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Various Parity-State Solution of Spin-One DKP Equation with Cornell and Exponential Interactions

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Abstract: We consider the spin-one DKP equation in the presence of vector Cornell and exponential interactions which are among successful interactions of particle physics. We obtain the exact analytical solutions of the former and the approximate solutions the latter via the Laplace integral transform method in terms of hyper geometric functions for arbitrary quantum number in three spatial dominations.

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1 Introduction

The relativistic Duffin-Kemmer-Petiau (DKP) equation describes spin-zero and spin-one bosons in a single unified basis [9,11,16]. From its formulation, the equation has been used to analyze various phenomena of particle and nuclear physics including K-meson decays, nucleus-meson scattering, etc [4]. The studies on DKP equation in particular play an important role in the study of spin-one bosons due to complicated nature of Proca equation. However, it ought to be mentioned that the DKP is definitely complicated in comparison with Schrödinger, Dirac and Klein-Gordon frameworks.

On the contrary to other wave equations of quantum mechanics, a few problems have been regarding spin-one DKP equation including linear, coulomb or harmonic terms [5]. In recent papers, the equation has been considered in generalizations of ordinary quantum mechanics including non-commutative and curved-space formalisms [6,17].

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On the other hand, the integral transforms, and in particular the Laplace transform, have been applied to a variety of quantum mechanical problems in both nonrelativistic and relativistic regimes.Schrödinger equation with the rather exponential-type Mie-type potential was studied in Ref [2]. A Laplace transform of Hypergeometric function was find in Ref. [13] to solve the associated Schrödinger equations. The Laplace transform has also been well applied to relativistic equations of quantum mechanics. In Dirac framework, the technique has been studied in connection with the so-called spin and pseudospin symmetries of equation in position-dependent formalism with a coulomb term [15].

In very recent works, the spin-one DKP equation has been discussed from new aspects. Y. Chargui and A. Dhahbi considered an extended version of the spin-one Duffin–Kemmer–Petiau oscillator by combining a Lorentz tensor spin-orbit coupling with the basic DKP oscillator one [7]. Hamil et al. considered the spin-one DKP equation with a nonminimal vector interaction with a generalized uncertainty principle [10]. De Montigny and Santos studied the Galilean Duffin-Kemmer-Petiau equation in arbitrary dimensions for a many-body purpose [12]. Sobhani and others considered bound and scattering states of the equation in q-deformed formulation [18]. Here, we intend to consider the spin-one DKP equation with Cornell and exponential terms and analyze the problem via Laplace transform approach. The most essential formulae of the spin-one DKP equation is quoted from[8,14,19].

2. DKP Equation with a Nonminimal Vector Interaction

2.1. General Formulae

Considering only the nonminimal vector interaction, the time-independent DKP equation can be written as [8]

(1) $$[\beta^0 E + i\beta^i \partial_i - (\mathrm{m} + i[\mathrm{P}, \beta^\mu]\mathrm{A}_\mu)]\varphi = 0.$$

where P is the projection operator such that $P^2 = P$ and $P^\dagger = P$ (c.f. Ref. [8] for more details) and β^μ matrices are [8,14,19]

$$\beta^0 = \begin{pmatrix} 0 & \bar{0} & \bar{0} & \bar{0} \\ \bar{0}^T & 0 & I & 0 \\ \bar{0}^T & I & 0 & 0 \\ \bar{0}^T & 0 & 0 & 0 \end{pmatrix}, \quad \beta^i = \begin{pmatrix} 0 & \bar{0} & e_i & \bar{0} \\ \bar{0}^T & 0 & 0 & -is_i \\ -e_i^T & 0 & 0 & 0 \\ \bar{0}^T & -is_i & 0 & 0 \end{pmatrix}, \tag{2}$$

where s_i are the 3×3 matrices, $(s_i)_{jk} = -i\,\varepsilon_{ijk}$,e_i stand for 1×3 matrices, $(s_i)_{1j} = \delta_{ij}$ and $\bar{0} = (0,0,0)$. I and 0 represent 3×3 unit and zero matrices, respectively, and T designates matrix transposition. The ten-component spinor can be written as $\varphi^T = (\varphi_1,...,\varphi_{10})$ and partitioned in three spatial dimensions as [8]

$$\varphi_1 = i\,\phi,$$

$$\vec{F} = \begin{pmatrix} \varphi_2 \\ \varphi_3 \\ \varphi_4 \end{pmatrix},$$

$$\vec{G} = \begin{pmatrix} \varphi_5 \\ \varphi_6 \\ \varphi_7 \end{pmatrix},$$

(3)

$$\vec{H} = \begin{pmatrix} \varphi_8 \\ \varphi_9 \\ \varphi_{10} \end{pmatrix},$$

or, equivalently [8,14,18,19]

$$i\,\vec{\nabla}\times\vec{F} - i\,\vec{A}\times\vec{F} = m\,\vec{H}, \tag{4}$$

$$\vec{\nabla}.\vec{G} + \vec{A}.\vec{G} = m\phi, \tag{5}$$

$$i\,\vec{\nabla}\times\vec{H} + i\,\vec{A}\times\vec{H} = m\,\vec{F} - (\mathrm{E} - \mathrm{iA}_0)\vec{G}, \tag{6}$$

$$\vec{\nabla}\phi - \vec{A}\phi = m\,\vec{G} - (\mathrm{E} + \mathrm{iA}_0)\vec{F}. \tag{7}$$

Using the elegant approach of Ref. [14], we put

$$\phi = \frac{\phi_{nj}(r)}{r} Y_j^{m_j}(\theta,\varphi), \tag{8}$$

$$\vec{F} = \sum_l \frac{F_{njl}(r)}{r} \vec{Y}_{jlm_j}(\theta,\varphi), \tag{9}$$

$$\vec{G} = \sum_l \frac{G_{njl}(r)}{r} \vec{Y}_{jlm_j}(\theta,\varphi), \tag{10}$$

$$\vec{H} = \sum_l \frac{H_{njl}(r)}{r} \vec{Y}_{jlm_j}(\theta,\varphi),$$

(11)

Now, introducing the notation [8]

$$\mathrm{F}_{n\,j\,j} = F_0,\qquad F_{n\,j\,j\pm1} = F_{\pm1}, \tag{12}$$

Using Eqs. (9) and (11) in (4) and considering $A_0 = A_0(r)$ and $\vec{A} = A_r(r)\hat{r}$, we have [14]

$$\left(\frac{dF_0}{dr} - \frac{j+1}{r}F_0 - A_r F_0\right) = -\frac{1}{\zeta_j} mH_{+1}, \tag{13}$$

$$\left(\frac{dF_0}{dr} + \frac{j}{r}F_0 - A_r F_0\right) = -\frac{1}{\alpha_j} mH_{-1}, \tag{14}$$

$$-\zeta_j\left(\frac{dF_{+1}}{dr} + \frac{j+1}{r}F_{+1} - A_r F_{+1}\right) - \alpha_j\left(\frac{dF_{-1}}{dr} - \frac{j}{r}F_{-1} - A_r F_{-1}\right) = mH_0, \tag{15}$$

where $\alpha_j = \sqrt{(j+1)/(2j+1)}$ and $\zeta_j = \sqrt{j/(2j+1)}$. Substituting (8) and (10) in (5), we obtain

$$-\alpha_j\left(\frac{dG_{+1}}{dr} + \frac{j+1}{r}G_{+1} + A_r G_{+1}\right) + \zeta_j\left(\frac{dG_{-1}}{dr} - \frac{j}{r}G_{-1} - A_r G_{-1}\right) = m\phi. \tag{16}$$

The radial equations obtained from (6) are

$$\left(\frac{dH_0}{dr} - \frac{j+1}{r}H_0 + A_r H_0\right) = -\frac{1}{\zeta_j}(\mathrm{m}F_{+1} - (\mathrm{E}-\mathrm{iA}_0)G_{+1}), \tag{17}$$

$$\left(\frac{dH_0}{dr} + \frac{j}{r}H_0 + A_r H_0\right) = -\frac{1}{\alpha_j}(\mathrm{m}F_{-1} - (\mathrm{E}-\mathrm{iA}_0)G_{-1}), \tag{18}$$

$$\left(\frac{dH_{+1}}{dr} + \frac{j+1}{r}H_{+1} + A_r H_{+1}\right) - \alpha_j\left(\frac{dH_{-1}}{dr} - \frac{j}{r}H_{-1} + A_r H_{-1}\right) = (mF_0 - (\mathrm{E}-\mathrm{iA}_0)G_0). \tag{19}$$

Finally, from (7) we find

$$(\mathrm{E}+\mathrm{iA}_0)\,\mathrm{F}_0 = mG_0, \tag{20}$$

$$\left(\frac{d\phi}{dr} - \frac{j+1}{r}\phi - A_r\phi\right) = -\frac{1}{\alpha_j}\left(\mathrm{mG}_{+1} - (\mathrm{E}+\mathrm{iA}_0)\,\mathrm{F}_{+1}\right), \tag{21}$$

$$\left(\frac{d\phi}{dr}+\frac{j}{r}\phi-A_r\phi\right)=\frac{1}{\zeta_j}(\mathrm{mG}_{-1}-(\mathrm{E}+\mathrm{iA}_0)\,\mathrm{F}_{-1}). \tag{22}$$

2.2. $(-1)^j$ Parity states

Using Eqs. (13), (14) and (20) we may write [8]

$$\frac{d^2F_0(\mathrm{r})}{dr^2}+\left[\kappa^2-\frac{dA_r}{dr}-\frac{j\,(\mathrm{j}+1)}{r^2}-A_r^2\right]F_0(\mathrm{r})=0, \tag{23}$$

where $\kappa^2=E^2-m^2+A_0^2$ and

$$\begin{pmatrix}\mathrm{F}_{+1}\\ G_{+1}\end{pmatrix}=\frac{1}{\kappa^2}\begin{pmatrix}(\mathrm{E}-\mathrm{iA}_0)\alpha_j\Delta_- & m\zeta_j\Delta_+\\ m\alpha_j\Delta_- & (\mathrm{E}+\mathrm{iA}_0)\zeta_j\Delta_+\end{pmatrix}\begin{pmatrix}\phi\\ H_0\end{pmatrix}, \tag{24}$$

where $\Delta_\pm=\frac{d}{dr}-\frac{j+1}{r}\pm A_r$

2.3. $(-1)^{j+1}$ Parity States

Using the Eqs. (17) and (21), we obtain [8]

$$\begin{pmatrix}\mathrm{F}_{-1}\\ G_{-1}\end{pmatrix}=\frac{1}{\kappa^2}\begin{pmatrix}-(\mathrm{E}-\mathrm{iA}_0)\zeta_j\Xi_- & m\alpha_j\Xi_+\\ -m\zeta_j\Xi_- & (\mathrm{E}+\mathrm{iA}_0)\alpha_j\Xi_+\end{pmatrix}\begin{pmatrix}\phi\\ H_0\end{pmatrix}, \tag{25}$$

where $\Xi_\pm=\frac{d}{dr}+\frac{j+1}{r}\pm A_r$. Considering $j=0$ and $A_0=0$, the equations (24) and (25) reduce to [8]

$$\begin{pmatrix}\mathrm{F}_{+1}\\ G_{+1}\end{pmatrix}=\frac{1}{\bar{\kappa}^2}\begin{pmatrix}E\alpha_j\Delta_- & m\zeta_j\Delta_+\\ m\alpha_j\Delta_- & E\zeta_j\Delta_+\end{pmatrix}\begin{pmatrix}\phi\\ H_0\end{pmatrix}, \tag{26}$$

$$\begin{pmatrix}\mathrm{F}_{-1}\\ G_{-1}\end{pmatrix}=\frac{1}{\bar{\kappa}^2}\begin{pmatrix}-E\zeta_j\Xi_- & m\alpha_j\Xi_+\\ -m\zeta_j\Xi_- & E\alpha_j\Xi_+\end{pmatrix}\begin{pmatrix}\phi\\ H_0\end{pmatrix}, \tag{27}$$

where $\bar{\kappa}^2=E^2-m^2$. In this case, we obtain [8]

$$\frac{d^2H_0}{dr^2}+\left[\bar{\kappa}^2+\frac{dA_r}{dr}-\frac{j\,(\mathrm{j}+1)}{r^2}-A_r^2\right]H_0=0, \tag{28}$$

$$\frac{d^2\phi}{dr^2}+\left[\bar{\kappa}^2-\frac{dA_r}{dr}-\frac{j\,(\mathrm{j}+1)}{r^2}-\frac{A_r}{r}-A_r^2\right]\phi=0.$$

(29)

3.1. Solutions for $(-1)^j$ Parity State Solution for Cornell Potential

We consider the Cornell potential (which consists of both confining and nonconfining terms and has successfully accounted for some data in particle physics) in our work

$$A_r = a\,r + \frac{b}{r}, \tag{30}$$

which brings Eq. (23) into the form

$$\frac{d^2}{dr^2} + \left[k^2 - \left(a - \frac{b}{r^2}\right) - \frac{j(j+1)}{r^2} - \left(a^2 r^2 + 2ab + \frac{b^2}{r^2}\right)\right] F_0(r) = 0. \tag{31}$$

For further simplicity, we introduce the parameters

$$\mu^2 = a^2 \;;\; \upsilon(\upsilon+1) = j(j+1) + b^2 - b \;\;;\varepsilon^2 = k^2 - 2ab - a\,, \tag{32}$$

In addition, we introduce the chance of variable and gauge transformation [2]

$$y = r^2,$$
$$F(y) = y^{\frac{-\upsilon}{2}} \phi(y), \tag{33}$$

and write Eq. (32) as

$$\left\{ y\frac{d^2}{dy^2} - \left(\upsilon - \frac{1}{2}\right)\frac{d}{dy} - \frac{1}{4}\left(\mu^2 y - \varepsilon^2\right) \right\} \phi(y) = 0. \tag{34}$$

Now, applying the Laplace transform

$$L\{\phi(y)\} = f(s) = \int_0^\infty dy\, e^{-sy} \phi(y), \tag{35}$$

Eq. (32) appears in Laplace space as

$$\left(s^2 - \frac{\mu^2}{4}\right)\frac{df(s)}{ds} + \left\{\left(\upsilon + \frac{3}{2}\right)s - \frac{\varepsilon^2}{4}\right\} f(s) = 0. \tag{36}$$

The latter possesses the solution

$$f(s) = N\left(s + \frac{\mu}{2}\right)^{\frac{-\varepsilon^2}{4\mu} - \frac{1}{2}\left(\upsilon + \frac{3}{2}\right)} \left(s - \frac{\mu}{2}\right)^{\frac{\varepsilon^2}{4\mu} - \frac{1}{2}\left(\upsilon + \frac{3}{2}\right)}. \tag{37}$$

To have a single-valued wave function, we consider

$$\frac{\varepsilon^2}{4\mu} - \frac{1}{2}\left(\upsilon + \frac{3}{2}\right) = n \quad ,(n = 0, 1, 2, \ldots). \tag{38}$$

Let us now rewrite a series expansion of the from

$$f\left(s\right)=N'\sum_{k=0}^{n}\left(-1\right)^{k}\frac{n!\left(s+\frac{\mu}{2}\right)^{-\left(\upsilon+\frac{3}{2}+k\right)}}{k!\left(n-k\right)!} \tag{39}$$

where N' is a constant. We can now apply the inverse Laplace transform and write

$$\phi\left(y\right)=N''\sum_{K=0}^{n}\frac{\left(-1\right)^{k}n!\,\Gamma\left(\upsilon+\frac{3}{2}\right)}{k!\left(n-k\right)!\Gamma\left(\upsilon+\frac{3}{2}+k\right)}y^{\left(\upsilon+\frac{1}{2}+k\right)}e^{-\mu\frac{y}{2}}. \tag{40}$$

Now, recalling the form of Hypergeometric function [1]:

$${}_1F_1\left(-n,\sigma;y\right)=\sum_{m=0}^{n}\frac{\left(-1\right)^{m}n!\Gamma\left(\sigma\right)}{m!\left(n-m\right)!\Gamma\left(\sigma+m\right)}y^{m}, \tag{41}$$

we may write

$$\phi\left(y\right)=N'''e^{-\mu\frac{y}{2}}y^{\upsilon+\frac{1}{2}}{}_1F_1\left(-n,\upsilon+\frac{3}{2};y\right). \tag{42}$$

We have thus succeeded in finding the component according to Eq. (33) and the other components can be simply calculated via the equations of the text. It should be mentioned that the other parity state can be simply followed via similar steps.

3.2. Solutions for $(-1)^j$ Parity State Solution for Exponential Potential

Considering the exponential interaction [20]

$$A_r=v_0e^{-a\left(r-r_0\right)}, \tag{43}$$

which resembles Coulomb interaction in some ranges and has accounted for some data, the q. (23) becomes

$$\frac{d^2F_0\left(r\right)}{dr^2}+\left[k^2+\left(v_0ae^{-a\left(r-r_0\right)}\right)-\frac{j\left(j+1\right)}{r^2}-v_0^2e^{-2a\left(r-r_0\right)}\right]F_0\left(r\right)=0. \tag{44}$$

We have to use an approximate scheme to obtain an arbitrary-state solution for Eq. (44). Here, we consider the approximation [3]

$$\frac{1}{r^2}\approx\left(c_0+c_1e^{-\alpha x}+c_2e^{-2\alpha x}\right),$$

(45-a)

Where

$$x = \frac{r - r_0}{r_0} \tag{45-b}$$

$$\alpha = a\, r_0 \tag{45-c}$$

$$c_0 = \frac{1}{r_0^2}\left(1 - \frac{3}{\alpha} + \frac{3}{\alpha^2}\right) \tag{45-d}$$

$$c_1 = \frac{1}{r_0^2}\left(\frac{4}{\alpha} - \frac{6}{\alpha^2}\right), \tag{45-e}$$

$$c_2 = \frac{1}{r_0^2}\left(\frac{-1}{\alpha} + \frac{3}{\alpha^2}\right). \tag{45-f}$$

Substituting the approximation, equation appears as

$$-\frac{d^2 F_0(x)}{dx^2} + \left[\left(j\,(j+1)c_2 + v_0^2\right)e^{-2\alpha x} - \left(v_0 a - j\,(j+1)c_1\right)e^{-\alpha x}\right]F_0(x) = \left(k^2 - j\,(j+1)c_0\right)F_0(x), \tag{46}$$

Where

$$c = \frac{\sqrt{\left(j\,(j+1)c_2 + v_0^2\right)}}{\alpha},$$
$$\lambda = \frac{\sqrt{-\left(k^2 - j\,(j+1)\right)}}{\alpha}. \tag{47}$$

Now, applying the chance of variable

$$y = 2ce^{-\alpha x}, \tag{48}$$

Eq. (46) appears as

$$\frac{d^2 F_0(y)}{dy^2} + \frac{1}{y}\frac{dF_0(y)}{dy} + \left(-\frac{1}{4} + \frac{c}{y} - \frac{\lambda^2}{y^2}\right)F_0(y) = 0. \tag{49}$$

A gauge transformation of the form

$$F_0(y) = y^{\lambda}\upsilon(y), \tag{50}$$

brings Eq. (49) into the form

$$y\frac{d^2\upsilon(y)}{dy^2} + (2\lambda + 1)\frac{d\upsilon(y)}{dy} - \left(\frac{y}{4} - c\right)\upsilon(y) = 0.$$

(51)

Using zero boundary conditions for the function and its first derivative, we obtain the first-order Laplace-space equation

$$\left(-s^2+\frac{1}{4}\right)\frac{dV(s)}{ds}+(2\lambda-1)sV(s)+cV(s)=0, \tag{52}$$

which simply gives

$$V(s)=c\left(s-\frac{1}{2}\right)^{\lambda-\frac{1}{2}+c}\left(s+\frac{1}{2}\right)^{\lambda-\frac{1}{2}-c}. \tag{53}$$

The inverse Laplace transform of the latter is

$$\upsilon=c\int_{\Gamma}\left(s-\frac{1}{2}\right)^{\lambda-\frac{1}{2}+c}\left(s+\frac{1}{2}\right)^{\lambda-\frac{1}{2}-c}e^{s\xi}\,ds. \tag{54}$$

A simple comparison of Eq. (54) with the integral representation of Hypergeometric function, i.e.

$${}_1F_1(a,b;y)=c\int_{\Gamma}t^{a-1}(1-t)^{b-a-1}e^{ty}\,dt \tag{55}$$

indicates that

$$\upsilon(y)=ce^{-\frac{y}{2}}\,{}_1F_1(a,b;y) \tag{56}$$

where

$$a=\lambda+\frac{1}{2}-c\,,\qquad b=2\lambda+1. \tag{57}$$

which simply determines the component via Eq. (50). By analogy to the approach of previous section, the energy in this case appears as

$$E=\sqrt{j(j+1)+m^2-A_0{}^2-\left[\left(j(j+1)c_2+v_0{}^2\right)-\alpha\left(n+\frac{1}{2}\right)\right]^2} \tag{58}$$

5. Conclusions

We considered spin-one DKP equation with Cornell and exponential potentials. In our calculations, we used proper change of variables and Laplace integral transform. By applying the Laplace transform to the arising radial ordinary differential equation, we transformed the equation into Laplace space. Next, by solving the associated first-order differential equation as well as the series and integral representations of the

Hypergeometric function, we reported the component of the wave function. The energy was obtained from fundamental requirements of the wave functions. In the calculation, we had to use a Pekeris-type approximation in the case of exponential interaction to provide arbitrary-state solutions. The results can be used to study the characteristics of spin-one mesons including their mass spectrum, decay rates, charge radius, curvature parameter, etc. Also, the idea of using integral transforms can be extended to the fractional case and other dimensions as well.

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