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Numerical solution of eight order boundary value problems using Chebyshev polynomials

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Abstract : First-kind Chebyshev polynomials are used as the basis functions in this study to present the approximations to the eighth-order boundary-value problems. The problem is reduced using the suggested approach into a set of linear algebraic equations, which are then solved to determine the unknown constants. To demonstrate the application and effectiveness of the strategy, analytical results are provided using tables and graphs for three examples. The results obtained using the proposed method reveal that it is simple and outperforms comparable solutions in the literature.

Keywords: First kind Chebyshev polynomials; Boundary value problems; Collocation technique; Approximate solution
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1 Introduction

In the fields of applied mathematics, engineering, applied physics, fluid dynamics, astronomy, hydrodynamics and hydromagnetic stability, beam and long wave theory, and applied physics, Boundary Value Problems (BVPs) are encountered frequently. It is fairly challenging to solve these boundary value problems analytically, hence employing a numerical method is necessary to tackle these kinds of problems. Here are only a few authors who have provided numerical methods for solving the BVPs. The numerical solution of special eight-order BVPs by the modified decomposition method [30], differential quadrature solutions to boundary-value differential equations of eighth order [13], an effective method for approximating solutions to eighth-order BVPs [6], non polynomial spline methodology is used to solve eighth-order BVPs [23], the homotopy perturbation method is used to solve eighth-order BVPs [5], non-polynomial splines are used as solutions to sixth-order BVPs [25], utilizing the variational iteration technique, eighth order BVPs are solved in [24]. A new cubic B-spline method for linear fifth order BVPs [12], solution of sixth-order BVPs by Collocation method [3], numerical solutions of fifth and sixth order nonlinear BVPs by Daftardar Jafari method [4], Galerkin and collocation methods with Quintic B-splines are used to numerically solve eighth order boundary value problems [9, 15], for the solution of first and second-order

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ordinary differential equations, the tau-collocation approximation method [14] is used, numerical solution of two-point BVPs by interpolating subdivision schemes , subdivision schemes based collocation algorithms for solution of fourth order BVPs [8]. He's polynomials are used in a variational iteration method to solve a seventh-order BVPs [22], numerical solutions of a generalized nth order BVPs using power series approximation method [14], Cubic B-spline solution of nonlinear sixth order BVPs [10], Cubic B-spline numerical approximation for the solution of linear sixth-order BVPs [11], 12th-order boundary value problems are solved using the modified MVIM using Chebyshev polynomials [26]. Other related studies can be found in [28, 17, 7, 1, 29, 2, 20, 18, 19, 21]. As a result of the study mentioned above, this work take into consideration the numerical solution of eight order BVP:

$$\begin{aligned} &v^{\mathrm{viii}}(t)+\mu_1(t)v^{\mathrm{vii}}(t)+\mu_2(t)v^{\mathrm{vi}}(t)+\mu_3(t)v^{v}(t)+\mu_4(t)v^{\mathrm{iv}}(t)+\\ &\mu_5(t)v^{\mathrm{iii}}(t)+\mu_6(t)v^{\mathrm{ii}}(t)+\mu_7(t)v^{i}(t)+\mu_8(t)v(t)=g(t), t\in[a,b] \end{aligned} \tag{1.1}$$

with boundary conditions

$$v^{i}(a)=\alpha_i, v^{i}(b)=\beta_i, i=0,1,2, \tag{1.2}$$

where α_0 , α_1 , α_2 and β_0 , β_1, β_2 are given real constants, $\mu_i(t)$, $i=0,1,2,\cdots\ n$ and $g(t)$ are known functions on the an interval $t\in$ [a,b]and $v(t)$ is the unknown function to be determined.

2 Basic definitions

2.1 Chebyshev polynomials of the first kind

Definition 2.1: The Chebyshev polynomials of the first kind valid in internal [-1,1] denoted by $Q_n(t)$ is defined by the relation

$$Q_n(t)=cos[n\cos^{-1}(t)]. \tag{2.1}$$

Where n is the non-negative integers and the recurrence relation is given as

$$Q_{n+1}(t)=2xQ_n(t)-Q_{n-1}(t), n\geq 1 \tag{2.2}$$

$$Q_0(t)=1, Q_1(t)=t, Q_2(t)=2t^2-1, Q_3(t)=4t^3-3t, Q_4(t)=8t^4-8x^2+1. \tag{2.3}$$

The shifted equivalent of it that valid in $\in[0,1]$ are given as:

$$\begin{aligned} {Q^*}_0(t)&=1\\ {Q^*}_1(t)&=2t-1\\ {Q^*}_2(t)&=8t^2-8t+1\\ {Q^*}_3(t)&=32t^3-48t^2+18t-1\\ {Q^*}_4(t)&=128t^4-256t^3+160t^2-32t+1 \end{aligned} \tag{2.4}$$

Definition 2.2: We define absolute error as follows; Absolute Error=$|V(t)-v(t)|$; $0\leq t\leq 1$, where V(t) is the exact solution and $v(t)$ is the approximate solution

3 Method

The work assumed an approximate solution by means of the first kind Chebyshev polynomials in the form:

$$v(t)=\sum_{i=0}^{n}{Q^*}_i(t)a_i \tag{3.1}$$

The unknown constants to be determined are

$$a_{\mathrm{i}}\ ,\ i=0(1)n.$$

Thus, differentiating (3.1) with respect to nth-times as functions of t and substitute the resulted solution into (1.1) we have,

$$\begin{aligned}\sum_{i=0}^{n} Q^{\mathrm{viii}}(t)a_i+\mu_1(t)\sum_{i=0}^{n} Q^{\mathrm{vii}}(t)a_i+\mu_2(t)\sum_{i=0}^{n} Q^{\mathrm{vi}}(t)a_i+\\ \mu_3(t)\sum_{i=0}^{n} Q^{v}(t)a_i+\mu_4(t)\sum_{i=0}^{n} Q^{v}(t)a_i+\mu_5(t)\sum_{i=0}^{n} Q^{\mathrm{iv}}(t)a_i+\\ \mu_6(t)\sum_{i=0}^{n} Q^{\mathrm{iii}}(t)a_i+\mu_7(t)\sum_{i=0}^{n} Q^{\mathrm{ii}}(t)a_i+\mu_8(t)\sum_{i=0}^{n} Q^{i}(t)a_i=g(t)\end{aligned} \tag{3.2}$$

Let

$$\begin{aligned}\eta(t)=\sum_{i=0}^{n} Q^{\mathrm{viii}}(t)a_i,\tau^*(t)=\sum_{i=0}^{n} Q^{\mathrm{vii}}(t)a_i\tau(t)=\sum_{i=0}^{n} Q^{\mathrm{vi}}(t)a_i,\\ \varsigma^*(t)=\sum_{i=0}^{n} Q^{v}(t)a_i,\varsigma(t)=\sum_{i=0}^{n} Q^{\mathrm{iv}}(t)a_i,\xi(t)=\sum_{i=0}^{n} Q^{\mathrm{iii}}(t)a_i,\\ \gamma(t)=\sum_{i=0}^{n} Q^{\mathrm{ii}}(t)a_i,(\chi(t)=\sum_{i=0}^{n} Q^{i}(t)a_i),\omega(t)=\sum_{i=0}^{n} Q(t)a_i\end{aligned} \tag{3.3}$$

The linear algebraic system of equations in (n+1) unknown constants $a\text{'}_i s$ is obtained by Collocating (3.2) at the evenly spaced point $t_i=a+\frac{(b-a)i}{n},(i=0(1)n)$. Additional equations are obtained from Eq. (1.2), which are represented in matrix form:

$$\begin{pmatrix} W_{11} & W_{12} & W_{13} & W_{14} & \cdots & \mathrm{W}_{1n}\\ W_{21} & W_{22} & W_{23} & W_{24} & \cdots & \mathrm{W}_{2n}\\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots\\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots\\ W_{m1} & W_{m2} & W_{m3} & \mathrm{W}_{m4} & \dots & \mathrm{W}_{\mathrm{mn}}\\ {W_{11}}^0 & {W_{12}}^0 & {W_{13}}^0 & {W_{14}}^0 & \dots & {W_{1n}}^0\\ {W_{21}}^1 & {W_{22}}^1 & {W_{23}}^1 & {W_{24}}^1 & \dots & {W_{2n}}^0\\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots\\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots\\ {W_{m1}}^n & {W_{m2}}^{\mathrm{n}} & {W_{m3}}^{\mathrm{n}} & {W_{m4}}^{\mathrm{n}} & \dots & {\mathrm{W}_{\mathrm{mn}}}^n \end{pmatrix} \begin{pmatrix} a_0\\ a_1\\ \vdots\\ \vdots\\ \vdots\\ \vdots\\ \vdots\\ \vdots\\ \vdots\\ \vdots\\ \vdots\\ a_n \end{pmatrix} = \begin{pmatrix} X_{11}\\ X_{21}\\ \vdots\\ \vdots\\ X_{\mathrm{mn}}\\ {X_{11}}^0\\ {X_{22}}^1\\ \vdots\\ \vdots\\ {X_{\mathrm{mn}}}^n \end{pmatrix} \tag{3.4}$$

where W_{is} and ${W_{\mathrm{is}}}^*$ are the coefficients of a_{is} given as

$$\begin{aligned}W_{11},W_{12},W_{13},\cdots W_{1n}=\eta\left(t_1\right)+\mu_1\left(t_1\right)\tau^*\left(t_1\right)+\mu_2\left(t_1\right)\tau\left(t_1\right)\\ +\mu_3\left(t_1\right)\varsigma^*\left(t_1\right)+4\left(t_1\right)\varsigma\left(t_1\right)+\mu_4\left(t_1\right)\xi\left(t_1\right)+\mu_5\ \gamma\left(t_1\right)+\mu_6\left(t_1\right)\chi\left(t_1\right)+\\ \mu_7\left(t_1\right)\tau\left(t_1\right)+\mu_8\left(t_1\right)\omega\left(t_1\right)\end{aligned} \tag{3.5}$$

$$W_{21}, W_{22}, W_{23}, \cdots W_{2n} = \eta\left(t_2\right) + \mu_1\left(t_2\right)\tau^*\left(t_2\right) + \mu_2\left(t_2\right)\tau\left(t_2\right) + \mu_3\left(t_2\right)\varsigma^*\left(t_2\right) + \mu_4\left(t_2\right)\varsigma\left(t_2\right) + \mu_4\left(t_2\right)\xi\left(t_2\right) + \mu_5\ \gamma\left(t_2\right) + \mu_6\left(t_2\right)\chi\left(t_2\right) + \mu_7\left(t_2\right)\tau\left(t_2\right) + \mu_8\left(t_2\right)\omega\left(t_2\right). \tag{3.6}$$

${W_{11}}^0,\ {W_{12}}^0, {W_{13}}^0, \cdots\ {W_{1n}}^0$ are values of $v^i(a)$ and $v^i(b)$, and $X_{\rm is}$ are values of $f(t_i)$. Let equation (3.4) be:

$$G\left(t_i\right)A = B(t_i) \tag{3.7}$$

Where

$$G\left(t_i\right) = \begin{pmatrix} W_{11} & W_{12} & W_{13} & W_{14} & \cdots & \mathrm{W}_{1n} \\ W_{21} & W_{22} & W_{23} & W_{24} & \cdots & \mathrm{W}_{2n} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ W_{m1} & W_{m2} & W_{m3} & \mathrm{W}_{m4} & \ldots & \mathrm{W}_{\rm mn} \\ {W_{11}}^0 & {W_{12}}^0 & {W_{13}}^0 & {W_{14}}^0 & \ldots & {W_{1n}}^0 \\ {W_{21}}^1 & {W_{22}}^1 & {W_{23}}^1 & {W_{24}}^1 & \ldots & {W_{2n}}^0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ {W_{\rm mn}}^n & {W_{m2}}^{\rm n} & {W_{m3}}^{\rm n} & {W_{m4}}^{\rm n} & \ldots & {\mathrm{W}_{\rm mn}}^n \end{pmatrix}, A = \begin{pmatrix} a_0 \\ a_1 \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ a_n \end{pmatrix},\ \mathrm{B}\left(t_i\right) = \begin{pmatrix} X_{11} \\ X_{21} \\ \vdots \\ \vdots \\ X_{\rm mn} \\ {X_{11}}^0 \\ {X_{22}}^1 \\ \vdots \\ \vdots \\ {X_{\rm mn}}^n \end{pmatrix} \tag{3.8}$$

Multiply both sides of equation (3.7) by $G\left(t_i\right)^{-1}$ gives

$$A = G\left(t_i\right)^{-1}B(t_i) \tag{3.9}$$

The required approximate solution is obtained by solving Eq. (7) and then substituting the unknown constant values into the assumed approximate solution.

By Solving Eq. (7) and then substituting the unknown constant values into the assumed approximate solution, the required approximate solution is obtained.

4 Numerical examples

Example 4.1, [9]:Consider the eight Order boundary value problem

$$v^8(t) = -tv(t) - \left(48 + 15t + t^3\right)e^t, \qquad 0 \le t \le 1, \tag{4.1}$$

Subject to the boundary conditions

$$v(0) = 0, v^{'}(0) =, v^{''}(0) = 0,\ v^{'''}(0) = -3v(1) = 0, v^{'}(1) = -e, v^{''}(1) = -4e, v^{'''}(1) = -9e. \tag{4.2}$$

With the exact solution $v(t) = t(1-t)e^t$. We obtained the following unknown constants via the above-described method:

$$
\begin{aligned}
&a_0 = 0.212597929325208, a_1 = 0.0526032066414253, a_2 = -0.206056327199215 \\
&a_3 = -0.0520600553485351, a_4 = -0.00650768548259745, a_5 = -0.000541954722599436, \\
&a_6 = -0.0000338233219107422, a_7 = 0.00000167339660890609, a_8 = -7.03512868991061 \times 10^{-8} \\
&a_9 = -2.51098991700962 \times 10^{-9}, a_{10} = -7.84276058954119 \times 10^{-10}, a_{11} = -2.17758416704422 \times 10^{-12} \\
&a_{12} = -5.44193761168668 \times 10^{-14}, a_{13} = -1.23636050235751 \times 10^{-15} \\
&a_{14} = -2.57491429539680 \times 10^{-17}, a_{15} = -4.97695422982020 \times 10^{-19}, a_{16} = -8.92030280107478
\end{aligned}
\tag{4.3}
$$

Thus, the approximate solution is given as;

$$
\begin{aligned}
v(t) = {} & 5.022476988 \times 10^{-7} + 1.000004133t + 0.00001779243274t^2 - 0.5000563290t^3 \\
& - 0.3334986912x^4 - 0.1244686968t^5 - 0.03379927760t^6 - 0.006811000324t^7 - \\
& 0.001190476255t^8 - 0.0001736110549t^9 - 0.00002204591823t^{10} - 0.000002480077044t^{11} \\
& - 2.506300321 \times 10^{-7} t^{12} - 2.285004759 \times 10^{-8} t^{13} - 2..007534926 \times 10^{-9} t^{14} \\
& - 1.139485604 \times 10^{-10} t^{15} - 1.915620440 \times 10^{-11} t^{16}
\end{aligned}
\tag{4.4}
$$

Example 4.2 [9]:Consider the Boundary Value Problem

$$
v^8(t)+v^7(t)+2v^6(t)+2v^5(t)+2v^4(t)+2v^{'''}(t)+2v^{''}(t)+v^{'}(t)+v(t) = 14\cos x - 16\sin x - 4x\sin x, \quad 0 \le x \le 1,
\tag{4.5}
$$

Subject to the boundary conditions

$v(0) = 0, v^{'}(0) = -1, v^{''}(0) = 0, v^{'''}(0) = 7v(1) = 0, v^{'}(1) = 2\sin 1, v^{''}(1) = 4\cos 1 + 2\sin 1, v^{'''}(1) = 6\cos 1 - 6\sin 1$

with the exact solution $V(t) = (t^2 - 1)\sin t$

We obtained the following unknown constants via the above-described method:

$$
\begin{aligned}
&a_0 = -0.176733070536344, a_1 = -0.0218410911471154, a_2 = -0.179771723093299 \\
&a_3 = 0.0220918688055828, a_4 = -0.00305205301874017, a_5 = -0.000251586915500430 \\
&a_6 = 0.0000134350773212141 \ a_7 = 8.03951992959465 \times 10^{-7}, a_8 = -3.46104865643160 \times 10^{-8} \\
&a_9 = 5.24107004638911 \times 10^{-10}, a_{10} = 9.93311003903608 \times 10^{-12} a_{11} = -2.11438201087708 \times 10^{-12} \\
&a_{12} = 6.45175538035085 \times 10^{-14}, a_{13} = -8.32733591826538 \times 10^{-17} a_{14} = -5.23213015594760 \times 10^{-17} \\
&a_{15} = 1.86506446431401 \times 10^{-18}, a_{16} = -1.67744672326677 \times 10^{-20}
\end{aligned}
\tag{4.6}
$$

Thus, the approximate solution is given as;

$$
\begin{aligned}
v(t) = {} & 4.789345474 \times 10^{-9} - 1.000000004t + 2.320649076 \times 10^{-8} t^2 \\
& + 1.1666666270t^3 - 0.0004137822311x^4 - 0.1731128042 - 0.003409471809t^6 + 0.01156445848t^7 \\
& - 0.001298000304t^8 - 0.00002788952868t^9 + 0.00003780240300t^{10} - 0.000007380182020t^{11} \\
& + 3.516187681 \times 10^{-7} t^{12} + 7.397775300 \times 10^{-8} t^{13} \\
& - 1.557685160 \times 10^{-8} t^{14} + 1.289482013 \times 10^{-9} t^{15} - 1.602289408 \times 10^{-11} t^{16}
\end{aligned}
\tag{4.7}
$$

Table 1: Shows numerical outcomes for example 4.1 at n=16

t	Exact	App.Sol(proposed method)	Ab.Er of [9]	Ab.(proposed method) n=16
0.0	0.00000000000000	0.00000050224770	-	5.022E-07
0.1	0.09946538262000	0.09946640803000	5.215406E-08	1.025E-06
0.2	0.19542444130000	0.19542590850000	2.220273E-06	1.467E-06
0.3	0.28347034970000	0.28347181330000	7.003546E-06	1.464E-06
0.4	0.35803792750000	0.35803884210000	1.114607E-05	9.146E-07
0.5	0.41218031780000	0.41218032390000	1.227856E-05	6.128E-09
0.6	0.43730851200000	0.43730761260000	8.881092E-06	8.994E-07
0.7	0.42288806850000	0.42288662690000	2.533197E-06	1.442E-06
0.8	0.35608654850000	0.35608511150000	1.817942E-06	1.437E-06
0.9	0.22136428000000	0.22136329190000	2.041459E-06	9.881E-07
1	0.00000000000000	0.00000045626726	-	4.563E-07

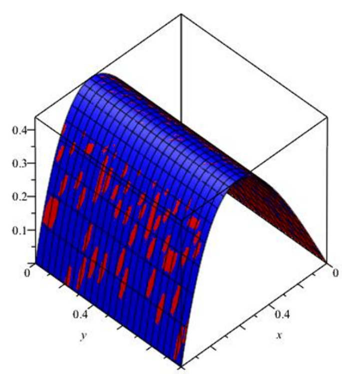

Figure 1: Demonstrates the graphical results for Example 4.1's exact solution and approximation solution

Table 2: Shows numerical outcomes for example 4.2 at n=16

t	Exact	App.Sol(proposed method)	Ab.Er of [9]	Ab.(proposed method) n=16
0.0	0.00000000000000	0.00000000478935	-	4.789e-09
0.1	-0.09883508248000	-0.09883510352000	3.576279E-07	2.104e-08
0.2	-0.19072255760000	-0.19072279380000	6.318092E-06	2.361e-07
0.3	-0.26892338810000	-0.26892405300000	1.895428E-05	6.649e-07
0.4	-0.32711140750000	-0.32711246920000	3.099442E-05	1.062e-06
0.5	-0.35956915400000	-0.35957031660000	3.641844E-05	1.163e-06
0.6	-0.36137118300000	-0.36137209690000	3.170967E-05	9.140e-07
0.7	-0.32855102050000	-0.32855151470000	1.925230E-05	4.944e-07
0.8	-0.25824819270000	-0.25824834830000	7.182360E-06	1.555e-07
0.9	-0.14883211280000	-0.14883213160000	1.460314E-06	1.872e-08
1	0.00000000000000	-0.00000000506857	-	5.069e-09

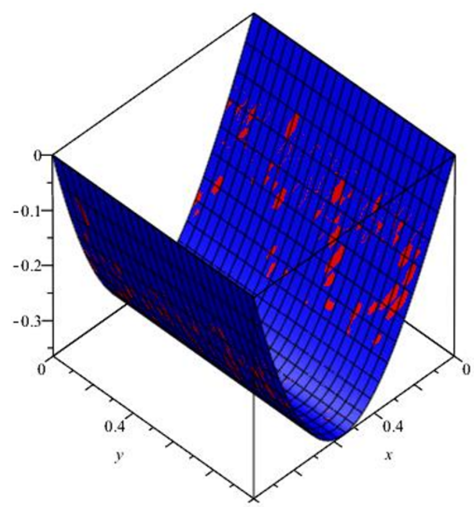

Figure 2: : Demonstrates the graphical results for Example 4.2's exact solution and approximation solution

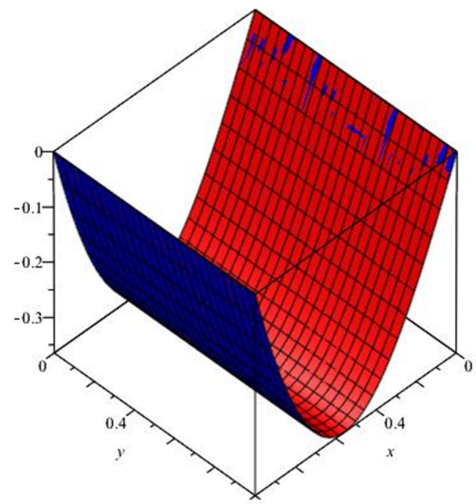

Figure 3: : Demonstrates the grapical results for Example 4.3's exact solution and approximation solution

Example 4.3 [15]: Consider the Boundary Value Problem

$$v^{8}(t)-v(t)=-8\left(2x\cos x+7\sin x\right)\qquad -1\leq x\leq 1, \tag{4.8}$$

Subject to the boundary conditions $v(-1)=0, v^{'}(-1)=2\sin 1, v^{''}(-1)=-4\cos 1-2\sin 1,\ v^{'''}(-1)=6\cos 1-6\sin 1 v(1)=0, v^{'}(1)=2\sin 1, v^{''}(1)=4\cos 1+2\sin 1, v^{'''}(1)=6\cos 1-6\sin 1$, with the exact solution $V(t)=(x^2-1)\sin x$

We obtained the following unknown constants via the above-described method:

$$\begin{aligned}
a_0&=3.71417586443736\times 10^{-14}, a_1=-0.229790548056755, a_2=-4.37980215644523\times 10^{-15}\\
a_3&=0.239707729827209 a_4=4.09108076378527\times 10^{-16}, a_5=-0.0100309819299826\\
a_6&=-2.90129201281697\times 10^{-17} a_7=0.000126260064388872, a_8=-1.72689281437172\times 10^{-19}\\
a_9&=-7.56408283331500\times 10^{-7} a_{10}=5.26202928132414\times 10^{-20} a_{11}=2.63491342883197\times 10^{-9}\\
a_{12}&=-9.76561066699262\times 10^{-21} a_{13}=-5.82115065874869\times 10^{-12}, a_{14}=8.62475644354010\times 10^{-22}
\end{aligned} \tag{4.9}$$

Thus, the approximate solution is given as;

$$\begin{aligned}
v(t)&=4.195944586\times 10^{-14}-0.9999593045t-1.254575677\times 10^{-14}t^2+1.166612473t^3+\\
&4.607250900\times 10^{-15}x^4-0.1749668697x^5-7.821561407\times 10^{-16}t^6+0.008523813327t^7\\
&-1.801414491\times 10^{-16}t^8-0.0002011573006t^9+1.209437417\times 10^{-16}t^{10}+0.000002775642509t^{11}\\
&-4.472887233\times 10^{-17}t^{12}-2.384343310\times 10^{-8}t^{13}+7.065400479\times 10^{-18}t^{14}
\end{aligned} \tag{4.10}$$

Table 3: Table 3 Shows numerical outcomes for example 4.3 at n=14

t	Exact	App.Sol(proposed method) n=14	Ab.Er(proposed method) n=14[***]
-1	0.00000000000000	-0.00001170662545	1.171e-05
-0.8	0.25824819270000	0.25823418990000	1.400E-05
-0.6	0.36137118300000	0.36135611720000	1.507E-05
-0.4	0.32711140750000	0.32709827140000	1.314E-05
-0.2	0.19072255760000	0.19071484150000	7.716e-06
0.0	0.00000000000000	0.00000000000004	4.196E-14
0.2	-0.19072255760000	-0.19071484150000	7.716E-06
0.4	-0.32711140750000	-0.32709827140000	1.324E-05s
0.6	-0.36137118300000	-0.36135611720000	1.507E-05
0.8	-0.25824819270000	-0.25823418990000	1.400E-05
1	0.00000000000000	0.00001170662551	1.171E-05

5 Conclusion

In this study, eight-order boundary value problems with first-kind Chebyshev polynomials are solved numerically. This suggested strategy was tested on a few examples and was found to generate more satisfying results than that of [9]. After considering the aforementioned elements, the study's findings support the suggested approach for solving further boundary value problems.

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