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Qom University of Technology

A linear mathematical model for the transmission dynamics of diabetes mellitus

Abayomi Ayotunde Ayoade [1], Sunday Olanrewaju Agboola [2]

Abstract: Diabetes mellitus is a global health problem, escalating at a disturbing rate due to unbalanced lifestyles and some underlying health issues. In this work, a system of first-order linear ordinary differential equations as well as numerical simulations were employed to gain insight into the dynamics of the disease. The theoretical outcome of the analysis was derived in terms of the model parameters while computer simulation was used to assess the behaviour of the model in terms of the parameters values. Both the theoretical and numerical studies of the model revealed lifestyles and effective treatment as the parameters to be targeted for effective reduction in both diabetes prevalence and mortality. It is therefore concluded that diabetes prevalence and mortality reduction is a function of adjustment in unbalanced lifestyles as well as improvement in diabetes treatment.

Keywords: Diabetes Mellitus; Lifestyle; Simulation; Mathematical Model; Treatment
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1 Introduction

Diabetes mellitus, popularly called diabetes, is a collection of many signs signifying chaotic metabolism, believed to be instigated by various issues based upon surrounding, environments or inheritance, which ultimately leads to hyperglycaemia, the condition of abnormal sugar levels in the bloodstream ([29]). In man, body cells require energy for proper functioning and the energy is supplied by glucose. The carbohydrates acquired during food consumption do brake down to form glucose. Therefore, the level of glucose in human bodies increases after each meal. At that moment, insulin is expected to perform its functions. Insulin is produced by the pancreas. It is the hormone that regulates the amount of glucose in the human bloodstream. The excess glucose in the body is usually checked and counterbalanced by the action of insulin. Diabetes therefore emanates when there is a failure in the process of controlling glucose level in the bloodstream ([15]). That is, diabetes occurs when the pancreas fails to secrete insulin or when the insulin secreted is not enough or not capable to check excess glucose in the body.

Diabetes has become a worldwide health crisis promoting the challenges of non-communicable illnesses, championed by declining levels of activity and rising prevalence of obesity ([11]). Recent reports by the International Diabetes Federation (IDF) indicates that over 370 million people are living with diabetes globally (the proportion of the adult population is 8.5%) and almost 463 million people are in pre-diabetes states (the proportion of the adult population is 6.5%), and over 625 million individuals are expected to develop diabetes by 2045 ([21]). The IDF reports also indicate that 80% of individuals suffering from

[1]Corresponding author: Department of Mathematics, University of Lagos, Nigeria, ayoadeabbayomi@gmail.com
[2]Department of Mathematical Sciences, Nigerian Army University, Biu, Nigeria, agboolasunday70@gmail.com

diabetes reside in low-and middle-income nations ([21, 34]). Besides, a person dies of diabetes mellitus every seven seconds, which accounts for 1.6 million deaths annually ([20, 33]).

Diabetes mellitus is generally categorised into two types namely type I and type II. Type I diabetes mellitus appears when the pancreas fails to secretes the required quantity of insulin to take in excess blood glucose ([34]). Type I diabetes mellitus is common among the children and people below 40 years of age ([21, 11]). Type II diabetes mellitus, on the other hand, occurs when the insulin secreted is unable to absorb free glucose from the bloodstream ([35]). Type II diabetes is connected with heredity and lifestyle - smoking, drinking, junk food and inadequate exercise ([25]). The glucose concentration in the bloodstream of a healthy person after eight hours fasting normally falls within a range of 60-100mg/dl (mg=milligram, dl=decilitre) ([35]). Nevertheless, the borderline between manageable and abnormal glucose levels in the human bloodstream is 126mg/dl and if an individual's glucose level attains 126mg/dl and more, in any two or more consecutive tests then the individual is supposed to be subjected to diabetes mellitus examination ([35]). Incidentally, the renal glucose threshold increases with advancement in age and there may be occasions when the glucose level in an individual might be as high as 200mg/dl whereas the individual might not be suffering from diabetes ([35]). Therefore, medical examination is needed to establish the presence of diabetes.

With insulin secretion failure or deficiency, patients suffer prolonged hyperglycaemia especially in type I diabetes mellitus and may require insulin therapy through pills or injections to survive or live normal lives. However, improper administration of insulin therapy could be disastrous particularly if the dosage is wrongly taken to the degree that the glucose presence in the bloodstream falls below 50mg/dl resulting in a condition known as hypoglycaemia. Both hypoglycaemia and hyperglycaemia are health problems that are associated with abnormalities in the blood glucose regulation. While hypoglycaemia can result in coma or sudden death, hyperglycaemia, on the other hand, can activate complications, such as vision problems and blindness, heart attack and stroke, kidney failure and amputations ([18]). The major symptoms of diabetes include incessant urination (polyuria), frequent thirst (polydipsia) and intense appetite (polyphagia). The symptoms and signs may occur quickly within some weeks in type 1 diabetes, especially in children. Diabetes mellitus of type I may instigate complex mental weakness in children and an instant weight loss regardless of usual or even frequent eating [medicinenet.com diabetes]. The risk of diabetes mellitus especially type II can be reduced by some simple interventional measures such as eating healthily, eating on time, exercising regularly, avoiding smoking, etc. However, the basis for effective management of diabetes mellitus is a timely diagnosis. Therefore, easy access to blood glucose testing should be on hand in every primary health care venue.

Mathematical model is described in [26] as the representation of the real world, characterised by the use of mathematics to represent the parts of the real world that are of interest and the relationship between those parts. The parts of the real world that are of interest may be economical, ecological or biological of which Epidemiology is a subset, hence Mathematical Modelling is a big tree with many branches. It has become an important scientific technique over the last two decades and is becoming more and more powerful tool to solve problems arising from Science, Engineering, Industries, and the Society in general ([26]). Mathematical models have been used both analytically and numerically to give insight into the dynamics of many real life phenomena e.g. poverty and crime [19, 1], blood flow and blood pressure [17], media impact on a new product innovation diffusion [16], agriculture, resource allocation, unemployment, out-of-school children and rumour ([5, 2, 6, 7, 3]). The essence of developing mathematical models is to gain insight into the real life phenomena and to use mathematical concepts and language to solve real life problems ([26, 8]).

Most studies on diabetes modelling are related to the dynamics of glucose and insulin [29, 18, 15, 24, 27], diabetes epidemiology [30, 12, 32] and optimal control strategies ([14, 23, 10, 28, 22]). While there are numerous modelling studies in the literature, the most important thing is to provide a simple method to get adequate understanding of diabetes dynamics. Based on this, a linear model is designed to expound the dynamics between glucose and insulin.

2 Model Formulation

The authours in [13] studied the transmission dynamics of diabetes and designed a nonlinear mathematical model to quantify the diabetic population as follows:

$$\begin{aligned}\dot{D}(t) &= I - (\lambda+\mu)D(t) + \gamma C(t),\\ \dot{C}(t) &= \lambda D(t) - (\gamma+\mu+\delta+\nu)C(t),\end{aligned} \tag{2.1}$$

where

- D(t) stands for the total diabetics with no complications.
- C(t) stands for the diabetics nursing complications.
- I represents the diabetes incidence.
- λ is the per capita probability of developing complications.
- μ represents the mortality rate unrelated to diabetes.
- δ represents the rate of mortality due to diabetes complications.
- γ represents the rate of cure of diabetes complications.
- ν represents the rate at which diabetics with complications become critically disabled

The authours developed numerical methods to solve the model and the model solutions revealed the behaviours of diabetes complications and various interventions against the complications. In [13], the factors that instigate diabetes incidence are not clearly defined. Therefore, the present analysis considered diabetes mellitus of type 2 and attributed its incidence to reckless lifestyles and heredity. The consideration modifies the model in [13] and the modification goes in Figure 1 thus:

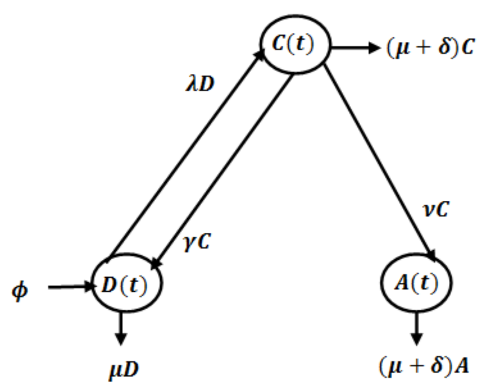

Figure 1: Flow chart of the modified model.

The following system of ODEs is obtained from Figure 1

$$\begin{aligned}\dot{D}(t) &= \phi - (\lambda+\mu)D(t) + \gamma C(t),\\ \dot{C}(t) &= \lambda D(t) - (\gamma+\mu+\delta+\nu)C(t),\\ \dot{A}(t) &= \nu C(t) - (\mu+\delta)A(t).\end{aligned}$$

Following [31, 9], the equation for changes in diabetics population who have become critically disabled (i.e. $\dot{A}(t)$) can be neglected from the analysis. The analysis is then based on the first two equations, i.e.,

$$\begin{aligned} \dot{D}(t) &= \phi - (\lambda + \mu)D(t) + \gamma C(t), \\ \dot{C}(t) &= \lambda D(t) - \alpha C(t), \end{aligned} \tag{2.2}$$

where

$$\alpha = \gamma + \mu + \delta + \nu.$$

The model in (2.2) is developed by following the same assumptions and terminology as in [13]. The definitions for the model parameters also remain as in [13] except that (I) is changed to ϕ and it denotes the recruitment rate of diabetics due to heredity and unbalanced lifestyles. In [13], the parameter (I), which stands for the recruitment rate of diabetics, is not attributed to a particular factor. Also, unlike in [13] where nonlinear scenario is considered for diabetes dynamics and where the analysis is more numerical, the present study considers a linear situation as in [29, 28, 4] for the sake of simplicity and subjects the model to both qualitative and quantitative analyses.

Given that $N(t) = C(t) + D(t)$, $N(t)$ is the sum of diabetes population and, $C(t)$ and $D(t)$ represent diabetics with and without complications respectively. Then the relation $N(t) = C(t) + D(t)$ leads to the IVP

$$\begin{aligned} \dot{C}(t) &= -(\lambda + \alpha)C(t) + \lambda N(t), t > 0; C(0) = C_\circ, \\ \dot{N}(t) &= \phi - (\nu + \delta)C(t) - \mu N(t), t > 0; N(0) = N_\circ. \end{aligned} \tag{2.3}$$

The system (2.3) governs the dynamics of diabetes in the population and the system is linear. The linearity comes from the assumption that, λ, the per capita probability of developing complications, is constant which implies that

$$\lambda = \frac{C_\circ}{N_\circ}. \tag{2.4}$$

Since the model is an extension and a modification of an existing model, the existence, positivity and boundedness of solutions properties are assumed for the model.

3 Model Analysis

The IVP 2.3 attains equilibrium when $\dot{C}(t)$ and $\dot{N}(t)$ vanish at once such that the non-trivial equilibrium point is obtained. The non-trivial equilibrium point is denoted by (C_*, N_*) and it is derived as

$$(C_*, N_*) = \left(\frac{\lambda\phi}{\lambda(\nu + \delta) + \mu(\lambda + \alpha)}, \frac{\phi(\lambda + \alpha)}{\lambda(\nu + \delta) + \mu(\lambda + \alpha)} \right). \tag{3.1}$$

3.1 Stability analysis of equilibrium point (C_*, N_*)

The Jacobian about the equilibrium (C_*, N_*) associated with the IVP 2.3 is derived as

$$J = \begin{pmatrix} -(\lambda + \alpha) & \lambda \\ -(\nu + \delta) & -\mu \end{pmatrix}. \tag{3.2}$$

The characteristic equation of (3.2) is quadratic in p and it is given as

$$p^2 + (\lambda + \alpha + \mu)p + \mu(\lambda + \alpha) + \lambda(\nu + \delta) = 0. \tag{3.3}$$

Generally, the IVP 2.3 is locally asymptotically stable if all the roots of 3.3 are real and negative or if the conditions trace $(J) < 0$ and $\det(J) > 0$ are satisfied simultaneously. It is observed from 3.2 and 3.3 that all the stated conditions are met. Therefore, the diabetes mellitus model represented by the IVP 2.3 is locally asymptotically stable.

4 Simulation and Discussion

The IVP 2.3 is to be quantified to observe the behaviours of the model in terms the values of the parameters. Following [13], assuming $\lambda = 0.8, \phi = 0.001, \nu = 0.05, \delta = 0.05, \gamma = 0.08, \mu = 0.02$, $C_\circ = 7,500$ and $N_\circ = 600,000$ initially then the IVP 2.3 is solved numerically to obtain the results displayed in Figures 2 and 3.

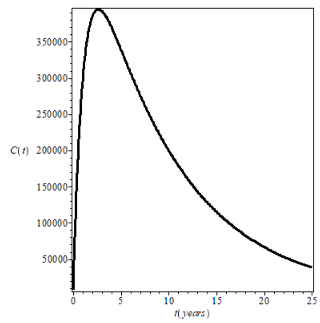

Figure 2: Simulation result for diabetes patients with complications.

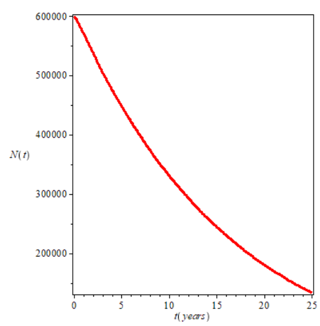

Figure 3: Simulation result for total diabetics population.

The equilibrium (C_*, N_*) derived for the model is endemic and the stability analysis conducted for the equilibrium revealed that the endemic equilibrium of the model is locally asymptotically stable. Since diabetes is non-communicable, the implication of the endemic equilibrium being locally asymptotically stable is that diabetes eradication in a population is not feasible. Diabetes is connected with both heredity and lifestyles. If unbalanced lifestyles are adjusted, heredity is there to account for the incidence of diabetes. However, based on the values chosen for the parameters, Figures 2 and 3 clearly show that the population of people living with diabetes either with or without complications in a population could reduce after a period of time or fall continuously over a period of time with availability and affordability of effective diabetes treatments as well as awareness campaigns on the importance of healthy lifestyles. Figures 4 and 5 even indicate that improvement in effective diabetes treatments and awareness campaigns on the importance of healthy lifestyles could yield instant results in reducing diabetes complications and diabetes incidence.

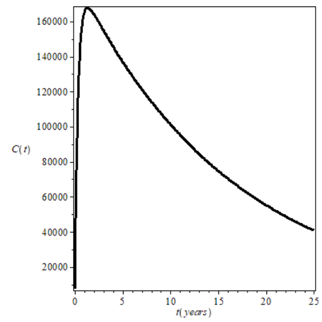

Figure 4: Simulation result for diabetes patients with complications with improvement in γ.

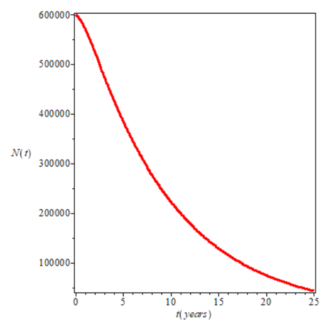

Figure 5: Simulation result for total diabetics population with improvement in ϕ.

5 Conclusion

In this study, a model was designed to gain insight into the dynamics of diabetes. The model was formulated in terms of a system of linear ordinary differential equations for simplicity and clarity sake. The equilibrium of the model was derived and the stability investigated. Numerical simulation was carried out by choosing a set of reasonable values for the model parameters. It was revealed from both the analytical and numerical outcomes of the study that while the incidence of diabetes is inevitable, the population of diabetics could be managed if the factors that contribute to diabetes incidence rate (ϕ) and the rate of cure of diabetes complications (γ) are handled with all seriousness. It is therefore concluded that diabetes prevalence and mortality reduction is a function of adjustment in unbalanced lifestyles as well as improvement in diabetes treatment.

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