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Spatial solitons in Schrodinger equation with a spatially modulated nonlinearity: Variational approach

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Abstract: In is paper, we have studied the propagation of spatial solitons in the medium with a spatially modulated nonlinearity. Wave equation includes the terms of diffraction and periodic self- focusing. To solve the wave equation, we have employed numerical method including Monte Carlo based Euler-Lagrange variational schema. The effect of the nonlinearity strength related to periodic Kerr self-focusing, on the physical properties of the systems such as maximum intensity and soliton width oscillations are considered.

Keywords: Spatial soliton; Variational approach; Monte Carlo; Periodic self-focusing
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1 Introduction

Over the past decades, solitons have been intense theoretical and experimental subject studies in almost all areas of physics including Hydrodynamics, plasma physics and nonlinear optics [6, 15, 20]. Optical solitons are self-guided beams that maintain their shape during propagation as a result of the interaction between linear and nonlinear effects [2, 3, 11]. If the nonlinear effects balance the dispersion, the soliton becomes localized in time, then called a temporal soliton. On the other hand, the soliton is spatially confined if the nonlinear effects balance the diffraction, then called a spatial soliton [2, 3, 11]. In this paper, we will only study the spatial solitons. In 1964, Chiao, Garmire, and Townes predicted spatial solitons, theoretically [10]. They showed that the expansion of an optical beam due to diffraction could be avoided in a nonlinear optical medium [10]. .Spatial solitons have also been observed experimentally [4, 5, 7]. One of the main applications is that of all-optical switching of light beams by means of spatial solitons, i.e. the control of light by light, [11, 21] which can occur at both the classical and quantum levels. The control of one light beam by another could be useful for optical communications and optical information processing. Another possible applications of spatial solitons are such as optical pulse compression [18], logic operations [22], etc. For discovery of all possible applications, it is very significant to realize the general characteristics of spatial soliton physics. Spatial solitons propagation through

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nonlinear waveguides with various transverse index profiles has previously been investigated. In 2001, Suryanto and van Groesen, investigated propagation of a spatial soliton in a waveguide with a triangular index profile. They found similar effects i.e. soliton beam oscillates in the waveguide (swing effect) [19]. In 2011, Darti and Surynto investigated spatial soliton propagation in a waveguide with Gaussian linear refractive index profile (Gaussian waveguide) and nonlocal nonlinearity. It is seen symmetric swing effect for soliton propagating in a Gaussian waveguide with local nonlinearity. The existence of nonlocal nonlinearity disturbs the soliton oscillation. For weak nonlocal nonlinearity, the soliton still oscillates but asymmetrically [12].

In our previous works, we had studied the spatial soliton propagation through waveguides with rectangular and parabolic rectangular index profile [14], saturation and refractive index geometry effects on localization of a spatial Soliton [13]. In the current work, we investigate the behavior of spatial solitons propagating in medium with a spatially modulated nonlinearity. We have used the variational approach to study the nonlinear Schrodinger equation in presence of periodic Kerr effect. We investigate effect of periodic nonlinearity on the behavior of the soliton.

2 Formalism

The propagation of spatial soliton is described by the nonlinear Schrödinger equation, which is written in the dimensionless form:

$$i\frac{\partial\psi}{\partial z}=\frac{1}{2}\frac{\partial^2\psi}{\partial x^2}-g(x)|\psi|^2\psi \qquad (2.1)$$

Here ψ denotes the slowly varying complex light field amplitude and z demonstrates the normalized propagation distance. In Eq. (2.1), the first term on the right represents diffraction and the second term shows self- focusing (SF). In this paper, we define the form of periodic nonlinearity as

$$g(x)=g_0\cos(x) \qquad (2.2)$$

Where g_0 is the nonlinearity strength. Using the Lagrangian formulation developed in Ref. [9], Eq. (2.1) can be rewritten as an Euler-Lagrange equation according to the variational raw [16]

$$\delta\int_0^\infty\int_{-\infty}^\infty \mathcal{L}\left[\psi,\psi^*,\frac{\partial\psi}{\partial z},\frac{\partial\psi^*}{\partial z},\frac{\partial\psi}{\partial x},\frac{\partial\psi^*}{\partial x}\right]dx\,dz=0 \qquad (3.2)$$

where the Lagrangian density $\mathscr{L}$ as follows

$$\mathscr{L}=\frac{i}{2}\left(\psi^*\frac{\partial\psi}{\partial z}-\psi\frac{\partial\psi^*}{\partial z}\right)+\frac{g_0\cos(x)}{2}\left(\frac{\partial\psi}{\partial x}\right)\left(\frac{\partial\psi^*}{\partial x}\right)-\frac{1}{2}\psi^2\psi^{*2} \qquad (4.2)$$

We can suggest a trial solution in the Gaussian form:

$$\psi(x,z)=A(z)\,exp\big(ib(z)\big)\,exp\left(-\frac{(1+iq(z))x^2}{4w^2}\right) \qquad (5.2)$$

where A, b, w and q are the amplitude, the phase of amplitude, the beam width, and the spatial chirp of the beam, respectively. Putting the mentioned trial function above in Eq. (4.2), and then integrating the consequence with respect to x, we obtain the reduced variational problem

$$\delta\int_0^\infty \mathscr{L}\,dz=0 \qquad (6.2)$$

Where $L=\int_{-\infty}^\infty \mathcal{L}\ dx$ is the Lagrangian. After some calculations, $\langle L\rangle$ can be analytically determined

$$\langle L\rangle = -\sqrt{\pi}A(z)^2w(z)b'(z) + \frac{1}{2}\sqrt{\frac{\pi}{2}}g_0A(z)^4e^{-\frac{1}{8}w(z)^2}w(z) + \frac{1}{4}\sqrt{\pi}A(z)^2w(z)q'(z) - \frac{1}{2}\sqrt{\pi}A(z)^2q(z)w'(z) - \frac{\sqrt{\pi}A(z)^2q(z)^2}{4w(z)} - \frac{\sqrt{\pi}A(z)^2}{4w(z)} \quad (7.2)$$

Then we acquire a set of variational equations governing the Gaussian parameters $p_j(z)$ (j=1,23,4) by using the Lagrangian $\langle L\rangle$

$$\frac{d}{dz}\left(\frac{\partial L}{\partial \dot{p}_j}\right) - \frac{\partial L}{\partial p_j} = 0 \quad (8.2)$$

Where $P_j = dP_j/dz$. After straightway calculation, we arrive the following equations:

$$A'(z) = \frac{A(z)q(z)}{2w(z)^2} \quad (9.1)$$

$$w'(z) = -\frac{q(z)}{w(z)} \quad (10.1)$$

$$q'(z) = \frac{g_0A(z)^2e^{-\frac{1}{8}w(z)^2}w(z)^2}{4\sqrt{2}} + \frac{g_0A(z)^2e^{-\frac{1}{8}w(z)^2}}{\sqrt{2}} - \frac{q(z)^2}{w(z)^2} - \frac{1}{w(z)^2} \quad (11.1)$$

$$b'(z) = \frac{g_0A(z)^2e^{-\frac{1}{8}w(z)^2}w(z)^2}{16\sqrt{2}} + \frac{5g_0A(z)^2e^{-\frac{1}{8}w(z)^2}}{4\sqrt{2}} - \frac{1}{2w(z)^2} \quad (12.1)$$

We solve Eqs. (9.2)-(12.2) using the Monte Carlo numerical method with the initial conditions A(0)=1, w (0) = 1, q (0) = 0, b(0)=1. The numerical results are given in the next section.

3 Results and discussion

Figure 1 shows width variations of soliton against the normalized propagation distance z for different values of the nonlinearity strength g_0=1, 1.1, 1.2. We see that the laser spot size have sine-like oscillations. The spot size oscillates about the z axis with a constant mean value. As shown in figure 1, for various values of the nonlinearity strength, we observe soliton-like behavior. We also see that by increasing nonlinearity strength g_0, the nonlinear Kerr effect becomes more dominant, and as a result, the number of oscillations increases and the period of oscillations decreases. Also, oscillations amplitude of soliton width decreases.

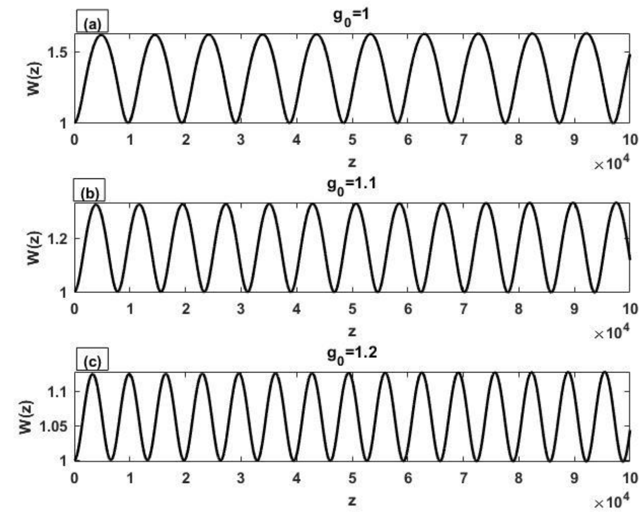

Figure 1: Width variations of soliton versus the normalized propagation distance z for different values of the nonlinearity strength (a)g_0=1, (b) g_0= 1.1 and (c) g_0=1.2.

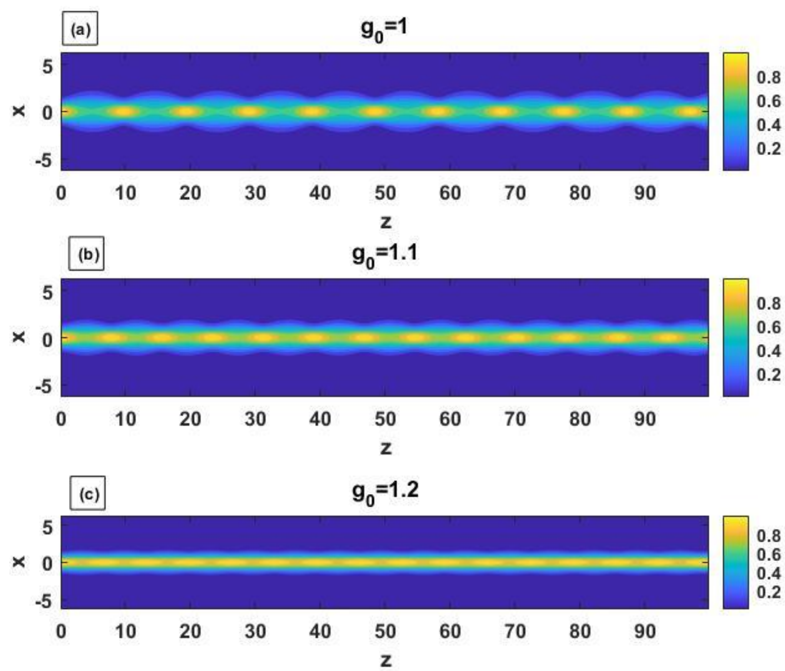

Figure 2: Numerical simulations of the soliton propagating z for different values of the nonlinearity strength (a)g_0=1,

(b) g_0= 1.1 and (c) g_0=1.2.

Figure 2 shows contour plots of intensity of the soliton propagating for different values of the nonlinearity strength (a)g_0=1, (b) g_0= 1.1 and (c) g_0=1.2. In figure 2, similar to figure 1, by increasing the nonlinearity strength g_0 from 1 to 1.2, the nonlinear Kerr effect becomes more dominant, and as a result, the number of oscillations increases the oscillation period becomes shorter but the maximum intensity of the soliton does not change. Furthermore, the width of soliton becomes narrower.

4 Conclusion

In this paper, we studied the propagation of spatial solitons in Schrodinger equation with a spatially modulated nonlinearity. For different values of the nonlinearity strength, the soliton-like behavior. We also saw that by increasing the nonlinearity strength, the oscillations number of soliton width increased and the oscillations amplitude of soliton width decreased.

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