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On the numerical solution of Fredholm-type integro-differential equations using an efficient modified Adomian decomposition method

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Abstract: The efficiency of the Adomian decomposition method in the solution of integro-differential equations cannot be overemphasized. However, improvement of the method is needed as its drawbacks have been analyzed and reported in recent literature. This present work develops a new modification of the method and its implementation on linear Fredholm type of integro-differential equations. The approach is based on the modification of the traditional Adomian decomposition method. The idea employs the Taylor series expansion of the source term whose resulting functions were combined in two terms for predicting the solution in each iteration. This approach yields a very high accuracy degree when compared to related methods in literature. The newly proposed method is said to accelerate and converges faster than the standard Adomian Decomposition Method. The procedure proves to be concise, effective and converges faster to the true solution of linear Fredholm Integro-differential problems.

Keywords: Source term, Adomian decomposition method, Fredholm Integro-differential equations, Taylor series, infinite series.

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1 Introduction

Due to its complexity, the numerical solution of Fredholm-type integro-differential equations frequently necessitates the use of advanced methods, such as the Adomian decomposition method (ADM). ADM solves these equations efficiently by decomposing the answer into a series. It combines the benefits of decomposition approach with an enhancement to improve convergence. The Adomian Decomposition Method (ADM) is a powerful tool for solving linear and nonlinear differential equations, as well as Boundary Value Problems (BVPs), in a variety of scientific and engineering fields (Keskin, 2019). This approach was revealed to be largely

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focused on the calculation of Adomian polynomials for nonlinear operators. The type of equation to be solved using ADM has recently created some obstacles to the formulation of these Adomian polynomials. As a result, a few modified ways have been created to solve the difficulties associated with ADM utilization. Various analytical solution approaches were used to address these polynomial problems. Hirota's bilinear approach (Hirota, 1980), the Dar transformation (Gu et al., 1999), the Adomian decomposition method and its modifications (Adomian 1994, Wazwaz 2001, Olayiwola & Kareem 2022, and Kareem & Olayiwola 2023), and numerous asymptotic methods (Oliver, 1986, Gollbabai & Javidi 2007). When dealing with a range of equations, both linear and nonlinear, the Adomian Decomposition technique is successful and trustworthy.

The Adomian Decomposition Method is also used to solve Boundary Value Problems (BVPs) in a variety of scientific and technical fields (Adomia, 1986). In Rohul Amin et al. (2021), Haar functions and integration process were used to obtain the expression of an unknown function and the derivatives of first-order derivatives. Shokri et al. (2018) study the relationship between closed Newton-Cotes formulas and efficient Schrodinger equation integration. The study of multistep Symplicit investigations has been relatively inadequate in recent decades, and the novel Symplicit methods have also been applied to the well-known radial schrodinger problem. Chriscella and Zanariah (2017) solved the second-order linear Fredholm integro-differential equation using a combination of the composite Simpson's 1/3 rule and the second order finite-difference technique, as well as the Gauss elimination strategy. Sunday et al. (2022) created the Non-Fixed Step-Size Algorithm (NFSSA) by integrating the Lagrange interpolation polynomial as a basis function at predefined limits. The NFSSA is also capable of integrating exceedingly stiff differential systems at short and large intervals. Mohd Karim et al. (2018) demonstrated how the Adomian decomposition technique may be used to recapitulate and quantitatively evaluate the generic form of linear second order Fredholm integro-differential equations. The series' noise terms are easily eliminated, resulting in less computing labour.

This present paper introduces a novel method to the formulation of ADM, which is use to solve Fredholm integro-differential equations. This change causes a disruption in the computation and application of the source term's expansion. As a result, the methodological approach is preferable than the old Adomian strategy. The unique modified Adomian Decomposition Method improves accuracy, convergence speed, and calculates efficiency greatly. Overall, the modified Adomian decomposition approach is a beneficial methodology for solving Fredholm-type integro-differential equations because it provides a good balance of efficiency and accuracy in numerical solutions.

2 Method

Consider the following integral equation

$$v(x) = g(x) + \lambda \int_a^x k(x,t)[L(u(t)) + N(v(T))]dt,\ \lambda \neq 0$$

where the kernel $k(x,t)$ and the function $g(x)$ are given real valued functions, λ is a numerical parameter, $L(v(t))$ and $N(v(t))$ are linear and non-linear operator of $v(x)$ respectively and the unknown function $v(x)$ is the solution to be determined.

If $A_j(x);\ j = 1,2,3,\cdots$ is called the Adomian polynomials which are evaluated by

$$A_r = \frac{1}{r!}\frac{d^r}{dx^r}N\left[\sum_{j=0}^{r}\lambda^j v_j\right];\ r = 0,1,2,\cdots$$

By the SADM,

$$v(x) = g(x),$$
$$v_{j+1}(x) = \lambda\int_a^x k(x,t)\left[L(v_j) + P_j\right]dt,\ j \geq 0$$

This section contains the procedures involved in the modification of the Adomian Decomposition Method. For the numerical solution of

$$v''(x) + k(x,t)\int_0^1 k_2(x,t)v(t)dt = \text{g(x)}.$$

The source term is presented in series form

$$g(x) = \sum_{j=0}^{+\infty} g_i(x).$$

This was obtained from its Taylor series expansion and the new recursive relation for $v(x)$ was obtained as:

$$v_0(x) = g_0(x), \tag{1}$$

$$v_1(x) = g_1(x) + g_2(x) + \lambda\int_a^x k(x,t)\left(L\big(v_0(x)\big) + A_0\right)dt, \tag{2}$$

$$v_2(x) = g_3(x) + g_4(x) + \lambda\int_a^x k(x,t)\left(L\big(v_0(x) + v_1(x)\big) + A_1\right)dt, \tag{3}$$

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$$v_{j+1}(x) = g_{2(j+1)}(x) + g_{2(j+1)-1}(x) + \lambda\int_a^x k(x,t)\left(L\left(v_j(x) + v_{j-1}(x)\right) + A_1\right)dt \tag{4}$$

The convergence spirit of the newly modified Adomian decomposition is highly displayed as the method accelerates the convergence of the solution faster than the Standard Adomian Decomposition Method (SADM). We assume that the nonlinear function is $F\big(v(x)\big)$ and therefore, below are few of Adomian polynomials.

$$A_0 = F(v_0), \tag{5}$$

$$A_1 = v_1F'(v_0), \tag{6}$$

$$A_2 = v_2F'(v_0) + \frac{1}{2!}v_1^2F''(v_0), \tag{7}$$

$$A_3 = v_3F'(v_0) + v_1v_2F''(v_0) + \frac{1}{3!}v_1^3F'''(v_0), \tag{8}$$

$$A_4 = v_4F'(v_0) + \left(\frac{1}{2!}v_2^2 + v_1v_3\right)F''(v_0) + \frac{1}{2!}v_1^2v_2F'''(v_0) + \frac{1}{4}v_1^4F^{(\text{iv})}(v_0), \tag{9}$$

There are two important points to note here. First, A_0 is dependent only on v_0, A_1 dependent only on v_0 and v_1. A_2 dependent only on v_0 , v_1 and v_2 , and so on.

Secondly, substituting these $A_j^{'}s$ in (3) gives:

$$F(v) = A_0 + A_1 + A_2 + A_3 + \cdots \tag{10}$$

$$= F(v_0) + (v_1 + v_2 + v_3 + \ \ldots) \mathrm{F}^{'}(v_0) + \frac{1}{2!}(v_1^2 + 2v_1v_2 + 2v_1v_3 + v_2^2)F^{''}(v_0)$$

$$+\frac{1}{3!}(v_1^3 + 3v_1^2v_3 + 6v_1v_2v_3 + \ \ldots) \mathrm{F}^{'''}(v_0) + \ \ldots$$

$$= F(v_0) + (v - v_0)F^{'}(v_0) + \frac{1}{2!}(v - v_0)^2F^{''}(v_0) + \ \ldots$$

In what follows, we will calculate a few of Adomian polynomials for some linear terms that may arise in several nonlinear integral equations.

Case 1.

The following are the Adomian's first four decomposition polynomials for $F(v) = v^2$

$$A_0 = v_0^2 \tag{11}$$

$$A_1 = 2v_0v_1 \tag{12}$$

$$A_2 = 2v_0v_2 + v_1^2 \tag{13}$$

$$A_3 = 2v_0v_3 + 2\mathrm{v}_1v_2 \tag{14}$$

Case 2.

The following are the Adomian's first four decomposition polynomials for $F(v) = v^3$

$$A_0 = v_0^3\ , \tag{15}$$

$$A_1 = 3v_0^2\ \mathrm{v}_1\ , \tag{16}$$

$$A_2 = 3v_0^2v_2 + 3\mathrm{v}_0v_1^2 \tag{17}$$

$$A_3 = 3v_0^2v_3 + 6\mathrm{v}_0v_1v_2 + v_1^3 \tag{18}$$

Case 3.

The first four Adomian polynomials for $F(v) = v^4$ are given by.

$$A_0 = v_0^4, \tag{19}$$

$$A_1 = 4v_0^3v_1\ , \tag{20}$$

$$A_2 = 4v_0^3v_2{+}6v_0^2v_1\ \ , \tag{21}$$

$$A_3 = 4v_0^3v_3 + 4\mathrm{v}_1^3v_0 + 12\mathrm{v}_0^2v_1 + v_2. \tag{22}$$

Case 4.

The following are the Adomian's first four decomposition polynomials for $F(v) = sin\, v$

$$A_0 = sin(v_0)\ \ , \tag{23}$$

$$A_1 = v_1\, cos\, v_0\ \ , \tag{24}$$

$$A_2 = v_2 \cos v_0 - \frac{1}{2!} v_1^2 \sin v_0 \ , \tag{25}$$

$$A_3 = v_3 \cos v_0 - v_1 v_2 \sin v_0 - \frac{1}{3!} v_1^3 \cos v_0 \ . \tag{26}$$

Case 5.

The following are the Adomian's first four decomposition polynomials for $F(v) = \cos v$

$$A_0 = \cos v_0 \ , \tag{27}$$

$$A_1 = -v_1 \sin v_0 \ , \tag{28}$$

$$A_2 = -v_2 \sin v_0 - \frac{1}{2!} \ v_1^2 \cos v_0 \ , \tag{29}$$

$$A_3 = -v_3 \sin v_0 - v_1 v_2 \cos v_0 + \frac{1}{3!} v_1^3 \sin v_0 \ , \tag{30}$$

Case 6.

The following are the Adomian's first four decomposition polynomials for $F(v) = exp(v)$

$$A_0 = exp(v_0) \ , \tag{31}$$

$$A_1 = v_1 \, exp(v_0) \ , \tag{32}$$

$$A_2 = \left(v_2 + \frac{1}{2!} v_1^2\right) \ exp(v_0) \ , \tag{33}$$

$$A_3 = \left(v_3 + v_1 v_2 + \frac{1}{3!} v_1^3\right) \ exp(v_0) \ , \tag{34}$$

3 Convergence Theorems and Analysis

Theorem 1: Let $f \in X$ and $\mathfrak{R}: X \to X$ be an analytic operator on X with infinite radius of convergence then the following relation

$$A_n := \frac{1}{n!} \frac{d^n}{d\epsilon^n} \left(\mathfrak{R} \left(\sum_{j=0}^{n} \epsilon^j u_j \right) \right)_{\epsilon=0}$$

holds for every $n \in N$.

Proof: Consider the vector equation

$$u = \mathfrak{R}(u) + f$$

The ADM for solving the vector equation consists in writing the unknown vector u in the form of an absolutely convergent vector series

$$u = \sum_{n=0}^{+\infty} u_n$$

and in splitting the nonlinear term $\mathfrak{R}(u)$ into an absolutely convergent vector series

$$N(u) = \sum_{n=0}^{+\infty} A_n$$

where the term A_n is obtained for all $n \in N$ by the formula

$$A_n := \frac{1}{n!}\frac{d^n}{d\epsilon^n}\left(\Re\left(\sum_{j=0}^{n}\epsilon^j u_j\right)\right)_{\epsilon=0}$$

Hence, the vector equation becomes

$$\sum_{n=0}^{+\infty} u_n = f + \sum_{n=0}^{+\infty} A_n.$$

Setting the formal identification,

$$\begin{cases} u_0 = f \\ u_{n+1} = A_n \end{cases}$$

Thus

$$A_n := \frac{1}{n!}\frac{d^n}{d\epsilon^n}\left(\Re\left(\sum_{j=0}^{n}\epsilon^j u_j\right)\right)_{\epsilon=0}$$

holds for every $n \in N$.

Theorem 2: The abstract functional equation

$$y = y_0 + f(y), y \in X,$$

where X is a Banach space and $f(y): X \to X$ is analytic near y_0.

Proof: Let

$$Y_n = y_0 + \sum_{k=1}^{n} y_k, \quad f_n(Y_n) = \sum_{k=0}^{n} A_k(y_0, y_1, \cdots, y_k).$$

The ADM is equivalent to determining a sequence of $\{Y_n\}_{n\in N}$ from $Y_0 = y_0, Y_{n+1} = y_0 + f_n(Y_n), n \geq 0$.
If there exist limits

$$Y = \lim_{n\to\infty} Y_n, \qquad f = \lim_{n\to\infty} f_n$$

in the Banach space X, then Y solves the fixed point equation

$$Y = y_0 + f(Y) \in X.$$

The convergence of the modified ADM was established under the following conditions:

$$\left||f(y)|\right|_X \leq 1, \qquad \forall\, y \in X$$

and

$$\left||f_n(Y_n) - f(Y)|\right|_X \to 0, \qquad as\ n \to \infty.$$

4 Numerical Experiments

Example 1: Consider the second order linear Fredholm Integro-differetial Equation:

$$v''(x) = e^x - \frac{4}{3}x + \int_0^1 xtv(t)dt \quad (35)$$

With initial condition $v(0) = 1, \quad v'(0) = 2$

The exact solution is $v(x) = e^x + x$

By direct integration principle,

$$\int_0^x \int_0^x v''dxdx = v(x) - 2x - 1 \quad (36)$$

$$\int_0^x \int_0^x e^x dxdx = e^x - x - 1 \quad (37)$$

$$\int_0^x \int_0^x -\frac{4}{3}xdxdx = -\frac{2}{9}x^3 \quad (38)$$

Then.

$$v(x) - 2x - 1 = e^x - x - 1 - \frac{2}{9}x^3 + \int_0^x \int_0^x \int_0^1 xtv(t)dtdxdx \quad (39)$$

$$v(x) = x - \frac{2}{9}x^3 + e^x + \int_0^x \int_0^x \int_0^1 xtv(t)dtdxdx \quad (40)$$

It is sufficient to state here that equation (40) is explored further using the New Adomian Decomposition Method (NADM)

$$If \ \ r = x - \frac{2}{9}x^3 + e^x$$

Expand taylor (r, x,10) to produce

$$1 + 2x + \frac{1}{2}x^2 - \frac{1}{18}x^3 + \frac{1}{24}x^4 + \frac{1}{120}x^5 + \frac{1}{720}x^6 + \frac{1}{5040}x^7 + \frac{1}{40320}x^8 + \frac{1}{362880}x^9 + 0(x^{10})$$

We take

$$a_0 = 1\,, \quad (41)$$

which implies

$$v_0 = 1\ \ , \quad (42)$$

and

$$g_0 = 2x + \frac{1}{2}x^2 \quad (43)$$

which translates to

$$a_1 = g_0 + \int_0^x \int_0^x \int_0^x xtv_0 dtdxdx \quad (44)$$

$$a_1 = 2x + \frac{1}{2}x^2 + \frac{1}{12}x^3 \quad (45)$$

Thus,

$$v_1 = 2t + \frac{1}{2}t^2 + \frac{1}{12}t^3 \quad (46)$$

Now,

$$g_1 = -\frac{1}{18}x^3 + \frac{1}{24}x^4 \quad (47)$$

which means

$$a_2 = g_1 + \int_0^x \int_0^x \int_0^1 xtv_1 dtdxdx \quad (48)$$

$$a_2 = \frac{19}{240}x^3 + \frac{1}{24}x^4 \quad (49)$$

$$v_2 = \frac{19}{240}t^3 + \frac{1}{24}t^4 \quad (50)$$

Now let

$$g_2 = \frac{1}{120}x^5 + \frac{1}{720}x^6 \quad (51)$$

This implies

$$a_3 = g_2 + \int_0^x \int_0^x \int_0^1 xtv_2 dtdxdx \quad (52)$$

$$a_3 = \frac{1}{120}x^5 + \frac{1}{720}x^6 + \frac{41}{10800}x^3 \quad (53)$$

$$v_3 = \frac{1}{120}t^5 + \frac{1}{720}t^6 + \frac{41}{10800}t^3 \quad (54)$$

Let

$$g_3 = \frac{1}{5040}x^7 + \frac{1}{40320}x^8 \quad (55)$$

This implies

$$a_4 = g_3 + \int_0^x \int_0^x \int_0^1 xtv_3 dtdxdx \quad (56)$$

$$a_4 = \frac{1}{5040}x^7 + \frac{1}{40320}x^8 + \frac{6421}{18144000}x^3 \quad (57)$$

$$v_4 = \frac{1}{5040}t^7 + \frac{1}{40320}t^8 + \frac{6421}{18144000}t^3 \quad (58)$$

Finally,

$$v_n = \sum_{j=0}^{+4} v_i(x) = v_0 + v_1 + v_2 + v_3 + v_4$$

Hence,

$$v_n(t) = 1 + 2t + \frac{1}{2}t^2 + \frac{3023701}{18144000}t^3 + \frac{1}{24}t^4 + \frac{1}{120}t^5 + \frac{1}{720}t^6 + \frac{1}{5040}t^7 + \frac{1}{40320}t^8 \quad (59)$$

$$v_n(x) = 1 + 2x + \frac{1}{2}x^2 + \frac{3023701}{18144000}x^3 + \frac{1}{24}x^4 + \frac{1}{120}x^5 + \frac{1}{720}x^6 + \frac{1}{5040}x^7 + \frac{1}{40320}x^8 \quad (60)$$

Equation (60) is obviously a convergent series to the series expansion of the exact solution $v(x) = e^x + x$.

Example 2: Consider the second order linear Fredholm Integro-differetial Equation:

$$v''(x) = e^x - x + \int_0^1 xtv(t)dt \quad (61)$$

With initial condition $v(0) = 1, \quad v(1) = e$.

The true solution is $v(x) = e^x$

By the New Adomian Decomposition Method (NADM) and following as outlined in Example 1, the series.

$$v_n(x) = 1 + x + \frac{1}{2}x^2 + \frac{9071663}{54432000}x^3 + \frac{1}{24}x^4 + \frac{1}{120}x^5 + \frac{1}{720}x^6 + \frac{1}{5040}x^7 + \frac{1}{40220}x^8 + \frac{1}{362880}x^9$$

is obtained which is a convergent series to the series expansion to the exact solution $v(x) = e^x$.

Example 3: Consider the second order linear Fredholm Integro-differetial Equation:

$$v''(x) = -e^x + \frac{1}{2}x + \int_0^1 xtv(t)dt \quad (62)$$

With initial condition $v(0) = 0, \quad v'(0) = -1$.

The exact solution is $v(x) = 1 - e^x$

By the procedure outline for the New Adomian Decomposition Method (NADM)

$$v_n(x) = -x - \frac{1}{2}x^2 - \frac{40319}{241920}x^3 - \frac{1}{24}x^4 - \frac{1}{120}x^5 - \frac{1}{720}x^6 - \frac{1}{5040}x^7 - \frac{1}{40320}x^8 - \frac{1}{362880}x^9.$$

4. Results

This section contains the display of numerical tables and graphs for purpose of comparing the new method with the standard ADM. It is worth noting that errors reported here are the absolute errors obtained from the absolute differences between the exact solution and the method of consideration.

Table 1: Numerical Comparison with SADM and Exact for Example 1

X	NADM	SADM	Rohul Amin *et. al.*, 2021	Exact	Error_SADM	Error_NADM
0	1	1	1	1	0	0
0.1	1.205170901	1.200000000	1.200000000	1.205170918	$5.17091800 \times 10^{-3}$	1.7000×10^{-8}
0.2	1.421402627	1.410000000	1.410000000	1.421402759	$1.14027580 \times 10^{-2}$	1.3200×10^{-7}
0.3	1.649858363	1.650000000	1.650000000	1.649858807	$1.41192000 \times 10^{-4}$	4.4400×10^{-7}
0.4	1.891823642	1.680000000	1.680000000	1.891824697	$1.18246980 \times 10^{-2}$	1.0550×10^{-6}
0.5	2.148719205	2.120000000	2.120000000	2.148721265	$2.87212710 \times 10^{-2}$	2.0600×10^{-6}
0.6	2.422115211	2.380000000	2.380000000	2.422118771	$4.21188000 \times 10^{-2}$	3.5600×10^{-6}
0.7	2.713746935	2.640000000	2.640000000	2.713752588	$7.37527070 \times 10^{-2}$	5.6530×10^{-6}
0.8	3.025532090	2.920000000	2.920000000	3.025540527	$1.05540928 \times 10^{-1}$	8.4370×10^{-6}
0.9	3.359589926	3.220000000	3.220000000	3.359601938	$1.39603111 \times 10^{-1}$	1.2012×10^{-5}
1.0	3.718262291	3.540000000	3.540000000	3.718278771	$1.78281828 \times 10^{-1}$	1.6480×10^{-5}

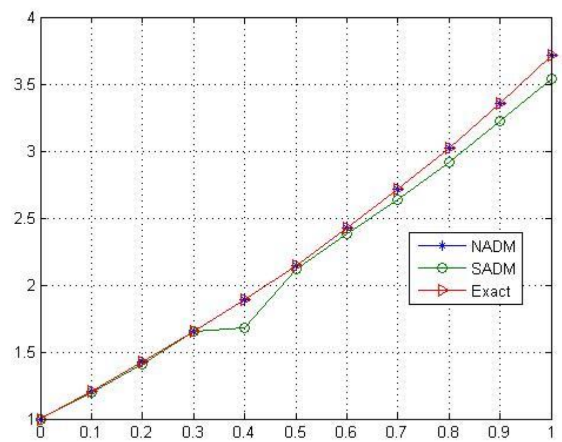

Figure 1: Numerical Comparison with SADM and Exact for Example 1

Table 2: Numerical Comparison with SADM and Exact for Example 2

X	NADM	SADM	Chriscella & Zanariah, 2017	EXACT	ERROR_SADM	ERROR_NADM
0	1	1	1	1	0	0
0.1	1.105170911	1.105349483	1.105349483	1.105170918	$1.78565000 \times 10^{-4}$	7.000×10^{-9}
0.2	1.221402710	1.222831275	1.222831275	1.221402759	$1.42851700 \times 10^{-3}$	4.900×10^{-8}
0.3	1.349858641	1.354680052	1.354680052	1.349858807	$4.82124400 \times 10^{-3}$	1.660×10^{-7}
0.4	1.491824301	1.503252833	1.503252833	1.491824697	$1.14281350 \times 10^{-2}$	3.960×10^{-7}
0.5	1.648720496	1.671041847	1.671041847	1.648721265	$2.23205760 \times 10^{-2}$	7.690×10^{-7}
0.6	1.822117462	1.860688756	1.860688756	1.822118771	$3.85699560 \times 10^{-2}$	1.309×10^{-6}
0.7	2.013750575	2.075000368	2.075000368	2.013752588	$6.12476610 \times 10^{-2}$	2.013×10^{-6}
0.8	2.225537727	2.316966008	2.316966008	2.225540527	$9.14250800 \times 10^{-2}$	2.800×10^{-6}
0.9	2.459598494	2.589776711	2.589776711	2.459601938	$1.30173600 \times 10^{-1}$	3.444×10^{-6}
1.0	2.718275336	2.896846437	2.896846437	2.718278771	$1.78564609 \times 10^{-1}$	3.435×10^{-6}

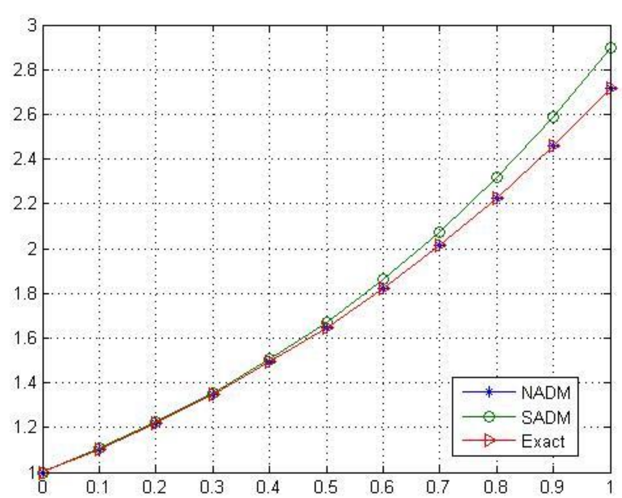

Figure 2: Numerical Comparison with SADM and Exact for Example 2

Table 3: Numerical Comparison with SADM and Exact for Example 3

X	NADM	SADM	Mohd *et. al.*, 2018	EXACT	ERROR_SADM	ERROR_NADM
0	0	0	0	0	0	0
0.1	-0.1051709139	-0.1052602003	-0.1052602003	-0.1051709181	$8.92823000 \times 10^{-5}$	4.200×10^{-9}
0.2	-0.2214027252	-0.2221170164	-0.2221170164	-0.2214027582	$7.14258400 \times 10^{-4}$	3.300×10^{-8}
0.3	-0.3498586959	-0.3522694302	-0.3522694302	-0.3498588075	$2.41062220 \times 10^{-3}$	1.116×10^{-7}
0.4	-0.4918244324	-0.3522694302	-0.3522694302	-0.4918246970	$5.71406750 \times 10^{-3}$	2.646×10^{-7}
0.5	-0.6487207484	-0.6598815591	-0.6598815591	-0.6487212651	$1.11602881 \times 10^{-2}$	5.167×10^{-7}
0.6	-0.8221178780	-0.8414037778	-0.8414037778	-0.8221187709	$1.92849778 \times 10^{-2}$	8.989×10^{-7}
0.7	-1.013751170	-1.044376537	-1.044376537	-1.013752588	$3.06238300 \times 10^{-2}$	1.418×10^{-6}
0.8	-1.225538411	-1.271253468	-1.271253468	-1.225540527	$4.57125400 \times 10^{-2}$	2.116×10^{-6}
0.9	-1.459598926	-1.524689911	-1.524689911	-1.459601938	$6.50868000 \times 10^{-2}$	3.012×10^{-6}
1.0	-1.718274637	-1.807564132	-1.807564132	-1.718278771	$8.92823040 \times 10^{-2}$	4.134×10^{-6}

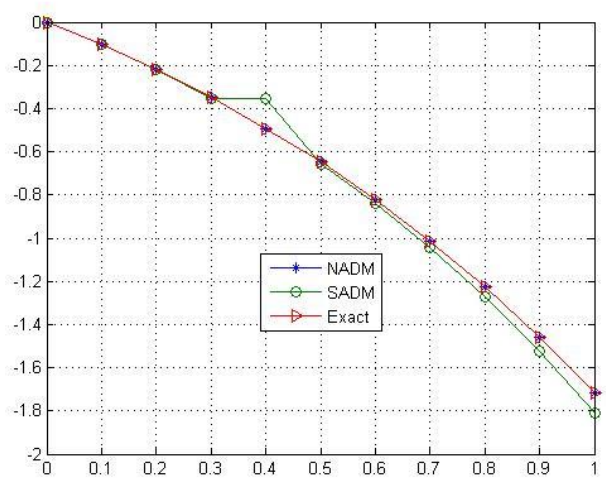

Figure 3: Numerical Comparison with SADM and Exact for Example 3

5. Discussion of Results

It could be observed in Table 1 that the new method accuracy outweighs the performance of the standard Adomian decomposition method and other method in literature. The standard Adomian decomposition method produces a maximum error of 1.7828×10^{-1} while the new method produces a maximum error of 1.6480×10^{-5} at $x = 1.0$. In Table 2, the standard Adomian decomposition method produces a maximum error of 1.7856 $\times 10^{-1}$ while the new method produces a maximum error of 3.435×10^{-6} at $x = 1.0$. And lastly in Table 3, the new method also outperforms the performance of the standard Adomian decomposition method. The standard Adomian decomposition method produces a maximum error of 8.9282×10^{-2} while the new method produces a maximum error of 4.134×10^{-6} at $x = 1.0$. From Figures 1-3, the behaviour of the standard Adomian Decomposition Method is well compared with the newly proposed method. Its superior strength is well displayed.

6. Conclusion

This new method introduced here applies effectively to the solution for the linear Fredholm Integro-Differential Equations. The proposed method takes the Taylor series expansion of the source term which needs to be of high order to make the selection as long as possible. The results obtained compared well with the exact and in some cases, they converge directly to the exact solutions with a smaller number of iterations. The proposed method is therefore concluded to be effective and efficient in the solution of linear Fredholm Integro-Differential Equations.

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