	
11.1%	11.0%	10.8%	10.7%	10.5%	10.4%	10.3%	10.1%	10.0%	9.9%	9.8%	-91.9%	1.9%

As can be seen in Table 4, AIRR is greater than the cost of capital ($r = 1.5\% \leq AIRR = 1.9\%$). Therefore, it can be said that buying this printer, with the said conditions, is profitable for the buyer and this investment is justifiable.

5 Conclusion

IRR is popular among both practitioners and academics. Although this method is associated with several disadvantages, most practitioners use it without considering the disadvantages. This can lead to incorrect decisions. In this article, a relatively new method called the average internal rate of return (AIRR) was introduced, which may also have challenges for decision-makers. Therefore, a new approach was provided to resolve the gap. Finally, numerical examples were presented and solved. The results indicated that the proposed method causes distribution of large size periodic rates between the rates of other periods so that the new values do not go far beyond reality. The new approach also enables a more accurate comparison of periodic rates of return for two or multiple projects.

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