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Extended rational techniques to resonant nonlinear Schrodinger equation

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Abstract: This paper studies advanced mathematical methods like sin-cos and sinh-cosh approaches to find precise solutions by considering inter-modal dispersion and spatio-temporal dynamics with kerr law nonlinearity in the Resonant shrödinger equation. These methods are useful for solving nonlinear partial differential equations. To obtain the ordinary differential equation for the traveling wave solution, we initially deal with the general partial differential equation (PDE). Then, a series of optical soliton solutions, including cusp and dark solitons, are derived for the Resonant shrödinger equation using this effective approach.

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1 Introduction

Nonlinear partial differential equations are instrumental in modeling various phenomena in nonlinear systems, spanning disciplines like fluid mechanics, plasma physics, and electromagnetism, among others [18, 13, 11, 14, 2, 8, 21, 17, 19, 15, 3, 12, 5, 10, 20, 16]. These equations involve nonlinear terms that make it challenging to find exact solutions [4, 9, 7, 6, 22, 23, 1]. The goal of this article is to consider the Resonant Shrödinger equation with inter-modal and spatio-temporal Kerr nonlinearity can be written in the form [22]:

$$i\varrho_t + \nu\varrho_{xx} + \iota u_{xt} + \tau\varrho^3 + \mu\left(\frac{|\varrho|xx}{|\varrho|}\right)\varrho = i\kappa\varrho x. \tag{1}$$

The imaginary unit i symbolizes in the equation, where the complex field amplitude is denoted by $\varrho(x,t)$. The temporal and spatial coordinates, t and x, respectively, are integral to the equation. The parameters ν and ι represent the coefficients for group velocity dispersion and spatio-temporal dispersion, respectively. Additionally, τ signifies nonlinearity in the Kerr law. To determine

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precise solutions, one can utilize the extended rational methods of sin-cos and sinh-cosh ansatz. These approaches suggest that the field amplitude $\varrho(x,t)$ can be expressed as a combination of trigonometric or hyperbolic functions multiplied by rational functions of t and x. Inserting these ansatz into the Resonant Schrödinger equation, applying trigonometric and hyperbolic identities, and solving for the unknown coefficients entails solving a system of algebraic equations to obtain exact solutions. These exact solutions obtained through extended rational techniques offer insights into the behavior of the resonant Shrödinger equation with inter-modal dispersion and spatio-temporal Kerr nonlinearity. These methods aid in comprehending the impact of various parameters on the system's dynamics and serve as a foundation for future analysis and interpretation. Various research methodologies have been used to arrive at exact solutions for the resonant Shrödinger equation, including Lie symmetry techniques, sine-cosine methods, the $\frac{G}{G'}$ expansion method, and the extended sinh-Gordan expansion technique, among others. This article opts to utilize the rational extended sinh-cosh technique and extended rational sin-cos procedure in exploring the resonant nonlinear Shrödinger equation.

The residual parts of this article are as follows: Section 2 uses techniques of extended rational sine-cosine and sinh-cosh [9, 7, 1] are employed. Section 3 adopts the equations mentioned earlier to resonant nonlinear Shrödinger equation. Section 4 finalizes the consequences of this article by drawing 2D and 3D graphical representations. Finally, Section 5 is dedicated to a brief conclusion.

2 The proposed method

Consider the nonlinear partial differential equation of the form as

$$P(u, u_t, u_x, u_{xx}, u_{tt}, ...) = 0, \tag{2}$$

where $u(x,t)$ represents the wave function. Now, we assume a traveling wave solution of the form:

$$u(x,t) = \Xi(\xi) \tag{3}$$

$$\begin{aligned} &\frac{\partial}{\partial t} = g\frac{d}{d\xi}, \quad \frac{\partial^2}{\partial t} = g^2\frac{d^2}{d\xi^2}, \\ &\frac{\partial}{\partial x} = \frac{d}{d\xi}, \quad \frac{\partial^2}{\partial x^2} = \frac{d^2}{d\xi^2}, ... \end{aligned} \tag{4}$$

g is the wave propogation speed, Eq. (2) can be turned into the following ordinary differential equation:

$$G(\Xi, \Xi', \Xi'', ...) = 0 \tag{5}$$

2.1 Extended rational sine-cosine procedure

Step 1. Assume that Eq. (4) has the solution in the form of

$$\Xi(\xi) = \frac{\beta_0 \sin(\eta\xi)}{\beta_2 + \beta_1 \cos(\eta\xi)}, \quad \cos(\eta\xi) \neq -\frac{\beta_2}{\beta_1}, \tag{6}$$

or

$$\Xi(\xi) = \frac{\beta_0\ \cos(\eta\xi)}{\beta_2 + \beta_1 \sin(\eta\xi)}, \quad \sin(\eta\xi) \neq -\frac{\beta_2}{\beta_1}, \tag{7}$$

in which η is a wave number and β_0,β_1 and β_2 are parameters to be resolved.
Step 2. In this step, we substitute one of the equations mentioned above in Eq. (4), then gathering all terms with the identical powers of $\cos(\eta\xi)^q$ or $\sin(\eta\xi)^q$ and equalling to zero. A set of algebraic equations is derived from all coefficients of $\cos(\eta\xi)^q$ or $\sin(\eta\xi)^q$. Algebraic equations' solutions will be found with mathematics software.
Step 3. In this part, we substitute the values of β_0, β_1, β_2 and η in Eq. (6) or Eq. (7). Then we will find the solutions of the equations.

2.2 Extended rational sinh-cosh procedure

Step1. Assume that Eq. (4) has the solution in the form of

$$\Xi(\xi) = \frac{\beta_0\ \sinh(\eta\xi)}{\beta_2 + \beta_1 \cosh(\eta\xi)}, \quad \cosh(\eta\xi) \neq -\frac{\beta_2}{\beta_1},$$
$$\Xi(\xi) = \frac{\beta_0\ \cosh(\eta\xi)}{\beta_2 + \beta_1 \sinh(\eta\xi)}, \quad \sinh(\eta\xi) \neq -\frac{\beta_2}{\beta_1}, \tag{8}$$

in which η is a wave number and β_0,β_1 and β_2 are parameters to be resolved.

Step 2. In this step, we substitute one of the equations mentioned above in Eq. (4), then gathering all terms with the identical powers of $\cosh(\eta\xi)^q$ or $\sinh(\eta\xi)^q$ and equalling to zero. A set of algebraic equations is derived from all coefficients of $\cosh(\eta\xi)^q$ or $\sinh(\eta\xi)^q$. Algebraic equations's solutions will be found with mathematics software.

Step 3. In this part, we substitute the values of β_0, β_1, β_2 and η in Eqs. (8) Then we will find the solutions of the equations.

3 The implementation of these methods

Suppose the resonant nonlinear Shrödinger equation, if we use the transformations

$$\varrho(x,t) = e^{i\phi(\xi)}\Xi(\xi), \quad \xi = x - vt, \quad \phi(\xi) = -\lambda x + \rho t + \theta, \tag{9}$$

where λ, ρ, θ and v are solutions frequency, wave number, phase constant, and velocity of the solution respectively. By substituting (9) in (2) the imaginery and real parts will decompose, and we can arrive

$$(\nu - \iota v + \mu)\Xi'' + (\iota\lambda\rho - \rho - \kappa\lambda - \nu\lambda^2)\Xi + \tau\Xi^3 = 0, \tag{10}$$

$$v = \frac{\kappa + 2\nu\lambda - \iota\rho}{\iota\lambda - 1}, \tag{11}$$

3.1 The implementation of the rational extended sin-cos technique

Assume that Eq. (9) has solutions in the formation of

$$\Xi(\xi)=\frac{\beta_0\ \sin(\eta\xi)}{\beta_2+\beta_1\cos(\eta\xi)}. \tag{12}$$

By replacing Eq. (12) in Eq. (10), then gathering all terms with the identical powers of $\cos(\eta\xi)^q$ and equalling to zero. A set of algebraic equations is derived from all coefficients of $\cos(\eta\xi)^q$. The next equations are gained:

$$\cos(\eta\xi)^2:\beta_0,\left(\left(\nu\lambda^2+(-\iota\rho+\kappa)\lambda+\rho\right)\beta_1{}^2+\tau\beta_0{}^2\right)$$
$$\cos(\eta\xi)^1:2\beta_0\beta_1,\left(\nu\lambda^2+(-\iota\rho+\kappa)\lambda+(1/2\iota v-\nu/2-\mu/2)\,\eta^2+\rho\right)\beta_2$$
$$\cos(\eta\xi)^0:-2\left(\left(\left(1/2\iota v-\nu/2-\mu/2\right)\eta^2-1/2,\nu\lambda^2+\left(1/2\iota\rho-\kappa/2\right)\lambda-\rho/2\right)\beta_2{}^2+\eta^2(-\iota v+\nu+\mu)\beta_1{}^2+1/2\tau\beta_0{}^2\right)\beta_0$$

These algebraic equations resolve, we will find the next solutions:
Part 1)

$$\eta=\pm\sqrt{-\frac{-\nu\lambda^2+\iota\lambda\,\rho-\lambda\,\kappa-\rho}{-2\,\iota v+2\,\nu+2\,\mu}},\quad \beta_0=\pm\sqrt{-\frac{\nu\lambda^2-\iota\lambda\,\rho+\lambda\,\kappa+\rho}{\tau}}\beta_1\quad \beta_1=\beta_1,\quad \beta_2=0 \tag{13}$$

Part 2)

$$\eta=\pm\sqrt{-\frac{-2\,\nu\lambda^2+2\,\lambda\,\iota\rho-2\,\lambda\,\kappa-2\,\rho}{-\iota v+\nu+\mu}},\quad \beta_0=\pm\sqrt{-\frac{\nu\lambda^2-\lambda\,\iota\rho+\lambda\,\kappa+\rho}{\tau}}\beta_2,\quad \beta_1=-\beta_2,\quad \beta_2=\beta_2 \tag{14}$$

part 3)

$$\eta=\pm\sqrt{-\frac{-2\,\nu\lambda^2+2\,\iota\lambda\,\rho-2\,\lambda\,\kappa-2\,\rho}{-\iota v+\nu+\mu}},\quad \beta_0=\pm\sqrt{-\frac{\nu\lambda^2-\iota\lambda\,\rho+\lambda\,\kappa+\rho}{\tau}}\beta_2,\ \beta_1=\beta_2,\quad \beta_2=\beta_2 \tag{15}$$

Case 1: Taking part 1 into consideration, the solution of Eq. (10) can be got as

$$\Xi_1(\xi)=\pm\sqrt{-\frac{\nu\lambda^2-\iota\lambda\,\rho+\lambda\,\kappa+\rho}{\tau}}\tan\left(\sqrt{-\frac{-\nu\lambda^2+\iota\lambda\,\rho-\lambda\,\kappa-\rho}{-2\,\iota v+2\,\mu+2\,\mu}}\xi\right) \tag{16}$$

By combining Equations (9) and (16) , we get

$$\varrho_1(x,t)=\pm\sqrt{-\frac{\nu\lambda^2-\iota\lambda\,\rho+\lambda\,\kappa+\rho}{\tau}}\tan\left(\sqrt{-\frac{-\nu\lambda^2+\iota\lambda\,\rho-\lambda\,\kappa-\rho}{-2\,\iota v+2\,\nu+2\,\mu}}\,(-tv+x)\right)\mathrm{e}^{i(-\lambda\,x+\rho\,t+\theta)} \tag{17}$$

Case 2: Likewise, for part 2, the solution of (10) can be got as

$$\Xi_2(\xi) = \pm\sqrt{-\frac{\nu\lambda^2 - \lambda\iota\rho + \lambda\kappa + \rho}{\tau}}\sin\left(\sqrt{-\frac{-2\nu\lambda^2 + 2\lambda\iota\rho - 2\lambda\kappa - 2\rho}{-\iota v + \nu + \mu}}\xi\right)$$
$$\times\left(1 + \cos\left(\sqrt{-\frac{-2\nu\lambda^2 + 2\lambda\iota\rho - 2\lambda\kappa - 2\rho}{-\iota v + \nu + \mu}}\xi\right)\right)^{-1} \tag{18}$$

By combining Equations (9) and (18), we get

$$\varrho_2(x,t) = \pm\sqrt{-\frac{\nu\lambda^2 - \lambda\,\iota\rho + \lambda\,\kappa + \rho}{\tau}}\sin\left(\sqrt{-\frac{-2\,\nu\lambda^2 + 2\,\lambda\,\iota\rho - 2\,\lambda\,\kappa - 2\,\rho}{-\iota v + \nu + \mu}}\,(-tv + x)\right)$$
$$\times\left(1 + \cos\left(\sqrt{-\frac{-2\,\nu\lambda^2 + 2\,\lambda\,\iota\rho - 2\,\lambda\,\kappa - 2\,\rho}{-\iota v + \nu + \mu}}\,(-tv + x)\right)\right)^{-1} \mathrm{e}^{i(-\lambda\,x + \rho\,t + \theta)} \tag{19}$$

Case 3: Likewise, for part 3, the solutions of Eq. (10) can be obtained as

$$\Xi_3(\xi) = \pm\sqrt{-\frac{\nu\lambda^2 - \iota\lambda\,\rho + \lambda\,\kappa + \rho}{\tau}}\sin\left(\sqrt{-\frac{-2\,\nu\lambda^2 + 2\,\iota\lambda\,\rho - 2\,\lambda\,\kappa - 2\,\rho}{-\iota v + \nu + \mu}}\,(-tv + x)\right)$$
$$\times\left(1 - \cos\left(\sqrt{-\frac{-2\,\nu\lambda^2 + 2\,\iota\lambda\,\rho - 2\,\lambda\,\kappa - 2\,\rho}{-\iota v + \nu + \mu}}\,(-tv + x)\right)\right)^{-1} \tag{20}$$

By combining Eqs. (9) and (20), we get

$$\varrho_3(x,t) = \pm\sqrt{-\frac{\nu\lambda^2 - \iota\lambda\,\rho + \lambda\,\kappa + \rho}{\tau}}\sin\left(\sqrt{-\frac{-2\,\nu\lambda^2 + 2\,\iota\lambda\,\rho - 2\,\lambda\,\kappa - 2\,\rho}{-\iota v + \nu + \mu}}\,(-tv + x)\right) \tag{21}$$
$$\times\left(1 - \cos\left(\sqrt{-\frac{-2\,\nu\lambda^2 + 2\,\iota\lambda\,\rho - 2\,\lambda\,\kappa - 2\,\rho}{-\iota v + \nu + \mu}}\,(-tv + x)\right)\right)^{-1} \mathrm{e}^{i(-\lambda\,x + \rho\,t + \theta)} \tag{22}$$

OR

Assume that Eq. (10) has the solutions in the formation of

$$\Xi(\xi) = \frac{\beta_0\,\cos(\eta\xi)}{\beta_2 + \beta_1\sin(\eta\xi)} \tag{23}$$

By replacing Eq.(22) in Eq.(10), then gathering all terms with the identical powers of $\sin(\eta\xi)^q$ and equalling to zero. A set of algebraic equations is derived from all coefficients of $\sin(\eta\xi)^q$. The next equations are gained:

$\sin(\eta\xi)^2 : \beta_0\left(-\left(\nu\lambda^2 - (-\iota\rho + \kappa)\lambda + \rho\right)\beta_1{}^2 - \tau\beta_0{}^2\right)$

$\sin(\eta\xi)^1 : -2\beta_0\beta_2\beta_1\left(\nu\lambda^2 - (-\iota\rho + \kappa)\lambda + \left(\frac{1}{2}\iota v - \frac{\nu}{2} - \frac{\mu}{2}\right)\eta^2 + \rho\right)$

$\sin(\eta\xi)^0 : 2\left(\left(\left(\frac{1}{2}\iota v - \frac{\nu}{2} - \frac{\mu}{2}\right)\eta^2 - \frac{1}{2}\nu\lambda^2 + \left(\frac{1}{2}\iota\rho - \frac{\kappa}{2}\right)\lambda - \frac{\rho}{2}\right)\beta_2{}^2 + \beta_1{}^2\left(-\iota v + \nu + \mu\right)\eta^2 + \frac{1}{2}\tau\beta_0{}^2\right)\beta_0$

These algebraic equations resolve, we will find the next solutions:
Part 4)

$$\eta=\pm\sqrt{-\frac{-\nu\lambda^2+\lambda\iota\rho-\lambda\kappa-\rho}{-2\iota v+2\nu+2\mu}},\quad \beta_0=\pm\sqrt{-\frac{\nu\lambda^2-\lambda\iota\rho+\lambda\kappa+\rho}{\tau}}\beta_1,\ \beta_1=\beta_1,\quad \beta_2=0 \tag{24}$$

Part 5)

$$\eta=\pm\sqrt{-\frac{-2\nu\lambda^2+2\iota\lambda\rho-2\kappa\lambda-2\rho}{-\iota v+\nu+\mu}},\quad \beta_0=\pm\sqrt{-\frac{\nu\lambda^2-\iota\lambda\rho+\kappa\lambda+\rho}{\tau}}\beta_2,\ \beta_1=\beta_2,\quad \beta_2=\beta_2 \tag{25}$$

Part 6)

$$\eta=\pm\sqrt{-\frac{-2\nu\lambda^2+2\iota\lambda\rho-2\kappa\lambda-2\rho}{-\iota v+\nu+\mu}},\quad \beta_0=\pm\sqrt{-\frac{\nu\lambda^2-\iota\lambda\rho+\kappa\lambda+\rho}{\tau}}\beta_2,\ \beta_1=-\beta_2,\quad \beta_2=\beta_2 \tag{26}$$

Case 4: Taking part 4 into consideration, the solution of Eq. (10) can be obtained as

$$\Xi_4(\xi)=\pm\sqrt{-\frac{\nu\lambda^2-\lambda\iota\rho+\lambda\kappa+\rho}{\tau}}\cot\left(\sqrt{-\frac{-\nu\lambda^2+\lambda\iota\rho-\lambda\kappa-\rho}{-2\iota v+2\nu+2\mu}}\xi\right) \tag{27}$$

By combining Eqs. (9) and (27), we get

$$\varrho_4(x,t)=\pm\sqrt{-\frac{\nu\lambda^2-\lambda\,\iota\rho+\lambda\,\kappa+\rho}{\tau}}\cot\left(\sqrt{-\frac{-\nu\lambda^2+\lambda\,\iota\rho-\lambda\,\kappa-\rho}{-2\,\iota v+2\,\nu+2\,\mu}}\,(-tv+x)\right)\mathrm{e}^{i(-\lambda\,x+\rho\,t+\theta)} \tag{28}$$

Case 5: Likewise, for part 5, the solution of Eq. (9) can be obtained as

$$\Xi_5(\xi)=\pm\sqrt{-\frac{\nu\lambda^2-\iota\lambda\,\rho+\kappa\,\lambda+\rho}{\tau}}\cos\left(\sqrt{-\frac{-2\,\nu\lambda^2+2\,\iota\lambda\,\rho-2\,\kappa\,\lambda-2\,\rho}{-\iota v+\nu+\mu}}\xi\right)$$
$$\times\left(1+\sin\left(\sqrt{-\frac{-2\,\nu\lambda^2+2\,\iota\lambda\,\rho-2\,\kappa\,\lambda-2\,\rho}{-\iota v+\nu+\mu}}\xi\right)\right)^{-1} \tag{29}$$

By combining Eqs. (9) and (29), we get

$$\varrho_5(x,t)=\pm\sqrt{-\frac{\nu\lambda^2-\iota\lambda,\rho+\kappa,\lambda+\rho}{\tau}}\cos\left(\sqrt{-\frac{-2,\nu\lambda^2+2,\iota\lambda,\rho-2,\kappa,\lambda-2,\rho}{-\iota v+\nu+\mu}}\,(-tv+x)\right)$$
$$\times\left(1+\sin\left(\sqrt{-\frac{-2,\nu\lambda^2+2,\iota\lambda,\rho-2\kappa,\lambda-2,\rho}{-\iota v+\nu+\mu}}\,(-tv+x)\right)\right)^{-1}e^{i(-\lambda,x+\rho,t+\theta)} \tag{30}$$

Case 6 : Likewise, for part 6, the solutions of Eq. (10) can be obtained as

$$\Xi_6(\xi) = \pm\sqrt{-\frac{a\lambda^2 - b\lambda\,\rho + \kappa\,\lambda + \rho}{c}}\cos\left(\sqrt{-\frac{-2\,a\lambda^2 + 2\,b\lambda\,\rho - 2\,\kappa\,\lambda - 2\,\rho}{-bv + a + \mu}}\xi\right) \times \left(1 - \sin\left(\sqrt{-\frac{-2\,\nu\lambda^2 + 2\,\iota\lambda\,\rho - 2\,\kappa\,\lambda - 2\,\rho}{-\iota v + \nu + \mu}}\xi\right)\right)^{-1} \tag{31}$$

By combining Eqs. (9) and (31), we get

$$\varrho_6(x,t) = \pm\sqrt{-\frac{\nu\lambda^2 - \iota\lambda\,\rho + \kappa\,\lambda + \rho}{\tau}}\cos\left(\sqrt{-\frac{-2\,\nu\lambda^2 + 2\,\iota\lambda\,\rho - 2\,\kappa\,\lambda - 2\,\rho}{-\iota v + \nu + \mu}}\,(-tv + x)\right) \times \left(1 - \sin\left(\sqrt{-\frac{-2\,\nu\lambda^2 + 2\,\iota\lambda\,\rho - 2\,\kappa\,\lambda - 2\,\rho}{-\iota v + \nu + \mu}}\,(-tv + x)\right)\right)^{-1} \mathrm{e}^{i(-\lambda\,x + \rho\,t + \theta)} \tag{32}$$

3.2 extended rational sinh-cosh technique

Assume that Eq. (10) has solutions in the formation of

$$\Xi(\xi) = \frac{\beta_0\,\sinh(\eta\xi)}{\beta_2 + \beta_1\cosh(\eta\xi)} \tag{33}$$

By replacing Eqs. (10) and (33), then gathering all terms with the identical powers of $\cosh(\eta\xi)^q$ and equalling to zero. A set of algebraic equations is derived from all coefficients of $\cosh(\eta\xi)^q$. The next equations are gained:

$$\begin{aligned}
&\cosh(\eta\xi)^2 : \beta_0\left(\left(\nu\lambda^2 + (-\iota\rho + \kappa)\,\lambda + \rho\right)\beta_1{}^2 - c\beta_0{}^2\right)\\
&\cosh(\eta\xi)^1 : 2\,\beta_0\,\beta_2\left(\nu\lambda^2 + (-\iota\rho + \kappa)\,\lambda + (-1/2\,\iota v + \nu/2 + \mu/2)\,\eta^2 + \rho\right)\beta_1\\
&\cosh(\eta\xi)^0 : 2\,\beta_0\left(\left((1/2\,\iota v - \nu/2 - \mu/2)\,\eta^2 + 1/2\,\nu\lambda^2 + (-1/2\,\iota\rho + \kappa/2)\,\lambda + \rho/2\right)\beta_2{}^2\right.\\
&\qquad\left. + \eta^2\,(-\iota v + \nu + \mu)\,\beta_1{}^2 + 1/2\,\tau\beta_0{}^2\right.
\end{aligned}$$

These algebraic equations resolve, we will find the next solutions:
Part 7)

$$\eta = \pm\sqrt{-\frac{\nu\lambda^2 - \lambda\,\iota\rho + \lambda\,\kappa + \rho}{-2\,\iota v + 2\,\nu + 2\,\mu}}, \quad \beta_0 = \pm\sqrt{-\frac{-\nu\lambda^2 + \lambda\,\iota\rho - \lambda\,\kappa - \rho}{\tau}}\beta_1 \quad \beta_1 = \beta_1, \quad \beta_2 = 0 \tag{34}$$

Part 8)

$$\eta = \pm\sqrt{-\frac{2\,\nu\lambda^2 - 2\,\iota\lambda\,\rho + 2\,\lambda\,\kappa + 2\,\rho}{-\iota v + \nu + \mu}}, \quad \beta_0 = \pm\sqrt{-\frac{-\nu\lambda^2 + \iota\lambda\,\rho - \lambda\,\kappa - \rho}{\tau}}\beta_2 \quad \beta_1 = \beta_1, \quad \beta_2 = \beta_2 \tag{35}$$

Part 9)

$$\eta=\pm\sqrt{-\frac{2\,\nu\lambda^2-2\,\iota\lambda\,\rho+2\,\lambda\,\kappa+2\,\rho}{-\iota v+\nu+\mu}},\quad \beta_0=\pm\sqrt{-\frac{-\nu\lambda^2+\iota\lambda\,\rho-\lambda\,\kappa-\rho}{\tau}}\beta_2\quad \beta_1=-\beta_1,\quad \beta_2=\beta_2. \tag{36}$$

Case 7: Taking part 7 into consideration, the solution of Eq. (9) can be obtained as

$$\Xi_7(\xi)=\pm\sqrt{-\frac{-\nu\lambda^2+\lambda\,\iota\rho-\lambda\,\kappa-\rho}{\tau}}\tanh\left(\sqrt{-\frac{\nu\lambda^2-\lambda\,\iota\rho+\lambda\,\kappa+\rho}{-2\,\iota v+2\,\nu+2\,\mu}}\xi\right). \tag{37}$$

By combining Equations Eqs. (9) and (37), we get

$$\varrho_7(x,t)=\pm\sqrt{-\frac{-\nu\lambda^2+\lambda\,\iota\rho-\lambda\,\kappa-\rho}{\tau}}\tanh\left(\sqrt{-\frac{\nu\lambda^2-\lambda\,\iota\rho+\lambda\,\kappa+\rho}{-2\,\iota v+2\,\nu+2\,\mu}}\,(-tv+x)\right)\mathrm{e}^{i(-\lambda\,x+\rho\,t+\theta)} \tag{38}$$

Case 8: Likewise, for part 8, the solution of (10) can be obtained as

$$\Xi_8(\xi)=\pm\sqrt{-\frac{-\nu\lambda^2+\iota\lambda\,\rho-\lambda\,\kappa-\rho}{\tau}}\sinh\left(\sqrt{-\frac{2\,\nu\lambda^2-2\,\iota\lambda\,\rho+2\,\lambda\,\kappa+2\,\rho}{-\iota v+\nu+\mu}}\xi\right)$$
$$\times\left(1+\cosh\left(\sqrt{-\frac{2\,\nu\lambda^2-2\,\iota\lambda\,\rho+2\,\lambda\,\kappa+2\,\rho}{-\iota v+\nu+\mu}}\xi\right)\right)^{-1}. \tag{39}$$

By combining Eqs. (9) and (39), we get

$$\varrho_8(x,t)=\sqrt{-\frac{-\nu\lambda^2+\iota\lambda\,\rho-\lambda\,\kappa-\rho}{\tau}}\sinh\left(\sqrt{-\frac{2\,\nu\lambda^2-2\,\iota\lambda\,\rho+2\,\lambda\,\kappa+2\,\rho}{-\iota v+\nu+\mu}}\,(-vt+x)\right)$$
$$\times\left(1+\cosh\left(\sqrt{-\frac{2\,\nu\lambda^2-2\,\iota\lambda\,\rho+2\,\lambda\,\kappa+2\,\rho}{-\iota v+\nu+\mu}}\,(-vt+x)\right)\right)^{-1}\mathrm{e}^{i(-\lambda\,x+\rho\,t+\theta)}. \tag{40}$$

Case 9 : Likewise, for part 9, the solutions of (10) can be obtained as

$$\Xi_9(\xi)=\pm\sqrt{-\frac{-\nu\lambda^2+\iota\lambda\rho-\lambda\kappa-\rho}{\tau}}\sinh\left(\sqrt{-\frac{2\nu\lambda^2-2\iota\lambda\rho+2\lambda\kappa+2\rho}{-\iota v+\nu+\mu}}\xi\right)$$
$$\times\left(1-\cosh\left(\sqrt{-\frac{2\nu\lambda^2-2\iota\lambda\rho+2\lambda\kappa+2\rho}{-\iota v+\nu+\mu}}\xi\right)\right)^{-1}. \tag{41}$$

By combining Eqs. (10) and (41), we get

$$\varrho_9(x,t)=\pm\sqrt{-\frac{-\nu\lambda^2+\iota\lambda\rho-\lambda\kappa-\rho}{\tau}}\sinh\left(\sqrt{-\frac{2\nu\lambda^2-2\iota\lambda\rho+2\lambda\kappa+2\rho}{-\iota v+\nu+\mu}}(-vt+x)\right)$$
$$\times\left(1-\cosh\left(\sqrt{-\frac{2\nu\lambda^2-2\iota\lambda\rho+2\lambda\kappa+2\rho}{-\iota v+\nu+\mu}}(-vt+x)\right)\right)^{-1}e^{i(-\lambda x+\rho t+\theta)}. \tag{42}$$

OR
Assume that Eq. (10) has solutions in the formation of

$$\Xi(\xi)=\frac{\beta_0\,\cosh(\eta\xi)}{\beta_2+\beta_1\sinh(\eta\xi)}. \tag{43}$$

By replacing Eqs. (10) and (43), then gathering all terms with the identical powers of $\sinh(\eta\xi)^q$ and equalling to zero. A set of algebraic equations is derived from all coefficients of $\sinh(\eta\xi)^q$. The next equations are gained:

$$\begin{aligned}
&\sinh(\eta\xi)^2 : \left(\left(\nu\lambda^2+(-\iota\rho+\kappa)\,\lambda+\rho\right)\beta_1{}^2-\tau\beta_0{}^2\right)\beta_0\\
&\sinh(\eta\xi)^1 : 2\,\beta_1,\left(\nu\lambda^2+(-\iota\rho+\kappa)\,\lambda+(-1/2\,\iota\upsilon+\nu/2+\mu/2)\,\eta^2+\rho\right)\beta_2\beta_0\\
&\sinh(\eta\xi)^0 : -2\,\Big(\left((-1/2\,\iota\upsilon+\nu/2+\mu/2)\,\eta^2-1/2\,\nu\lambda^2+(1/2\,\iota\rho-\kappa/2)\,\lambda-\rho/2\right)\beta_2{}^2\\
&\qquad +\beta_1{}^2\,(-\iota\upsilon+\nu+\mu)\,\eta^2+1/2\,\tau\beta_0{}^2\Big)\beta_0.
\end{aligned}$$

These algebraic equations resolve, we will find the next solutions:
set 10)

$$\eta=\pm\sqrt{-\frac{\nu\lambda^2-\lambda\,\iota\rho+\lambda\,\kappa+\rho}{-2\,\iota\upsilon+2\,\nu+2\,\mu}},\quad \beta_0=\pm\sqrt{-\frac{-\nu\lambda^2+\lambda\,\iota\rho-\lambda\,\kappa-\rho}{\tau}}\beta_1\quad \beta_1=\beta_1,\quad \beta_2=0. \tag{44}$$

Part 11)

$$\eta=\pm\sqrt{-\frac{2\,\nu\lambda^2-2\,\lambda\,\iota\rho+2\,\lambda\,\kappa+2\,\rho}{-\iota\upsilon+\nu+\mu}},\quad \beta_0=\pm\sqrt{-\frac{\nu\lambda^2-\lambda\,\iota\rho+\lambda\,\kappa+\rho}{\tau}}\beta_2\quad \beta_1=I\beta_2,\quad \beta_2=\beta_2. \tag{45}$$

Case 10: Taking part 10 into consideration, the solution of (10) can be obtained as

$$\Xi_{10}(\xi)=\pm\sqrt{-\frac{-\nu\lambda^2+\lambda\,\iota\rho-\lambda\,\kappa-\rho}{\tau}}\coth\left(\sqrt{-\frac{\nu\lambda^2-\lambda\,\iota\rho+\lambda\,\kappa+\rho}{-2\,\iota\upsilon+2\,\nu+2\,\mu}}\xi\right) \tag{46}$$

By combining Eqs. (9) and (46), we get

$$\varrho_{10}(x,t)=\pm\sqrt{-\frac{-\nu\lambda^2+\lambda\,\iota\rho-\lambda\,\kappa-\rho}{\tau}}\coth\left(\sqrt{-\frac{\nu\lambda^2-\lambda\,\iota\rho+\lambda\,\kappa+\rho}{-2\,\iota\upsilon+2\,\nu+2\,\mu}}\,(-t\upsilon+x)\right)\mathrm{e}^{i(-\lambda\,x+\rho\,t+\theta)}. \tag{47}$$

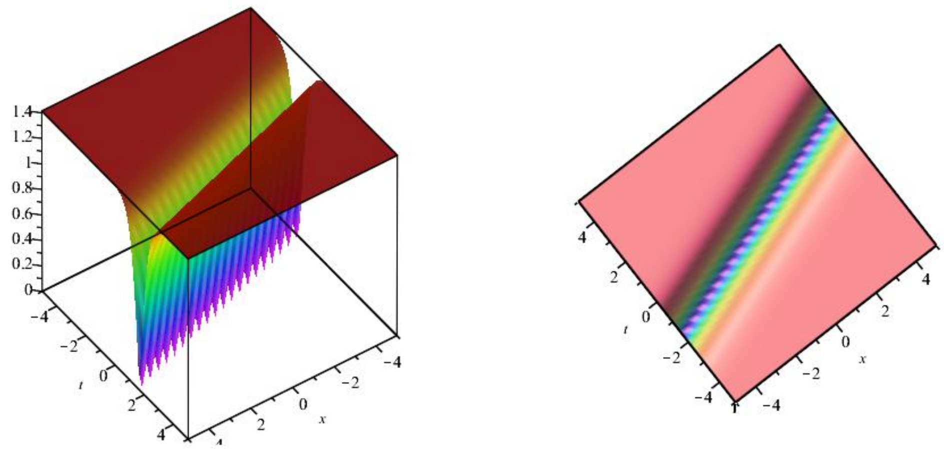
$$\begin{aligned}
\Xi_{11}(\xi)=\pm\sqrt{-\frac{\nu\lambda^2-\lambda\iota\rho+\lambda\kappa+\rho}{\tau}}\,\cosh\left(\sqrt{-\frac{2\nu\lambda^2-2\lambda\iota\rho+2\lambda\kappa+2\rho}{-\iota\upsilon+\nu+\mu}}\xi\right)\\
\times\left(1+i\sinh\left(\sqrt{-\frac{2\nu\lambda^2-2\lambda\iota\rho+2\lambda\kappa+2\rho}{-\iota\upsilon+\nu+\mu}}\xi\right)\right)^{-1}.
\end{aligned} \tag{48}$$

By combining Eqs. (10) and (48), we get

$$\varrho_{11}(x,t) = \pm\sqrt{-\frac{\nu\lambda^2 - \lambda\iota\rho + \lambda\kappa + \rho}{\tau}}\cosh\left(\sqrt{-\frac{2\nu\lambda^2 - 2\lambda\iota\rho + 2\lambda\kappa + 2\rho}{-\iota v + \nu + \mu}}\,(-tv + x)\right)$$
$$\times\left(1 + i\sinh\left(\sqrt{-\frac{2\nu\lambda^2 - 2\lambda\iota\rho + 2\lambda\kappa + 2\rho}{-\iota v + \nu + \mu}}\,(-tv + x)\right)\right)^{-1} e^{i(-\lambda x + \rho t + \theta)}. \quad (49)$$

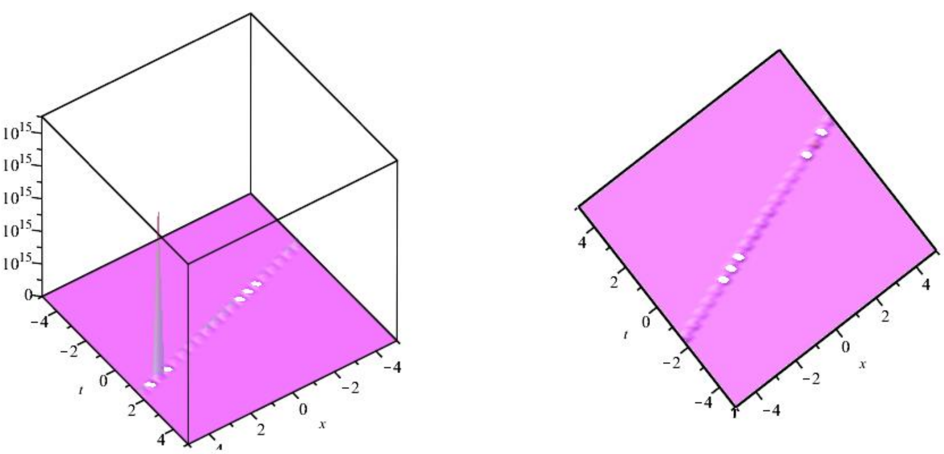
4 Graphical Representaion

In this section, 3D and contour surface visualizations were created using selected parameter values to showcase key findings from the analysis. The graphics portray the system's behavior and offer visual insights into the observed phenomena. Notably, the figures reveal the presence of sharp cusps in waveforms (Figures 2 and 4), which are distinctive pointed features associated with certain types of nonlinear waves. These cusps indicate nonlinear effects within the system and studying them aids in grasping the dynamics and characteristics of the waves. Additionally, the analysis highlights the existence of dark waves (Figures 1 and 3), which are localized structures or solitons propagating with reduced amplitudes compared to the surrounding waves. The formation and propagation of these dark and cusp waves are further illustrated through contour surface plots, offering valuable understanding of their behavior. By examining these visuals, one can delve deeper into the intricate dynamics and nonlinear influences at play in the system, facilitating the interpretation and analysis of the conclusions drawn.



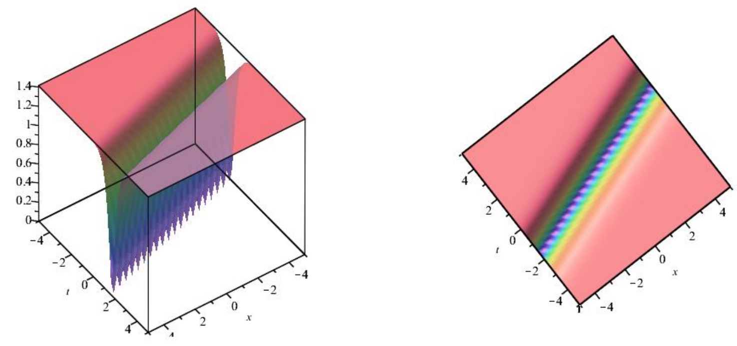
(a) (b)

Figure 1: The three-dimensional surfaces of Equation (17) with the values $\nu = 2$, $\iota = 2$, $\tau = -1$, $\mu = 1$, $\kappa = 1$, $\lambda = 1$, $\rho = 1$ in figures (a) and (b). The contour surfaces of Equation (17) are shown with $\nu = 2$, $\iota = 2$, $\tau = -1$, $\kappa = 1$, $\mu = 2$, $\lambda = 1$, $\rho = 1$ in figures (a) and (b).



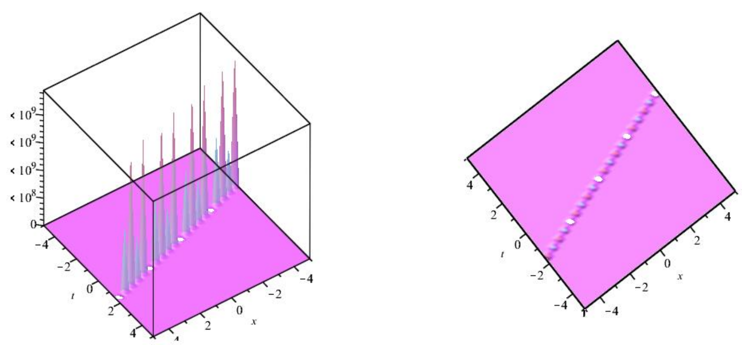
(a) (b)

Figure 2: The three-dimensional surfaces of Equation (27) with the values $\nu = 2$, $\iota = 2$, $\tau = -1$, $\mu = 1$, $\kappa = 1$, $\lambda = 1$, $\rho = 1$ in figures (a) and (b). The contour surfaces of Equation (27) are shown with $\nu = 2$, $\iota = 2$, $\tau = -1$, $\kappa = 1$, $\mu = 1$, $\lambda = 1$, $\rho = 1$ in figures (a) and (b).



(a) (b)

Figure 3: The three-dimensional surfaces of Equation (37) with the values $\nu = 2$, $\iota = 2$, $\tau = -1$, $\mu = 1$, $\kappa = 1$, $\lambda = 1$, $\rho = 1$ in figures (a) and (b). The contour surfaces of Equation (37) are shown with $\nu = 2$, $\iota = 2$, $\tau = -1$, $\kappa = 1$, $\mu = 1$, $\lambda = 1$, $\rho = 1$ in figures (a) and (b).



(a) (b)

Figure 4: The three-dimensional surfaces of Equation (46) with the values $\nu = 2$, $\iota = 2$, $\tau = -1$, $\mu = 1$, $\kappa = 1$, $\lambda = 1$, $\rho = 1$ in figures (c) and (d). The contour surfaces of Equation (46) are shown with $\nu = 2$, $\iota = 2$, $\tau = -1$, $\kappa = 1$, $\mu = 1$, $\lambda = 1$, $\rho = 1$ in figures (c) and (d).

5 Conclusions

This paper applied advanced mathematical techniques such as sin-cos and sinh-cosh methods to achieve accurate solutions while considering inter-modal dispersion and spatio-temporal dynamics with kerr law nonlinearity in the Resonant Shrödinger equation. These methodologies are effective for solving nonlinear partial differential equations. Initially addressing the general partial differential equation (PDE) to derive the ordinary differential equation for the traveling wave solution, the study then utilized these methods to obtain a series of optical soliton solutions, including cusp and dark solitons, for the Resonant Shrödinger equation.

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