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Introducing a simple method for detecting the path between two different vertices in the Graphs

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Abstract: The problem of path detection in graphs has been proposed from the past up to present, and various solutions have been proposed for this purpose, but it is often not an easy task to implement these methods on a computer. In this paper, a technique for detecting paths in a graph will be introduced using matrix algebra, which makes it possible to implement this rule on a computer. This method can be helpful the optimization of tree-spanning trees in networks. At the end of this study, a numerical example is solved using the proposed method.

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1 Introduction and Statement of Problem

Routing is transferring packets from the source node to the destination node at a minimum cost. Hence, the routing algorithm receives, organizes, and distributes information about the network status [8]. This algorithm has been designed to create practical routes between nodes, send traffic data between selected routes, and yield high performance [6]. Routing, along with congestion control and acceptance control, define network performance. The routing algorithm should have a general purpose of routing strategy based on local useful information. This algorithm also has to keep the user satisfied with the quality of the service [3].
In some other optimization issues, engineers are looking for a tree that, in addition to full coverage of a network, has the lowest cost routes [5].
But it is always before researchers solve the problem either by way of exact or through meta-heuristic methods, they need to make sure whether the issue has a valid answer or not. This paper tries to address this issue and ultimately proves the validity of the proposed approach with the help of algebraic theorems.

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2 Literature Review

In 2015, Patel and Jhaveri examined the issue of safe routes. These researchers have proposed a new approach to find routes using Cryptographic information [10].
In 2006, Riemann and Laumanns, by using an artificial ant algorithm (ACO), introduced a meta-heuristic method to find a spinning tree with the lowest weight in a grid. In this algorithm, after several repetitions, artificial ants learned to use optimal edges in designing routes [11].
In 2017, Ahmed and Paulus, in an article, discussed the way to optimize the routs in wireless networks. They used multiple routes for network coverage to reduce traffic volumes [1].
In 2015, Cong and Zhao in an article used a minimum spanning tree method (MST) to design an urban water supply network as a case study so that they could be able to reduce the time of construction of the water supply network and also the cost of plumbing in all the routs [4].
In 2014, Martin et al. in an article, developed the issue of locating routing. To accomplish this issue, these researchers used Branch and Bound algorithm. [9]
In 2016, Yoo et al. in an article, contributed to the issue of the nonlinear fixed-charge capacitated network design problem (NFCNDP). The proposed method for these researchers is to use a hybrid meta-heuristic approach. The results of the research show that the proposed algorithm has high efficiency [12].
In 2016, Yousefikhoshbakht et al in an article, contributed to the issue of Vehicle Routing Problem (VRP). This paper presents a balance based on the vehicles' traveled route called the balanced vehicle routing problem (BCVRP), and then a model integer linear programming is proposed for the BCVRP. Because solving of this problem as same as VRP is difficult, an effective rank-based ant system (ERAS) algorithm is proposed in this like paper. In addition, to show the efficiency of the proposed ERAS, some test problems in both small and large scales involving 10 to 199 customers have been considered and solved. The computational results show that the proposed algorithm results are better than results of classical rank-based and system (RAS) and exact algorithms for solving the BCVRP within a comparatively shorter period of time [13].
Jafari et al. (2020) investigated the optimization of transportation routes through optimal facility layout in an article. The results of this study showed that optimizing facility layout using Meta-heuristic Algorithms (MA) has optimized transportation costs between work stations [7].

3 Graph

A graph contains two sets: a non-empty set of nodes or vertices and a set of edges connecting the vertices [2].

4 Weighted graph

A graph whose edges are weighted is called a weighted graph. The weight can indicate the cost, distance, time or any other characteristic of the edge [2].

5 Walk

A Walk of G = (V, E) is a finite sequence of vertices and G's edges, such as $W = v_0 e_1 v_1 e_2 \dots e_k v_k$, for which $1 \leq i \leq k$, the vertices $v_{i-1}$and $v_i$are the two edges of e_i. In this case, it can be said that W is a Walk from v_0 to $v_k$or, in other words (v_0, v_k)- is a Walk [6].

6 Connectivity

An undirected graph is called a connected graph, if there is at least one Walk between the two different vertices. The connection is more complex in the directed graph because the direction must be considered. The vertex a may be connected to the vertex b, but there is no Walk from vertex b to the vertex a [12].

7 Tree

The tree is a simple connected graph without a cycle. Tree is a graph tree that contains only one routes between its two vertices. A tree with one n is a vertex when it contains $n - 1$ edges [12].

8 Graph adjacency matrix

There are several ways to display the graph on a computer. The two base data structures used to display the graph are called the adjacency matrix and adjacent list. The adjacency matrix of the graph G with n vertices (whose vertices are respectively named fromv_1 to v_n) is a bit matrix of $n \times n$ called A in which the entry $a_{i,j}$ is equal to 1 if there is an edge from v_i to v_j, then $a_{i,j}$ is equal to 0 if there is no edge from v_i to v_j [12].

Theorem (1): Suppose that $G = (V, E)$ is a multiple-valued graph without edges. $V = \{v_1, \dots, v_n\}$ and $A = (a_{ij})_{n \times n}$ is the adjacency matrix. In this case, the entry matrix of (i, j) in the power k of matrix A is equal to the number of (v_0, v_k)-Walks of length k in G [12].

Theorem (2): Suppose that $G = (V, E)$ is a multiple-valued graph without edges. $V = \{v_1, \dots, v_n\}$ and $A = (a_{ij})_{n \times n}$ is the adjacency matrix. If we construct the matrix S as $S = \sum_{j=0}^{n} A^j$, then:

$$\begin{cases} s_{ij} = 0 \Leftrightarrow \text{There is no Walk from vertex } i \text{ to vertex } j \\ s_{ij} > 0 \Leftrightarrow \text{There is at least one Walk from vertex } i \text{ to vertex } j \end{cases} \tag{1}$$

Proof:

Given that in the adjacency matrix, we denote connections with numbers zero and one, therefore, all the entries of power k-th of the matrix A are A^k for $k = 0,1,..,n$ is negative (they are either zero or positive number). Therefore, it is obvious that all the entries of the matrix S are also not negative.

Proof of the first mode:

According to Theorem (1), the entry (i,j) −th in the power of k-th of matrix A is equal to (v_i, v_j)−Walks of length k in G. Thus, if there is no Walk from vertex i to vertex j, it means that entry of (i,j) −th in the power of k-th of the matrix A is equal to zero. It can be concluded that,

$$s_{ij} = 0 \tag{2}$$

Proof of the second mode:

If $s_{ij} = 0$, then all the powers of k-th of the matrix A in entry of (i, j) −th contain the zero value. On the other hand, according to theorem (1), entry of (i, j) −th in the power of k-th of the matrix A is equal to the number of (v_i, v_j) −Walks of length k in G. Therefore, it can be concluded that there is no Walk from i to vertex j.

Proof of the third mode:

According to theorem (1), entry of (i, j) −th in the power of k-th of the matrix A is equal to the number of (v_i, v_j) −Walks of length k in G. So if there is only one Walk from vertex i to vertex j, it means that entry of (i, j) −th, in one of the powers of the matrix A has a value greater than zero. Thus, it can be concluded that:

$$s_{ij} > 0 \tag{3}$$

Proof of the fourth mode:

If $s_{ij} > 0$, then one of the powers in the entry of (i, j) −th contains a value greater than zero. On the other hand, according to theorem (1), entry of (i, j) −th in the power of k-th of the matrix A is equal to the number of (v_i, v_j)−Walks of length k in G. Thus, it can be concluded that there is only one Walk from vertex i to vertex$j$.

The proof is complete, and the verdict is established.

Theorem (3): Assume that $A = \left(a_{ij}\right)_{n\times n}$ is a matrix with real entries, and we make the matrix S as $S = \sum_{j=0}^{n} A^j$, then, if the matrix $(A - I)$ is inverse, it can be concluded that,

$$S = \frac{A^{n+1} - I_n}{A - I_n} \tag{4}$$

Proof:

We define the matrix S as follows:

$$\mathrm{S} = I_n + A + A^2 + A^3 + \cdots + A^n \tag{5}$$

We multiply the relation (5) in the matrix A from the right, and the following equation is obtained:

$$\mathrm{S} \times A = A + A^2 + A^3 + \cdots + A^n + A^{n+1} \tag{6}$$

Now we add and subtract the Identity matrix (I_n) to the right of equation (6):

$$\mathrm{S} \times A = I_n + A + A^2 + A^3 + \cdots + A^n + A^{n+1} - I_n \tag{7}$$

According to the relations of (5) and (7) it can be concluded that:

$$\mathrm{S} \times A = S + A^{n+1} - I_n \tag{8}$$

By simplifying the relation (8) we have:

$$\mathrm{S} \times (A - I_n) = A^{n+1} - I_n \tag{9}$$

According to the assumption, the matrix $(A - I_n)$ is invertible. Therefore, we multiply each side of the relation (9) from the right in $(A - I_n)^{-1}$:

$$S = \frac{A^{n+1} - I_n}{A - I_n} \tag{10}$$

The proof is complete, and the verdict is established.

Numerical Example 1

Consider the graph G with the adjacency matrix A. Now, by using a new approach, we can determine whether there is a routes from the third vertex to the fifth vertex or not.

$$A=\begin{bmatrix}0&1&1&1&0&0\\1&0&1&1&0&0\\1&1&0&1&0&0\\1&1&1&0&0&0\\0&0&0&0&1&1\\0&0&0&0&1&0\end{bmatrix}$$

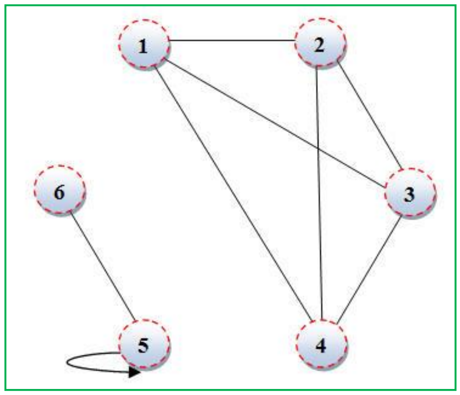

Figure1. Graph related to the adjacency matrix A

In the following, we calculate the matrix S with the aid of equation (10), which results in a calculation as follows:

$$S=\begin{bmatrix}274&273&273&273&0&0\\273&274&273&273&0&0\\273&273&274&273&0&0\\273&273&273&274&0&0\\0&0&0&0&33&20\\0&0&0&0&20&13\end{bmatrix}$$

Given that $s_{3,5}=0$, these two vertices are separated from each other, and there is no route between these two vertices. Therefore, there is no algorithm that can find a route to connect the third vertex to the fifth vertex.

Numerical Example 2

Consider the graph G with the adjacency matrix A in Numerical Example (1). Now, by using a new approach, we can determine whether there is a route from the second vertex to the fourth vertex or not.

Given that $s_{2,4}=273>0$, so these two vertices are not separated from each other, and we can find a route that can connect the second vertex to the fourth vertex.

Numerical Example 3

Consider the graph G with the adjacency matrix A in Numerical Example (1). Now, by using a new approach, we can determine whether there is a route from the second vertex to the third vertex or not.

Given that $s_{2,3} = 273 > 0$, so these two vertices are not separated from each other, and we can find a route that can connect the second vertex to the third vertex.

Numerical Example 4

Consider the graph G with the adjacency matrix A in Numerical Example (1). Now, by using a new approach, we can determine whether there is a route from the sixth vertex to the third vertex or not.
Given that $s_{6,3} = 0$, these two vertices are separated from each other, and there is no route between these two vertices. Therefore, there is no algorithm that can find a route to connect the third vertex to the fifth vertex.

9 Conclusion

In many optimization issues, engineers are looking for optimal routes to reduce transportation costs, etc. This is while we must first make sure that the issue to be reviewed is justified or not (possible). This article addresses this issue. The results show that the proposed method can detect whether there is a route between two nodes or not. Finally, a numerical example was introduced and solved using the new method.

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