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Parametric distance measure for trapezoidal intuitionistic fuzzy numbers and application in Multi-criteria group decision-making

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Abstract: It is important to have an intuitionistic fuzzy set that allows each set element to have a membership value, a non-membership value, and a hesitancy value. This is because each element of the set can possess all three values. We will focus on one type of continuous intuitionistic fuzzy number, called trapezoidal intuitionistic fuzzy numbers, because they are more flexible in representing information about membership and non-membership functions and are continuous. This research aims to introduce a parametric ranking and distance measure to compare and obtain the distinction value between intuitionistic trapezoidal fuzzy numbers. Parametric measures offer more flexibility than deterministic measurement tools in modeling real-world problems by considering a suitable variety of responses based on different levels of parameters. After presenting the structure and effective indicators of the proposed tool, we have detailed its features and basic principles. Moreover, based on this measure, a hybrid process is designed for multi-criteria group decision-making (MCGDM) problems with trapezoidal intuitionistic fuzzy data. A numerical example is also examined to elucidate the implementation process of this integrated methodology. Additionally, comparative analysis with some related methods confirms the adequate performance of the new parametric measure in combined methods with similar subjects.

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1 Introduction

Incomplete and inaccurate information is an inseparable part of many practical problems. Without appropriate adaptive tools, bias and rigid thinking can lead to unsuitable solutions. Classical analysis methods are primarily compatible with numerical techniques. However, human understanding and knowledge, at their core, focus more on processing information beyond exact numbers. To achieve a constructive

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match between understanding and solution patterns, fuzzy set theory (FS), introduced by [70], has been used as an efficient tool to address certain types of ambiguity and uncertainty. Numerous applications of this theory have emerged in mathematical modeling, industrial engineering, and decision-making fields [11, 15, 25].

The most essential feature of any fuzzy set is the assignment of a value called the membership degree to each element in the set. In recent decades, various theoretical and practical advancements have been made in basic fuzzy sets. The intuitionistic fuzzy set (IFS), introduced by Atanassov in 1986, is one of the most significant generalizations of the fuzzy set [33]. The characteristics of the membership degree, non-membership degree, and hesitancy degree distinguish the elements of this type of set. A key topic in intuitionistic fuzzy numbers has been the comparison, ranking, and determination of their distance and similarity measures. In general, the theory of measures, by introducing various criteria for evaluation, has enabled the comparison of the information available in intuitionistic fuzzy sets. Distance, similarity, and entropy measures are recognized as effective tools for solving many decision-making problems, including medical diagnosis, pattern recognition, machine learning, and image processing.

Multi-criteria decision-making (MCDM) is one of the most widely used and essential tools for managers when selecting the best alternative based on the value of the options compared to the relevant criteria in environments with intuitionistic fuzzy data. Over time, the size and complexity of MCDM problems have increased significantly. Consequently, experts have faced various challenges in decision-making methods due to uncertainty and subjective judgments. Structurally, broad categories such as numerical, outranking, distance-based, pairwise comparison, and other strategies have been explored as solution approaches. Among these, the distance-based approach—more popular than other methods—includes techniques such as Complex Proportional Assessment (COPRAS) [14], Data Envelopment Analysis (DEA) [21], Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) [42, 22], Compromise Programming (CP) [24], and VlseKriterijumska Optimizacija I KOmpromisno Resenje (VIKOR) [10], among others [27].

Among the distance-based processes for solving MCDM problems, the TOPSIS approach is widely recognized as the most frequently applied. Numerous researchers have studied this method or developed it in combination with other approaches to address MCDM challenges. The primary concept of the TOPSIS method was first described by Hwang and Chen [36].

Over the years, many modifications and improvements have been made to the theoretical foundations and practical applications of the IF-TOPSIS method. For example, in 2009, Boran et al. [5] generalized the TOPSIS technique to MCGDM problems involving intuitionistic fuzzy (IF) data and weights. Several authors have also extended variations of IF-TOPSIS using IF or linguistic weights [1, 7, 8, 23]. Furthermore, real-life decision-making problems have been simulated using these methods [34, 26, 64, 71].

Figure 1 illustrates the general structure, advantages, and limitations of the IF-TOPSIS method.

It is clear that, to choose the best alternative under uncertainty, an appropriate comparative tool from information theory is required. Along with the development of methods, various measurement operators have been proposed by researchers to address multi-criteria decision-making (MCDM) problems in environments with intuitionistic fuzzy data.

In 2009, Jianqiang and Zhong [13] extended the concepts of the weighted arithmetic averaging and weighted geometric averaging operators to TrIFNs, applying them to MADM problems. Later, Wei [62] generalized some arithmetic aggregation operators and applied them to MAGDM problems based on TrIFNs. Researchers such as Wan and Dong [29] constructed the expectation rank and various aggregation operators for TrIFNs.

On the other hand, Ye [67] introduced the expected value method for intuitionistic trapezoidal fuzzy multi-criteria decision-making problems. Additionally, Ye (Jun Ye, 2012b) described the MAGDM method using vector similarity measures for TrIFNs. Furthermore, Wu and Cao [63] generalized some geometric aggregation operators with TrIFNs. In another study, Wan [28] extended the power average operators of TrIFNs and applied them to MAGDM.

After defining the principles of similarity measures for IFSs by Li and Cheng [47], Xu [65] proposed

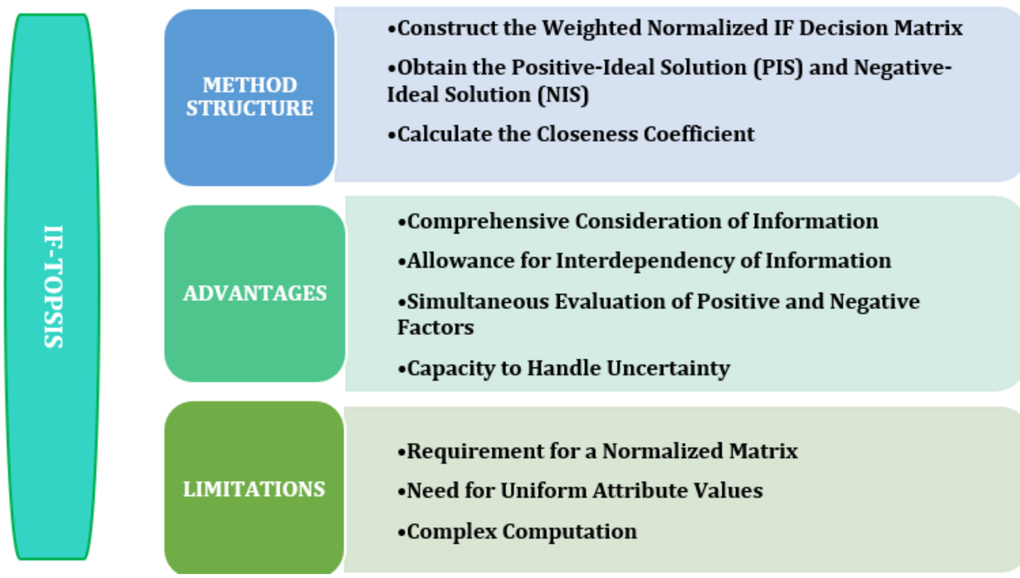


Figure 1: Overview of the IF-TOPSIS Method.

several similarity measure definitions for this type of number and demonstrated their applications in solving the MADM problem. Xu and Chen [66] defined continuous distance and similarity measures for IFSs based on the geometric distance model, including the weighted Hamming distance, weighted Euclidean distance, and weighted Hausdorff distance.

Upon closer examination of the existing measures, we find that developing a measure, especially of the parametric and robust type, is quite complex. Few measures can fully satisfy the axiomatic definition of a measure while addressing non-intuitionistic cases across all problems. Additionally, some measures either lack physical meaning or do not have simplified expressions. Therefore, it is essential to define a more intuitive and conceptual measure for trapezoidal intuitionistic fuzzy numbers. Considering these challenges and the flexibility of parametric distance measures, this research aims to introduce a parametric distance measure for trapezoidal intuitionistic fuzzy numbers that adheres to the principles of measurement. We will apply this measure within the TOPSIS method to solve multi-criteria decision-making (MCDM) problems.

To achieve this goal, the structure of this research is as follows: In Section 2, we discuss the basic concepts of intuitionistic fuzzy numbers and the principles related to distance measures. In Section 3, the ranking of trapezoidal intuitionistic fuzzy numbers is presented, along with the introduction of the new parametric distance measure and supporting theorems. Section 4 introduces the problem and presents a hybrid algorithm to investigate the use of the proposed measure in multi-criteria decision-making. In Section 5, a numerical example is provided, followed by validation tests and sensitivity analysis. The results of this example are analyzed and compared with existing methods. Finally, the conclusions and key findings of this study are presented in Section 6.

2 Preliminaries

This section provides a brief review of the basic concepts and operators of IT2FS. Specifically, we cover the fundamental concepts of fuzzy numbers, intuitionistic fuzzy sets, and the principles of distance measures, all of which are essential for the subsequent discussions and the development of a new structure.

2.1 Fuzzy sets and Intuitionistic fuzzy numbers

The essential definitions of fuzzy sets, intuitionistic fuzzy sets, and trapezoidal intuitionistic fuzzy numbers, along with key operators, are provided briefly below.

Definition 2.1. *Suppose M is the universal set, then $\tilde{A}$ is a fuzzy set by the following representation (Zadeh, 1965):*

$$\tilde{A} = \{\langle x, \mu_{\tilde{A}}(x)\rangle | 0 \leq \mu_{\tilde{A}}(x) \leq 1, x \in \mathcal{M}\}, \tag{2.1}$$

where $\mu_{\tilde{A}}(x)$ corresponds to membership value of each element of the universal set with respect to $\tilde{A}$ and is defined according to the relation (2.2)*:*

$$\mu_{\tilde{A}} X \to [0,1], (x \in \mathcal{M}, \mu_{\tilde{A}}(x) \in [0,1]) \tag{2.2}$$

Definition 2.2. *The intuitionistic fuzzy set $\tilde{A}$, assuming $\mathcal{M}$ be the universal set, has the following representation [3]:*

$$\tilde{A} = \{\langle x, \mu_{\tilde{A}}(x), \nu_{\tilde{A}}(x)\rangle | 0 \leq \mu_{\tilde{A}}(x) + \nu_{\tilde{A}}(x) \leq 1, x \in \mathcal{M}\}, \tag{2.3}$$

Where $\mu_{\tilde{A}}(x)$ and $\nu_{\tilde{A}}(x)$ respectively correspond to membership and non-membership values and are defined according to equations (2.4) *and* (2.5)

$$\mu_{\tilde{A}} : M \to [0,1], (x \in X, \mu_{\tilde{A}}(x) \in [0,1]), \tag{2.4}$$

$$\nu_{\tilde{A}} : M \to [0,1], (x \in X, \nu_{\tilde{A}}(x) \in [0,1]), \tag{2.5}$$

A function $\pi_{\tilde{A}}(x)$ is called the hesitancy function for each $x \in \mathcal{M}$ can be represented by the relation:

$$\pi_{\tilde{A}}(x) = 1 - (\mu_{\tilde{A}}(x) + \nu_{\tilde{A}}(x)), \tag{2.6}$$

Such that $\pi_{\tilde{A}}(x)$ is a number between zero and one.

Definition 2.3. *For two intuitionistic fuzzy sets $\tilde{A}_1$ and $\tilde{A}_2$ in the universal set $\mathcal{M}$, the following propositions are valid [3]:*

1. $\tilde{A}_1 \subseteq \tilde{A}_2 \Leftrightarrow \forall x \in \mathcal{M}, \mu_{\tilde{A}_1}(x) \leq \mu_{\tilde{A}_2}(x), \nu_{\tilde{A}_1}(x) \geq \nu_{\tilde{A}_2}(x)$
2. $\tilde{A}_1 = \tilde{A}_2 \Leftrightarrow \tilde{A}_1 \subseteq \tilde{A}_2, \tilde{A}_2 \subseteq \tilde{A}_1$
3. $\tilde{A}_1^c = \{\langle x, \nu_{\tilde{A}_1}(x), \mu_{\tilde{A}_1}(x)\rangle | x \in \mathcal{M}\}$

Definition 2.4 ([**?**]D.-F. Li, 2014)**.** *An arbitrary intuitionistic fuzzy number (IFN) such as $\tilde{A}$ defines an intuitionistic fuzzy set on the axis of real numbers where membership and non-membership functions are introduced, corresponding to relations* (2.7) *and* (2.8)

$$\mu_{\tilde{A}}(x) = \begin{cases} f_{\tilde{A}}(x), & a \leq x \leq b \\ 1, & b \leq x \leq c \\ g_{\tilde{A}}(x), & c \leq x \leq d \\ 0, & o.w \end{cases} \tag{2.7}$$

And

$$\nu_{\tilde{A}}(x) = \begin{cases} h_{\tilde{A}}(x), & a' \leq x \leq b' \\ 0, & b' \leq x \leq c' \\ r_{\tilde{A}}(x), & c' \leq x \leq d' \\ 1, & o.w \end{cases} \tag{2.8}$$

Where $f_{\tilde{A}}$, and $r_{\tilde{A}}$ are two continuous and non-decreasing functions from $\mathbb{R}$ to $[0,1]$, $g_{\tilde{A}}$ and $h_{\tilde{A}}$ are two continuous and non-increasing functions from $\mathbb{R}$ to $[0,1]$. Also, we have:

$$a' \leq a, b' \leq b \leq c \leq c', d \leq d', 0 \leq \mu, \nu \leq 1, 0 \leq \mu + \nu \leq 1.$$

Trapezoidal intuitionistic fuzzy numbers (TraIFN), which are known as an important class of intuitionistic fuzzy numbers, are defined as follows:

Definition 2.5 ([48]). *$\tilde{A} = \langle [a,b,c,d],[a',b',c',d'] \rangle$ is an arbitrary trapezoidal intuitionistic fuzzy number (TraIFN), if the corresponding membership and non-membership functions are introduced by relations* (2.9) *and* (2.10)

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{(x-a)}{b-a}, & a \leq x < b \\ 1, & b \leq x < c \\ \frac{(d-x)}{d-c}, & c \leq x < d \\ 0, & o.w \end{cases} \tag{2.9}$$

And

$$\nu_{\tilde{A}}(x) = \begin{cases} \frac{b'-x}{b'-a'}, & a' \leq x < b' \\ 0, & b' \leq x < c' \\ \frac{d'-x}{d'-c'}, & c' \leq x < d' \\ 1, & o.w \end{cases} \tag{2.10}$$

Where a,b,c,d,a',b',c' and d' are real numbers. Furthermore $0 \leq \mu, \nu \leq 1, 0 \leq \mu + \nu \leq 1$.

If $b = c$ and $b' = c'$, then a trapezoidal intuitionistic fuzzy number (TraIFN) is converted to a triangular intuitionistic fuzzy number (TriIFN). Figure 2 -a,b depictured TraIFN and TriIFN respectively.

Definition 2.6 ([9]). *let $\tilde{A}_1 = \langle [a_1,b_1,c_1,d_1],[a'_1,b'_1,c'_1,d'_1] \rangle$, and $\tilde{A}_2 = \langle [a_2,b_2,c_2,d_2],[a'_2,b'_2,c'_2,d'_2] \rangle$ are two trapezoidal intuitionistic fuzzy numbers and λ is an arbitrary positive number. Then,*

$$1)\ \tilde{A}_1 \oplus \tilde{A}_2 = \langle (a_1+a_2, b_1+b_2, c_1+c_2, d_1+d_2), (a'_1+a'_2, b'_1+b'_2, c'_1+c'_2, d'_1+d'_2) \rangle, \tag{2.11}$$

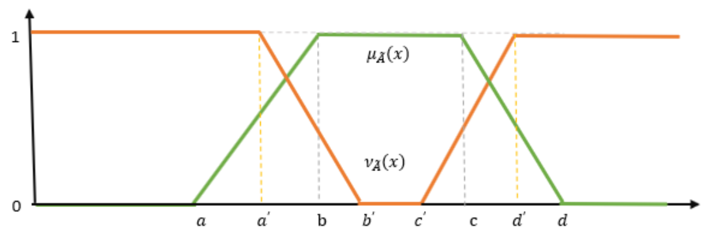
$$2)\ \lambda\tilde{A}_1 = \langle (\lambda a_1, \lambda b_1, \lambda c_1, \lambda d_1), (\lambda a'_1, \lambda b'_1, \lambda c'_1, \lambda d'_1) \rangle, \tag{2.12}$$

Definition 2.7 ([32]). *α-cut, β-cut and (α,β)-cut for the number $\tilde{A} = \langle [a,b,c,d],[a',b',c',d'] \rangle$ respectively are:*

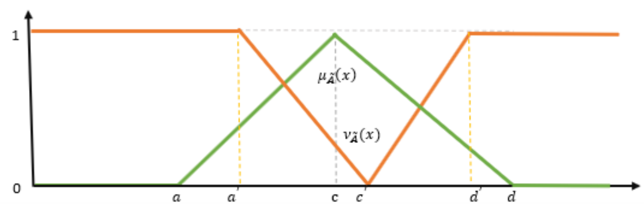
$$\tilde{A}_\alpha = \{x | \mu_{\tilde{A}}(x) \geq \alpha\} = [a + \alpha(b-a), d + \alpha(c-d)]$$

$$\tilde{A}_\beta = \{x | \nu_{\tilde{A}}(x) \leq \beta\} = [a' + (1-\beta)(b'-a'), d' + (1-\beta)(c'-d')]$$

$$\tilde{A}_{\alpha,\beta} = \{x | \mu_{\tilde{A}}(x) \geq \alpha, \nu_{\tilde{A}}(x) \leq \beta\} = [a + \alpha(b-a), d + \alpha(c-d)] \cap [a' + (1-\beta)(b'-a'), d' + (1-\beta)(c'-d')]$$



(a) Trapezoidal intuitionistic fuzzy number



(b) Triangular intuitionistic fuzzy number

Figure 2:

2.2 Distance measure for trapezoidal intuitionistic fuzzy numbers

One of the most important theoretical and practical tools in the theory of information, particularly under trapezoidal intuitionistic fuzzy numbers, is the concept of the distance measure [66]. This measure can be used to evaluate the difference and distance between sets.

Definition 2.8. *If $\tilde{A}_1, \tilde{A}_2$ and $\tilde{A}_3$ are three trapezoidal intuitionistic fuzzy sets in $\mathcal{M}$, we represent the distance measure between the two sets $\tilde{A}_1$ and $\tilde{A}_2$ by $D(\tilde{A}_1, \tilde{A}_2)$, which is the measure in axiomatic principles The following applies:*

A 1) $0 \leq D(\tilde{A}_1, \tilde{A}_2) \leq 1$

A 2) $\tilde{A}_1 = \tilde{A}_2 D(\tilde{A}_1, \tilde{A}_2) = 0$

A 3) $D(\tilde{A}_1, \tilde{A}_2) = D(\tilde{A}_2, \tilde{A}_1)$

A 4) $\tilde{A}_1 \subseteq \tilde{A}_2 \subseteq \tilde{A}_3 \Rightarrow D(\tilde{A}_1, \tilde{A}_2) \leq D(\tilde{A}_1, \tilde{A}_3), D(\tilde{A}_2, \tilde{A}_3) \leq D(\tilde{A}_1, \tilde{A}_3)$

Although the aforementioned axioms for the measure of a relationship are fundamental, incorporating additional properties can enhance the comprehensiveness of the proposed measure.

3 A new parametric distance measure for trapezoidal intuitionistic fuzzy numbers

The most important indicator for evaluating a trapezoidal intuitionistic fuzzy set is its degree of membership. This value plays a crucial role in determining the value of each set and, consequently, the distance between sets. Another influential factor is the complement of non-membership, which also significantly affects the value of these numbers. In other words, the higher this value, the higher the rank of the number. Therefore, the coefficients of these two key factors directly impact the final value of the set and the distance between sets. Based on this concept, we proceed to the next topic for each trapezoidal intuitionistic fuzzy number.

3.1 Ranking trapezoidal intuitionistic fuzzy numbers

Suppose we want to rank the following n trapezoidal intuitionistic fuzzy numbers:

$$\tilde{A}_i = \langle [a_i, b_i, c_i, d_i], [a_i', b_i', c_i', d_i'] \rangle \qquad i = 1, 2, 3, \ldots, n.$$

The steps of the proposed ranking method are as follows:

Step 1: Normalize the trapezoidal intuitionistic fuzzy numbers. To do, for each

$$\tilde{A}_i we have : \tilde{A}_i = \left\langle \left[\frac{a_i}{m}, \frac{b_i}{m}, \frac{c_i}{m}, \frac{d_i}{m}\right], \left[\frac{a_i'}{m}, \frac{b_i'}{m}, \frac{c_i'}{m}, \frac{d_i'}{m}\right] \right\rangle$$

where

$$m = \max\{|a_i|, |b_i|, |c_i|, |d_i|, |a_i'|, |b_i'|, |c_i'|, |d_i'|\}$$

Step 2: Obtain the left and right formulas corresponding to the membership function and the complement non-membership function of the standard trapezoidal intuitionistic fuzzy numbers with respect to the lines $r = -1$ and $r = 1$, respectively.
left and right formulas corresponding to the membership function;$[\underline{f}_i(r), \bar{f}_i(r)]$
left and right formulas corresponding to complement non-membership function; $[\underline{g}_i'(r), \bar{g}_i'(r)]$

Step 3: a) Considering the α level from the decision maker, we define the following rank for the membership function:

$$S_\alpha(\tilde{A}_i) = \frac{1}{2}\int_\alpha^1 (\underline{f}_i(r) - \bar{f}_i(r))\,\mathrm{d}\,r = \frac{1}{2}(Q_L^\mu(\alpha) - Q_R^\mu(\alpha))$$

b) Considering the level $\beta' = 1 - \beta$ from the decision maker, we define the following rank for the non-membership function:

$$S_{\beta'}(\tilde{A}_i) = \frac{1}{2}\int_{\beta'}^1 (\underline{g}_i'(r) - \bar{g}_i'(r))\,\mathrm{d}\,r = \frac{1}{2}(Q_L^{\nu'}(\beta') - Q_R^{\nu'}(\beta'))$$

c) Considering the weights of ω_1, and ω_2, the final value or rank of each intuitionistic trapezoidal fuzzy number is introduced as follows:

$$S_{\alpha,\beta'}(\tilde{A}) = \frac{1}{2}(\omega_1 S_\alpha(\tilde{A}) + \omega_2 S_{\beta'}(\tilde{A}))$$

Or

$$S_{\alpha,\beta'}(\tilde{A}) = \frac{1}{2}\Big\{\omega_1(Q_L^\mu(\alpha) - Q_R^\mu(\alpha)) + \omega_2(Q_L^{\nu'}(\beta') - Q_R^{\nu'}(\beta'))\Big\},$$

Where $\omega_1 + \omega_2 = 1, 0 \le \alpha \le 1, 0 \le \beta' \le 1$, If $\omega_1 = \omega_2$

$$S_{\alpha,\beta'}(\tilde{A}) = \frac{1}{4}\Big\{(Q_L^\mu(\alpha) - Q_R^\mu(\alpha)) + (Q_L^{\nu'}(\beta') - Q_R^{\nu'}(\beta'))\Big\} \tag{3.1}$$

As an example, for standard and positive trapezoidal intuitionistic fuzzy numbers, we have:

$$[\underline{f}_i(r), \bar{f}_i(r)] = [(a_i + 1) + r(b_i - a_i), (1 - d_i) + r(d_i - c_i)]$$
$$[\underline{g}_i'(r), \bar{g}_i'(r)] = [(a_i' + 1) + r(b_i' - a_i'), (1 - d_i') + r(d_i' - c_i')]$$

Hence

$$S_{\alpha,\beta'}(\tilde{A}) = \frac{1}{4}\Big\{(((a_i+1)(1-\alpha)+\frac{(1-\alpha^2)}{2}(b_i-a_i)) - ((1-d_i)(1-\alpha)+\frac{(1-\alpha^2)}{2}(d_i-c_i))) + (((a_i'+1)(1-\beta')+\frac{1-\beta\prime^2}{2}(b_i'-a_i')) - ((1-d_i')(1-\beta')+\frac{1-\beta\prime^2}{2}(d_i'-c_i')))\Big\},$$

Where $0 \leq \alpha \leq 1, 0 \leq \beta' \leq 1$.

Step 4: After calculating the rank of each standard trapezoidal intuitionistic fuzzy number, the comparison between the ranks is done as follows:

1. $S_{\alpha,\beta'}(\tilde{A}_1) < S_{\alpha,\beta'}(\tilde{A}_2) \Leftrightarrow \tilde{A}_1 \prec_{\alpha,\beta'} \tilde{A}_2$
2. $S_{\alpha,\beta'}(\tilde{A}_1) = S_{\alpha,\beta'}(\tilde{A}_2) \Leftrightarrow \tilde{A}_1 \sim_{\alpha,\beta'} \tilde{A}_2$
3. $S_{\alpha,\beta'}(\tilde{A}_1) > S_{\alpha,\beta'}(\tilde{A}_2) \Leftrightarrow \tilde{A}_1 \succ_{\alpha,\beta'} \tilde{A}_2$

Remark 3.1. $\tilde{A}_1 \sim_{\alpha,\beta'} \tilde{A}_2 \Leftrightarrow \Big(Q_L^{\mu_{\tilde{A}_1}}(\alpha) = Q_L^{\nu'_{\tilde{A}_2}}(\beta'), Q_R^{\mu_{\tilde{A}_1}}(\alpha) = Q_R^{\nu'_{\tilde{A}_1}}(\beta'), Q_L^{\nu'_{\tilde{A}_1}}(\beta') = Q_L^{\mu_{\tilde{A}_2}}(\alpha), Q_R^{\nu'_{\tilde{A}_1}}(\beta') = Q_R^{\mu_{\tilde{A}_2}}(\alpha)\Big)$ *Or* $(Q_L^{\mu_{\tilde{A}_1}}(\alpha) = Q_L^{\mu_{\tilde{A}_2}}(\alpha), Q_R^{\mu_{\tilde{A}_1}}(\alpha) = Q_R^{\mu_{\tilde{A}_2}}(\alpha), Q_L^{\nu'_{\tilde{A}_1}}(\beta') = Q_L^{\nu'_{\tilde{A}_2}}(\beta'), Q_R^{\nu'_{\tilde{A}_1}}(\beta') = Q_R^{\nu'_{\tilde{A}_2}}(\beta'))$ *In this context, the comparison between two trapezoidal intuitionistic fuzzy numbers becomes feasible. Acceptable ranking methods exhibit logical characteristics and often maintain a linear structure.*
Property 1*: If* $\tilde{A} = \langle[0,0,0,0],[0,0,0,0]\rangle$ *then,* $S_{\alpha,\beta'}(\tilde{A}) = 0$.
Property 2*: If* $\tilde{A} = \langle[a,a,a,a],[a,a,a,a]\rangle$ *then,* $S_{\alpha,\beta'}(\tilde{A}) = \frac{a}{2}(2-\alpha-\beta')$.

Proof. $S_{\alpha,\beta'}(\tilde{A}) = \frac{1}{4}((2a)(1-\alpha)) + ((2a)(1-\beta')) = \frac{2a}{4}(2-\alpha-\beta')$. If $\alpha, \beta = 0$, then

$$S_{0,1}(\tilde{A}) = a.$$

Property 3: If $\tilde{A} = \langle[1,1,1,1],[1,1,1,1]\rangle$ then, $S_{0,1}(\tilde{A}) = 1$.
Property 4: If $\tilde{A} = \langle[-1,-1,-1,-1],[-1,-1,-1,-1]\rangle$ then, $S_{0,1}(\tilde{A}) = -1$. □

Theorem 3.2 (linearity of the index structure)**.** *Suppose* $\tilde{A}_1 = \langle[a_1,b_1,c_1,d_1],[a_1',b_1',c_1',d_1']\rangle$ *, and* $\tilde{A}_2 = \langle[a_2,b_2,c_2,d_2],[a_2',b_2',c_2',d_2']\rangle$ *be two trapezoidal intuitionistic fuzzy numbers and* $\gamma \in \mathbb{R}^+$*, then*

$$S_{\alpha,\beta'}(\gamma\tilde{A}_1 \oplus \tilde{A}_2) = \gamma S_{\alpha,\beta'}(\tilde{A}_1) \oplus S_{\alpha,\beta'}(\tilde{A}_2). \tag{3.2}$$

Proof. Based on definition 2.6, we have:

$$\gamma\tilde{A}_1 = \langle(\gamma a_1, \gamma b_1, \gamma c_1, \gamma d_1), (\gamma a_1', \gamma b_1', \gamma c_1', \gamma d_1')\rangle$$

Therefore

$$\gamma\tilde{A}_1 \oplus \tilde{A}_2 = \langle(\gamma a_1 + a_2, \gamma b_1 + b_2, \gamma c_1 + c_2, \gamma d_1 + d_2), (\gamma a_1' + a_2', \gamma b_1' + b_2', \gamma c_1' + c_2', \gamma d_1' + d_2')\rangle$$

So, from the left side of the relation (3.2)

$$\begin{aligned} S_{\alpha,\beta'}(\gamma\tilde{A}_1 \oplus \tilde{A}_2) = &\frac{1}{4}\Big\{((\gamma a_1 + a_2 + 1)(1-\alpha) + \frac{(1-\alpha^2)}{2}(\gamma b_1 + b_2 - \gamma a_1 - a_2) \\ &- ((1-\gamma d_1 - d_2)(1-\alpha) + \frac{(1-\alpha^2)}{2}(\gamma d_1 + d_2 - \gamma c_1 - c_2))) \\ &+ (((\gamma a_1' + a_2') + 1)(1-\beta') + \frac{1-\beta'^2}{2}(\gamma b_1' + b_2' - (\gamma a_1' + a_2') \\ &- ((1-\gamma d_1' - d_2')(1-\beta') + \frac{1-\beta'^2}{2}(\gamma d_1' + d_2' - \gamma c_1' - c_2')))\Big\}, \end{aligned} \tag{3.3}$$

$$0 \leq \alpha \leq 1, 0 \leq \beta' \leq 1$$

On the other hand

$$\begin{aligned} S_{\alpha,\beta'}(\tilde{A}_1) =& \frac{1}{4}\Big\{((a_1+1)(1-\alpha)+\frac{(1-\alpha^2)}{2}(b_1-a_1)-((1-d_1)(1-\alpha) \\ &+\frac{(1-\alpha^2)}{2}(d_1-c_1)))+((a_1'+1)(1-\beta')+\frac{1-\beta'^2}{2}(b_1'-a_1')-((1-d_1')(1-\beta') \\ &+\frac{1-\beta'^2}{2}(d_1'-c_1')))\Big\}, \end{aligned}$$

Hence

$$\begin{aligned} \gamma S_{\alpha,\beta'}(\tilde{A}_1) =& \frac{1}{4}\Big\{(\gamma a_1+\gamma)(1-\alpha)+\frac{(1-\alpha^2)}{2}(\gamma b_1-\gamma a_1)-((\gamma-\gamma d_1)(1-\alpha)+\frac{(1-\alpha^2)}{2}(\gamma d_1-\gamma c_1)) \\ &+(\gamma a_1'+\gamma)(1-\beta')+\frac{1-\beta\prime^2}{2}(\gamma b_1'-\gamma a_1')-((\gamma-\gamma d_1')(1-\beta')+\frac{1-\beta\prime^2}{2}(\gamma d_1'-\gamma c_1')))\Big\} \\ =& \frac{1}{4}\Big\{(\gamma a_1)(1-\alpha)+\frac{(1-\alpha^2)}{2}(\gamma b_1-\gamma a_1)-((-\gamma d_1)(1-\alpha)+\frac{(1-\alpha^2)}{2}(\gamma d_1-\gamma c_1)) \\ &+((\gamma a_1')(1-\beta')+\frac{1-\beta\prime^2}{2}(\gamma b_1'-\gamma a_1')-((-\gamma d_1')(1-\beta')+\frac{1-\beta\prime^2}{2}(\gamma d_1'-\gamma c_1')))\Big\} \end{aligned}$$

Therefore, the right side of the relation (3.2) is equal to

$$\begin{aligned} \gamma S_{\alpha,\beta'}(\tilde{A}_1)+S_{\alpha,\beta'}(\tilde{A}_2) =& \frac{1}{4}\{((\gamma a_1+(a_2+1))(1-\alpha)+\frac{(1-\alpha^2)}{2}(\gamma b_1-\gamma a_1+(b_2-a_2)) \\ &-(((-\gamma d_1+(1-d_2))(1-\alpha)+\frac{(1-\alpha^2)}{2}(\gamma d_1-\gamma c_1+(d_2-c_2)))) \\ &+((\gamma a_1'+(a_2'+1))(1-\beta')+\frac{1-\beta\prime^2}{2}(\gamma b_1'-\gamma a_1'+(b_2'-a_2')) \\ &-((-\gamma d_1'+(1-d_1'))(1-\beta')+\frac{1-\beta\prime^2}{2}(\gamma d_1'-\gamma c_1'+(d_2'-c_2'))))\} \end{aligned} \tag{3.4}$$

The equality between relations (3.3) and (3.4) completes the proof. □

3.2 A proposed parametric distance measure for trapezoidal intuitionistic fuzzy numbers

We now apply the aforementioned ranking scheme to determine the distance measure between two intuitionistic fuzzy numbers. Some of the key advantages of using this approach are:

- Many researchers have used three indices—membership degree, non-membership degree, and hesitation degree—simultaneously in their formulas to create new measures. By using only two values (membership degree and the degree of non-belonging complementary function), we can significantly reduce the computational complexity.
- The determination of weights for the primary factors by the decision maker provides greater flexibility and results in a variety of possible solutions.
- It demonstrates effective performance in various applications, such as multi-criteria decision-making.
- The proposed measure adheres to the fundamental principles of distance measurement.

Based on the concepts introduced in the previous section, the suggested distance measure for two intuitionistic fuzzy numbers is defined by the following rule:

$$\begin{aligned} D_{\alpha,\beta'}(\tilde{A}_1,\tilde{A}_2) =& \frac{1}{4}\Big\{\omega_1(|Q_L^{\mu_{\tilde{A}_1}}(\alpha) - Q_L^{\mu_{\tilde{A}_2}}(\alpha)| + |Q_R^{\mu_{\tilde{A}_1}}(\alpha) - Q_R^{\mu_{\tilde{A}_2}}(\alpha)|) \\ &+ \omega_2(|Q_L^{\nu'_{\tilde{A}_1}}(\beta') - Q_L^{\nu'_{\tilde{A}_2}}(\beta')| + |Q_R^{\nu'_{\tilde{A}_1}}(\beta') - Q_R^{\nu'_{\tilde{A}_2}}(\beta')|)\Big\} \end{aligned}$$

If $\omega_1 = \omega_2$

$$\begin{aligned} D_{\alpha,\beta'}(\tilde{A}_1,\tilde{A}_2) =& \frac{1}{8}\Big\{(|Q_L^{\mu_{\tilde{A}_1}}(\alpha) - Q_L^{\mu_{\tilde{A}_2}}(\alpha)| + |Q_R^{\mu_{\tilde{A}_1}}(\alpha) - Q_R^{\mu_{\tilde{A}_2}}(\alpha)|) \\ &+ (|Q_L^{\nu'_{\tilde{A}_1}}(\beta') - Q_L^{\nu'_{\tilde{A}_2}}(\beta')| + |Q_R^{\nu'_{\tilde{A}_1}}(\beta') - Q_R^{\nu'_{\tilde{A}_2}}(\beta')|)\Big\} \end{aligned} \tag{3.5}$$

Theorem 3.3. *Show that relation* (3.5) *applies to the principle of distance measure.*

Proof. 1) Considering that the result of each of the absolute magnitudes is smaller than 2, the structure of the relationship, and the fact that the sum of the weights is one, thematic principle 1 is the result. Therefore:
$0 \leq D_{\alpha,\beta'}(\tilde{A}_1,\tilde{A}_2) \leq 1$

2) If $D_{\alpha,\beta'}(\tilde{A}_1,\tilde{A}_2) = 0$, we have:

$$Q_L^{\mu_{\tilde{A}_1}}(\alpha) = Q_L^{\mu_{\tilde{A}_2}}(\alpha), Q_R^{\mu_{\tilde{A}_1}}(\alpha) = Q_R^{\mu_{\tilde{A}_2}}(\alpha), Q_L^{\nu'_{\tilde{A}_1}}(\beta') = Q_L^{\nu'_{\tilde{A}_2}}(\beta'), Q_R^{\nu'_{\tilde{A}_1}}(\beta') = Q_R^{\nu'_{\tilde{A}_2}}(\beta')$$

As a result

$$\tilde{A}_1 \sim_{\alpha,\beta'} \tilde{A}_2$$

The converse of principle 2 is also shown similarly.

3) Principle 3 is the result considering that each of the expression is in absolute value. Therefore:

$$D_{\alpha,\beta'}(\tilde{A}_1,\tilde{A}_2) = D_{\alpha,\beta'}(\tilde{A}_2,\tilde{A}_1)$$

4) Assuming that $\tilde{A}_1 \subseteq \tilde{A}_2 \subseteq \tilde{A}_3$, we have:

$$(1).|Q_L^{\mu_{\tilde{A}_1}}(\alpha) - Q_L^{\mu_{\tilde{A}_2}}(\alpha)| \leq |Q_L^{\mu_{\tilde{A}_1}}(\alpha) - Q_L^{\mu_{\tilde{A}_3}}(\alpha)|, |Q_R^{\mu_{\tilde{A}_1}}(\alpha) - Q_R^{\mu_{\tilde{A}_2}}(\alpha)| \leq |Q_R^{\mu_{\tilde{A}_1}}(\alpha) - Q_R^{\mu_{\tilde{A}_3}}(\alpha)|$$

Moreover

$$(2).|Q_L^{\nu'_{\tilde{A}_1}}(\beta') - Q_L^{\nu'_{\tilde{A}_2}}(\beta')| \leq |Q_L^{\nu'_{\tilde{A}_1}}(\beta') - Q_L^{\nu'_{\tilde{A}_3}}(\beta')|, |Q_R^{\nu'_{\tilde{A}_1}}(\beta') - Q_R^{\nu'_{\tilde{A}_2}}(\beta')| \leq |Q_R^{\nu'_{\tilde{A}_1}}(\beta') - Q_R^{\nu'_{\tilde{A}_3}}(\beta')|$$

Hence

$$\begin{aligned} D_{\alpha,\beta'}(\tilde{A}_1,\tilde{A}_2) =& \frac{1}{8}\Big\{(|Q_L^{\mu_{\tilde{A}_1}}(\alpha) - Q_L^{\mu_{\tilde{A}_2}}(\alpha)| + |Q_R^{\mu_{\tilde{A}_1}}(\alpha) - Q_R^{\mu_{\tilde{A}_2}}(\alpha)|) \\ &+ (|Q_L^{\nu'_{\tilde{A}_1}}(\beta') - Q_L^{\nu'_{\tilde{A}_2}}(\beta')| + |Q_R^{\nu'_{\tilde{A}_1}}(\beta') - Q_R^{\nu'_{\tilde{A}_2}}(\beta')|)\Big\} \\ \leq& \frac{1}{8}\Big\{(|Q_L^{\mu_{\tilde{A}_1}}(\alpha) - Q_L^{\mu_{\tilde{A}_3}}(\alpha)| + |Q_R^{\mu_{\tilde{A}_1}}(\alpha) - Q_R^{\mu_{\tilde{A}_3}}(\alpha)|) \\ &+ (|Q_L^{\nu'_{\tilde{A}_1}}(\beta') - Q_L^{\nu'_{\tilde{A}_3}}(\beta')| + |Q_R^{\nu'_{\tilde{A}_1}}(\beta') - Q_R^{\nu'_{\tilde{A}_3}}(\beta')|)\Big\} = D_{\alpha,\beta'}(\tilde{A}_1,\tilde{A}_3) \end{aligned}$$

As a result

$$D_{\alpha,\beta'}(\tilde{A}_1,\tilde{A}_2) \leq D_{\alpha,\beta'}(\tilde{A}_1,\tilde{A}_3).$$

It can be shown in a similar way

$$D_{\alpha,\beta'}(\tilde{A}_2,\tilde{A}_3) \leq D_{\alpha,\beta'}(\tilde{A}_1,\tilde{A}_3).$$

Therefore, the relation (3.5) applies to all measure properties.

□

Theorem 3.4. *Show*

$$D_{\alpha,\beta'}(\tilde{A}_1,\tilde{A}_3) \leq D_{\alpha,\beta'}(\tilde{A}_1,\tilde{A}_2) + D_{\alpha,\beta'}(\tilde{A}_2,\tilde{A}_3) \tag{3.6}$$

Proof. Starting from the left side of (4.1), we have:

$$\begin{aligned} D_{\alpha,\beta'}(\tilde{A}_1,\tilde{A}_3) =& \frac{1}{8}\Big\{(|Q_L^{\mu_{\tilde{A}_1}}(\alpha) - Q_L^{\mu_{\tilde{A}_3}}(\alpha)| + |Q_R^{\mu_{\tilde{A}_1}}(\alpha) - Q_R^{\mu_{\tilde{A}_3}}(\alpha)|) \\ &+ (|Q_L^{\nu'_{\tilde{A}_1}}(\beta') - Q_L^{\nu'_{\tilde{A}_3}}(\beta')| + |Q_R^{\nu'_{\tilde{A}_1}}(\beta') - Q_R^{\nu'_{\tilde{A}_3}}(\beta')|)\Big\} \\ =& \frac{1}{8}\Big\{(|Q_L^{\mu_{\tilde{A}_1}}(\alpha) - Q_L^{\mu_{\tilde{A}_2}}(\alpha) + Q_L^{\mu_{\tilde{A}_2}}(\alpha) - Q_L^{\mu_{\tilde{A}_3}}(\alpha)| \\ &+ |Q_R^{\mu_{\tilde{A}_1}}(\alpha) - Q_R^{\mu_{\tilde{A}_2}}(\alpha) + Q_R^{\mu_{\tilde{A}_2}}(\alpha) - Q_R^{\mu_{\tilde{A}_3}}(\alpha)|) \\ &+ (|Q_L^{\nu'_{\tilde{A}_1}}(\beta') - Q_L^{\nu'_{\tilde{A}_2}}(\beta') + Q_L^{\nu'_{\tilde{A}_2}}(\beta') - Q_L^{\nu'_{\tilde{A}_3}}(\beta')| \\ &+ |Q_R^{\nu'_{\tilde{A}_1}}(\beta') - Q_R^{\nu'_{\tilde{A}_2}}(\beta') + Q_R^{\nu'_{\tilde{A}_2}}(\beta') - Q_R^{\nu'_{\tilde{A}_3}}(\beta')|)\Big\} \end{aligned}$$

Based on the Triangular inequality property of absolute value

$$\begin{aligned} D_{\alpha,\beta'}(\tilde{A}_1,\tilde{A}_3) \leq& \frac{1}{8}\Big\{(|Q_L^{\mu_{\tilde{A}_1}}(\alpha) - Q_L^{\mu_{\tilde{A}_2}}(\alpha)| + |Q_R^{\mu_{\tilde{A}_1}}(\alpha) - Q_R^{\mu_{\tilde{A}_2}}(\alpha)|) \\ &+ (|Q_L^{\nu'_{\tilde{A}_1}}(\beta') - Q_L^{\nu'_{\tilde{A}_2}}(\beta')| + |Q_R^{\nu'_{\tilde{A}_1}}(\beta') - Q_R^{\nu'_{\tilde{A}_2}}(\beta')|)\Big\} \\ &+ \frac{1}{8}\Big\{(|Q_L^{\mu_{\tilde{A}_2}}(\alpha) - Q_L^{\mu_{\tilde{A}_3}}(\alpha)| + |Q_R^{\mu_{\tilde{A}_2}}(\alpha) - Q_R^{\mu_{\tilde{A}_3}}(\alpha)|) \\ &+ (|Q_L^{\nu'_{\tilde{A}_2}}(\beta') - Q_L^{\nu'_{\tilde{A}_3}}(\beta')| + |Q_R^{\nu'_{\tilde{A}_2}}(\beta') - Q_R^{\nu'_{\tilde{A}_3}}(\beta')|)\Big\} \\ &= D_{\alpha,\beta'}(\tilde{A}_1,\tilde{A}_2) + D_{\alpha,\beta'}(\tilde{A}_2,\tilde{A}_3). \end{aligned}$$

□

4 Application: Multi-criteria decision-making based on the proposed parametric distance measure for trapezoidal intuitionistic fuzzy numbers

In this section, we will highlight the performance and efficiency of the combined process using the parametric measures presented in the previous section, demonstrated through a practical decision-making example.

4.1 Introduction of the problem

Suppose a set of alternatives $\tilde{A} = \{\tilde{A}_1, \tilde{A}_2, \ldots, \tilde{A}_m\}$, a set of criteria $\tilde{C} = \{\tilde{C}_1, \tilde{C}_2, \ldots, \tilde{C}_n\}$, and a set of decision-makers $\{dm_k | k = 1, 2, \ldots, t\}$ in such a way that the amount of each alternative in each criterion is a trapezoidal intuitionistic fuzzy number, and has a representation as follows:

$$\begin{aligned}\tilde{A}_{ijk} =&\{\langle \tilde{C}_j, \langle [a_{ijk}, b_{ijk}, c_{ijk}, d_{ijk}], [a'_{ijk}, b'_{ijk}, c'_{ijk}, d'_{ijk}] \rangle\rangle | \tilde{C}_j \in \tilde{C}\},\ \text{Where } i = 1, 2, \ldots, m, j = 1, 2, \ldots, n,\\ &k = 1, 2, \ldots, t.\end{aligned}$$

The decision-makers initially recorded their opinions about each alternative for each criterion as a linguistic variable (see Table 1 in [19]). Additionally, from the decision-makers' perspective, each criterion is associated with an important factor, which is indicated and its value selected from Table 1. Since the criteria are not necessarily aligned—some may relate to cost, while others relate to profit—we aim to select the optimal candidate from the available options. Generally, methods for solving multi-criteria group decision-making (MCGDM) problems begin with intuitionistic fuzzy data by constructing a decision matrix that includes both decision-making data and intuitionistic fuzzy information. Subsequently, the alternatives are evaluated based on the selected criteria (such as profit or cost) using aggregation operators, distance measures, and similarity measures. Finally, the candidates are ranked, and the optimal decision is made.

Table 1: Linguistic variables corresponding to the value and weight of each criterion.

Linguistic terms	TraIF
Very low (VL)	$\langle(0.0, 0.0, 0.0, 0.0), (0.0, 0.0, 0.0, 0.0)\rangle$
low (L)	$\langle(0.0, 0.1, 0.2, 0.3), (0.0, 0.1, 0.2, 0.3)\rangle$
Medium low (ML)	$\langle(0.1, 0.2, 0.3, 0.4), (0.0, 0.2, 0.3, 0.5)\rangle$
Medium (M)	$\langle(0.3, 0.4, 0.5, 0.6), (0.2, 0.4, 0.5, 0.7)\rangle$
Medium High (MH)	$\langle(0.5, 0.6, 0.7, 0.8), (0.4, 0.6, 0.7, 0.9)\rangle$
High (H)	$\langle(0.7, 0.8, 0.9, 1.0), (0.7, 0.8, 0.9, 1.0)\rangle$
Very High (VH)	$\langle(1.0, 1.0, 1.0, 1.0), (1.0, 1.0, 1.0, 1.0)\rangle$

4.2 Solution method based on proposed parametric distance measure

In this section, we apply the proposed measure within a hybrid model [19] to solve the multi-criteria decision-making problem. The conceptual diagram in Figure 3 provides an overview of this method, while the following algorithm solves the multi-criteria group decision-making problem using Equation (3.5): **Step 1**:

a) Obtain the linguistic matrix based on the decision maker's preferences and the weights assigned to each criterion.

b) Convert the linguistic matrix from Step 1, along with the weights for each criterion, into trapezoidal intuitionistic fuzzy numbers using Table 1.

Step 2:

a) Form the matrix based on the average opinions of the decision-makers using the following relation:

$$[\tilde{A}_{ij}]_{m*n} = \Big[\frac{1}{t}\sum_{k=1}^{t} \tilde{A}_{ijk}\Big]_{m*n}$$

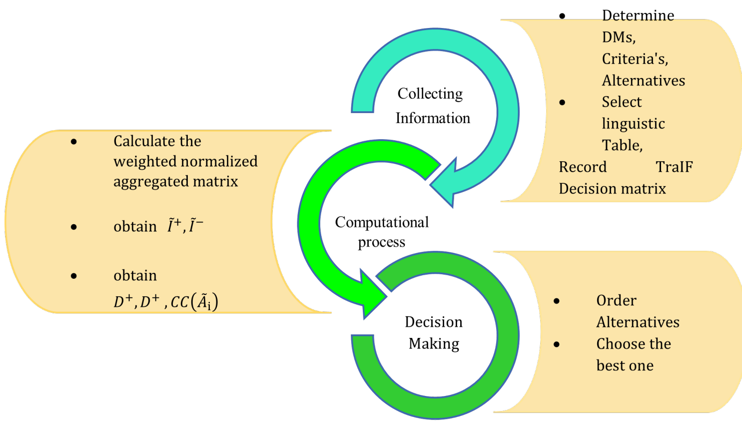


Figure 3: Conceptual diagram of the solution method based on the proposed parametric distance measure.

b) Form the matrix based on the average weights assigned to each criterion by the decision-makers:

$$[\tilde{w}_j]_{m*n} = \left[\frac{1}{t}\sum_{k=1}^{t}\tilde{w}_{ijk}\right]_{m*n}$$

Step 3: Normalize the aggregative mean matrix of decision makers' opinions: If $\tilde{C}_j$ is the cost criterion, then

$$\tilde{A}_{ij}^{nor} = \left\langle \left[\frac{\delta_j^-}{d_{ij}}, \frac{\delta_j^-}{c_{ij}}, \frac{\delta_j^-}{b_{ij}}, \frac{\delta_j^-}{a_{ij}}\right], \left[\frac{\delta'_{j-}}{d'_{ij}}, \frac{\delta'_{j-}}{c'_{ij}}, \frac{\delta'_{j-}}{b'_{ij}}, \frac{\delta'_{j-}}{a'_{ij}}\right] \right\rangle$$

Where $\delta_j^- = \min i\{a_{ij}\} and \delta'_{j-} = \min i\{a'_{ij}\}$ for every j.
And, if $\tilde{C}_j$ is the profit criterion, then

$$\tilde{A}_{ij}^{nor} = \left\langle \left[\frac{a_{ij}}{\delta_j^+}, \frac{b_{ij}}{\delta_j^+}, \frac{c_{ij}}{\delta_j^+}, \frac{d_{ij}}{\delta_j^+}\right], \left[\frac{a'_{ij}}{\delta_j^+}, \frac{b'_{ij}}{\delta_j^+}, \frac{c'_{ij}}{\delta_j^+}, \frac{d'_{ij}}{\delta_j^+}\right] \right\rangle$$

Where $\delta_j^+ = \max_i \{d_{ij}\}$ and $\delta_j'^+ = \max_i \{d'_{ij}\}$ for every j.

Step 4: Calculate the weighted normalized trapezoidal intuitionistic fuzzy matrix. For this purpose, use the following relation for each level of the previous step matrix:

$$\tilde{A}_{ij}^{norw} = \tilde{w}_j . \tilde{A}_{ij}^{n}$$

Step 5:

a) Specify the positive ideal, as follows:

$$\tilde{I}^{+} = \{\tilde{A}_1^+, \tilde{A}_2^+, \ldots, \tilde{A}_n^+\}$$

Where for each profit criteria

$$\tilde{A}_j^+ = \left\langle \begin{array}{c} [\max_i a_{ij}, \max_i b_{ij}, \max_i c_{ij}, \max_i d_{ij}], \\ [\max_i a'_{ij}, \max_i b'_{ij}, \max_i c'_{ij}, \max_i d'_{ij}] \end{array} \right\rangle$$

For each cost criteria

$$\tilde{A}_j^+ = \left\langle \begin{array}{c} [\min_i a_{ij}, \min_i b_{ij}, \min_i c_{ij}, \min_i d_{ij}], \\ [\min_i a'_{ij}, \min_i b'_{ij}, \min_i c'_{ij}, \min_i d'_{ij}] \end{array} \right\rangle$$

b) Specify the negative ideal as follows:

$$\tilde{I}^- = \{\tilde{A}_1^-, \tilde{A}_2^-, \ldots, \tilde{A}_n^-\}$$

Where for each profit criteria

$$\tilde{A}_j^- = \left\langle \begin{array}{c} [\min_i a_{ij}, \min_i b_{ij}, \min_i c_{ij}, \min_i d_{ij}], \\ [\min_i a'_{ij}, \min_i b'_{ij}, \min_i c'_{ij}, \min_i d'_{ij}] \end{array} \right\rangle$$

And, for each cost criteria

$$\tilde{A}_j^- = \left\langle \begin{array}{c} [\max_i a_{ij}, \max_i b_{ij}, \max_i c_{ij}, \max_i d_{ij}], \\ [\max_i a'_{ij}, \max_i b'_{ij}, \max_i c'_{ij}, \max_i d'_{ij}] \end{array} \right\rangle$$

Step 6: By choosing arbitrary values for α, β', the distance between each alternative and the positive ideal, $D_{\alpha,\beta'}(\tilde{A}_i, \tilde{I}^+)$ and also the distance between each alternative and the negative ideal , find $D_{\alpha,\beta'}(\tilde{A}_i, \tilde{I}^-)$ using equation (3.5).

Step 7: Using the values obtained in the previous step, for i=1,2,...,m, we find the degree of relative closeness based on the equation (4.1):

$$CC(\tilde{A}_i) = \frac{D_{\alpha,\beta'}(\tilde{A}_i, \tilde{I}^-)}{D_{\alpha,\beta'}(\tilde{A}_i, \tilde{I}^-) + D_{\alpha,\beta'}(\tilde{A}_i, I^+)} \quad (4.1)$$

Step 8: The highest degree of closeness obtained from step 7 indicates the best choice.

5 Numerical examples

The software selection problem has been investigated and solved in some issues [19, 20]. Assume that, among the four candidates, $\tilde{A} = \{\tilde{A}_1, \tilde{A}_2, \tilde{A}_3, \tilde{A}_4\}$ we are looking to choose the best one to increase the efficiency of system performance. The four-person decision-making team $\{dm_1, dm_2, dm_3, dm_4\}$ evaluates candidates based on the three following criteria;

$\tilde{C}_1$, the cost of hardware and software

$\tilde{C}_2$, the ability to develop organizational performance

$\tilde{C}_3$, the ability to develop software.

The data related to the evaluation of each of the candidates in each of the three criteria mentioned by the experts with linguistic data are given, according to Table 2.

According to step 1, convert the linguistic matrix and the weights corresponding to each criterion into a matrix with trapezoidal intuitionistic fuzzy numbers. (Table 3)

The matrix related to the average opinions of decision-makers and weights according to step 2, is calculated and the data is recorded in Table 4.

The normalized aggregative average matrix of decision makers' opinions is given in Table 5. Using the average weights and information in Table 5, the normalized aggregative information is recorded in Table 6. Now, we find the positive and negative ideals based on the information in Table 6 as follows:

$$\tilde{I}^+ = \big\{\langle(0.18, 0.25, 0.35, 0.50), (0.10, 0.19, 0.27, 0.52)\rangle, \langle(0.17, 0.28, 0.40, 0.55), (0.10, 0.27, 0.40, 0.65)\rangle, \\ \langle(0.19, 0.31, 0.47, 0.65), (0.10, 0.27, 0.40, 0.73)\rangle\big\}$$

Table 2: Evaluation information of candidates in three criteria and the corresponding weight of each criterion by four decision makers with linguistic variables.

		C_1	C_2	C_3
dm_1	A_1	ML	MH	ML
	A_2	M	MH	MH
	A_3	H	MH	L
	A_4	MH	M	MH
	W_1	MH	M	M
dm_2	A_1	M	MH	ML
	A_2	H	L	M
	A_3	ML	H	MH
	A_4	H	H	M
	W_2	M	M	MH
dm_3	A_1	MH	ML	MH
	A_2	ML	H	M
	A_3	M	H	M
	A_4	MH	M	MH
	W_3	MH	ML	M
dm_4	A_1	H	H	H
	A_2	M	MH	M
	A_3	ML	H	M
	A_4	ML	ML	L
	W_4	MH	M	M

Table 3: Evaluation information of candidates in three criteria and the corresponding weight of each criterion by four decision makers with trapezoidal intuitionistic fuzzy numbers.

DMs		C_1	C_2	C_3
dm_1	A_1	$\langle(0.1,0.2,0.3,0.4),(0.0,0.2,0.3,0.5)\rangle$	$\langle(0.5,0.6,0.7,0.8),(0.4,0.6,0.7,0.9)\rangle$	$\langle(0.1,0.2,0.3,0.4),(0.0,0.2,0.3,0.5)\rangle$
	A_2	$\langle(0.3,0.4,0.5,0.6),(0.2,0.4,0.5,0.7)\rangle$	$\langle(0.5,0.6,0.7,0.8),(0.4,0.6,0.7,0.9)\rangle$	$\langle(0.5,0.6,0.7,0.8),(0.4,0.6,0.7,0.9)\rangle$
	A_3	$\langle(0.7,0.8,0.9,1.0),(0.7,0.8,0.9,1.0)\rangle$	$\langle(0.5,0.6,0.7,0.8),(0.4,0.6,0.7,0.9)\rangle$	$\langle(0.0,0.1,0.2,0.3),(0.0,0.1,0.2,0.3)\rangle$
	A_4	$\langle(0.5,0.6,0.7,0.8),(0.4,0.6,0.7,0.9)\rangle$	$\langle(0.3,0.4,0.5,0.6),(0.2,0.4,0.5,0.7)\rangle$	$\langle(0.5,0.6,0.7,0.8),(0.4,0.6,0.7,0.9)\rangle$
dm_2	W_1	$\langle(0.5,0.6,0.7,0.8),(0.4,0.6,0.7,0.9)\rangle$	$\langle(0.3,0.4,0.5,0.6),(0.2,0.4,0.5,0.7)\rangle$	$\langle(0.3,0.4,0.5,0.6),(0.2,0.4,0.5,0.7)\rangle$
	A_1	$\langle(0.3,0.4,0.5,0.6),(0.2,0.4,0.5,0.7)\rangle$	$\langle(0.5,0.6,0.7,0.8),(0.4,0.6,0.7,0.9)\rangle$	$\langle(0.1,0.2,0.3,0.4),(0.0,0.2,0.3,0.5)\rangle$
	A_2	$\langle(0.7,0.8,0.9,1.0),(0.7,0.8,0.9,1.0)\rangle$	$\langle(0.0,0.1,0.2,0.3),(0.0,0.1,0.2,0.3)\rangle$	$\langle(0.3,0.4,0.5,0.6),(0.2,0.4,0.5,0.7)\rangle$
	A_3	$\langle(0.1,0.2,0.3,0.4),(0.0,0.2,0.3,0.5)\rangle$	$\langle(0.7,0.8,0.9,1.0),(0.7,0.8,0.9,1.0)\rangle$	$\langle(0.5,0.6,0.7,0.8),(0.4,0.6,0.7,0.9)\rangle$
	A_4	$\langle(0.7,0.8,0.9,1.0),(0.7,0.8,0.9,1.0)\rangle$	$\langle(0.7,0.8,0.9,1.0),(0.7,0.8,0.9,1.0)\rangle$	$\langle(0.3,0.4,0.5,0.6),(0.2,0.4,0.5,0.7)\rangle$
dm_3	W_2	$\langle(0.3,0.4,0.5,0.6),(0.2,0.4,0.5,0.7)\rangle$	$\langle(0.3,0.4,0.5,0.6),(0.2,0.4,0.5,0.7)\rangle$	$\langle(0.5,0.6,0.7,0.8),(0.4,0.6,0.7,0.9)\rangle$
	A_1	$\langle(0.5,0.6,0.7,0.8),(0.4,0.6,0.7,0.9)\rangle$	$\langle(0.1,0.2,0.3,0.4),(0.0,0.2,0.3,0.5)\rangle$	$\langle(0.5,0.6,0.7,0.8),(0.4,0.6,0.7,0.9)\rangle$
	A_2	$\langle(0.1,0.2,0.3,0.4),(0.0,0.2,0.3,0.5)\rangle$	$\langle(0.7,0.8,0.9,1.0),(0.7,0.8,0.9,1.0)\rangle$	$\langle(0.3,0.4,0.5,0.6),(0.2,0.4,0.5,0.7)\rangle$
	A_3	$\langle(0.3,0.4,0.5,0.6),(0.2,0.4,0.5,0.7)\rangle$	$\langle(0.7,0.8,0.9,1.0),(0.7,0.8,0.9,1.0)\rangle$	$\langle(0.3,0.4,0.5,0.6),(0.2,0.4,0.5,0.7)\rangle$
	A_4	$\langle(0.5,0.6,0.7,0.8),(0.4,0.6,0.7,0.9)\rangle$	$\langle(0.3,0.4,0.5,0.6),(0.2,0.4,0.5,0.7)\rangle$	$\langle(0.5,0.6,0.7,0.8),(0.4,0.6,0.7,0.9)\rangle$
dm_4	W_3	$\langle(0.5,0.6,0.7,0.8),(0.4,0.6,0.7,0.9)\rangle$	$\langle(0.1,0.2,0.3,0.4),(0.0,0.2,0.3,0.5)\rangle$	$\langle(0.3,0.4,0.5,0.6),(0.2,0.4,0.5,0.7)\rangle$
	A_1	$\langle(0.7,0.8,0.9,1.0),(0.7,0.8,0.9,1.0)\rangle$	$\langle(0.7,0.8,0.9,1.0),(0.7,0.8,0.9,1.0)\rangle$	$\langle(0.7,0.8,0.9,1.0),(0.7,0.8,0.9,1.0)\rangle$
	A_2	$\langle(0.3,0.4,0.5,0.6),(0.2,0.4,0.5,0.7)\rangle$	$\langle(0.5,0.6,0.7,0.8),(0.4,0.6,0.7,0.9)\rangle$	$\langle(0.3,0.4,0.5,0.6),(0.2,0.4,0.5,0.7)\rangle$
	A_3	$\langle(0.1,0.2,0.3,0.4),(0.0,0.2,0.3,0.5)\rangle$	$\langle(0.7,0.8,0.9,1.0),(0.7,0.8,0.9,1.0)\rangle$	$\langle(0.3,0.4,0.5,0.6),(0.2,0.4,0.5,0.7)\rangle$
	A_4	$\langle(0.1,0.2,0.3,0.4),(0.0,0.2,0.3,0.5)\rangle$	$\langle(0.1,0.2,0.3,0.4),(0.0,0.2,0.3,0.5)\rangle$	$\langle(0.0,0.1,0.2,0.3),(0.0,0.1,0.2,0.3)\rangle$
	W_4	$\langle(0.5,0.6,0.7,0.8),(0.4,0.6,0.7,0.9)\rangle$	$\langle(0.3,0.4,0.5,0.6),(0.2,0.4,0.5,0.7)\rangle$	$\langle(0.3,0.4,0.5,0.6),(0.2,0.4,0.5,0.7)\rangle$

And

$$\tilde{I}^- = \big\{\langle(0.23, 0.33, 0.49, 0.75), (0.12, 0.25, 0.37, 0.87)\rangle, \langle(0.10, 0.17, 0.26, 0.38), (0.04, 0.16, 0.25, 0.48)\rangle,$$
$$\langle(0.15, 0.26, 0.40, 0.58), (0.07, 0.38, 0.48, 0.65)\rangle\big\}$$

The results related to the distance between each alternative and the positive ideal, as well as the distance

Table 4: Average evaluation of candidates and weights.

	C_1	C_2	C_3
A_1	$\langle(0.5, 0.6, 0.7, 0.8), (0.33, 0.5, 0.6, 0.78)\rangle$	$\langle(0.45, 0.55, 0.65, 0.75), (0.38, 0.55, 0.65, 0.83)\rangle$	$\langle(0.35, 0.45, 0.55, 0.65), (0.28, 0.45, 0.55, 0.73)\rangle$
A_2	$\langle(0.35, 0.45, 0.55, 0.65), (0.28, 0.45, 0.55, 0.73)\rangle$	$\langle(0.43, 0.53, 0.63, 0.73), (0.38, 0.53, 0.63, 0.78)\rangle$	$\langle(0.35, 0.45, 0.55, 0.65), (0.25, 0.45, 0.55, 0.75)\rangle$
A_3	$\langle(0.3, 0.4, 0.5, 0.6), (0.23, 0.4, 0.5, 0.68)\rangle$	$\langle(0.65, 0.75, 0.85, 0.95), (0.63, 0.75, 0.85, 0.98)\rangle$	$\langle(0.28, 0.38, 0.48, 0.58), (0.2, 0.38, 0.48, 0.65)\rangle$
A_4	$\langle(0.45, 0.55, 0.65, 0.75), (0.38, 0.55, 0.65, 0.83)\rangle$	$\langle(0.35, 0.45, 0.55, 0.65), (0.28, 0.45, 0.55, 0.73)\rangle$	$\langle(0.33, 0.43, 0.53, 0.63), (0.25, 0.43, 0.53, 0.7)\rangle$
W	$\langle(0.45, 0.55, 0.65, 0.75), (0.35, 0.55, 0.65, 0.85)\rangle$	$\langle(0.25, 0.35, 0.45, 0.55), (0.15, 0.35, 0.45, 0.65)\rangle$	$\langle(0.35, 0.43, 0.53, 0.63), (0.25, 0.43, 0.53, 0.7)\rangle$

Table 5: Normalized aggregative information of candidates.

	C_1	C_2	C_3
A_1	$\langle(0.43, 0.50, 0.60, 0.75), (0.30, 0.38, 0.46, 0.71)\rangle$	$\langle(0.47, 0.58, 0.68, 0.79), (0.38, 0.56, 0.66, 0.84)\rangle$	$\langle(0.54, 0.69, 0.85, 1.0), (0.37, 0.60, 0.63, 0.97)\rangle$
A_2	$\langle(0.46, 0.55, 0.67, 0.86), (0.32, 0.42, 0.51, 0.84)\rangle$	$\langle(0.45, 0.55, 0.66, 0.76), (0.38, 0.54, 0.64, 0.79)\rangle$	$\langle(0.54, 0.69, 0.85, 1.0), (0.33, 0.60, 0.73, 1.0)\rangle$
A_3	$\langle(0.50, 0.60, 0.75, 1.0), (0.34, 0.46, 0.58, 1.0)\rangle$	$\langle(0.68, 0.79, 0.89, 1.0), (0.64, 0.77, 0.87, 0.99)\rangle$	$\langle(0.42, 0.58, 0.73, 0.88), (0.27, 0.50, 0.63, 0.87)\rangle$
A_4	$\langle(0.40, 0.46, 0.55, 0.67), (0.28, 0.35, 0.42, 0.61)\rangle$	$\langle(0.37, 0.47, 0.58, 0.68), (0.28, 0.46, 0.56, 0.74)\rangle$	$\langle(0.50, 0.65, 0.81, 0.96), (0.33, 0.57, 0.70, 0.93)\rangle$

Table 6: Weighted normalized aggregative information of candidates.

	C_1	C_2	C_3
A_1	$\langle(0.19, 0.28, 0.39, 0.56), (0.10, 0.21, 0.30, 0.60)\rangle$	$\langle(0.12, 0.20, 0.31, 0.43), (0.06, 0.20, 0.30, 0.55)\rangle$	$\langle(0.19, 0.31, 0.47, 0.65), (0.10, 0.27, 0.40, 0.73)\rangle$
A_2	$\langle(0.21, 0.30, 0.43, 0.64), (0.11, 0.23, 0.33, 0.71)\rangle$	$\langle(0.11, 0.19, 0.30, 0.42), (0.06, 0.19, 0.29, 0.51)\rangle$	$\langle(0.35, 0.45, 0.55, 0.65), (0.25, 0.45, 0.55, 0.75)\rangle$
A_3	$\langle(0.23, 0.33, 0.49, 0.75), (0.12, 0.25, 0.37, 0.87)\rangle$	$\langle(0.17, 0.28, 0.40, 0.55), (0.10, 0.27, 0.40, 0.65)\rangle$	$\langle(0.15, 0.26, 0.40, 0.58), (0.07, 0.38, 0.48, 0.65)\rangle$
A_4	$\langle(0.18, 0.25, 0.35, 0.50), (0.10, 0.19, 0.27, 0.52)\rangle$	$\langle(0.10, 0.17, 0.26, 0.38), (0.04, 0.16, 0.25, 0.48)\rangle$	$\langle(0.18, 0.29, 0.44, 0.63), (0.08, 0.26, 0.39, 0.70)\rangle$

between each alternative and the negative ideal, along with the degree of relative closeness and the best alternative, are given in Table 7.

Table 7: Summary results of example ($\alpha = 0.1, \beta' = 0.9$).

$\tilde{A}_i$	$D_{\alpha,\beta'}(\tilde{A}_i, \tilde{I}^-)$	$D_{\alpha,\beta'}(\tilde{A}_i, \tilde{I}^+)$	$CC(\tilde{A}_i)$	Ordering result	Best choice
$\tilde{A}_1$	0.0298	0.0461	0.6070	1	
$\tilde{A}_2$	0.0422	0.0337	0.4439	3	$\tilde{A}_1$
$\tilde{A}_3$	0.0454	0.0315	0.4095	4	
$\tilde{A}_4$	0.0371	0.0403	0.5210	2	

The rank of the best alternative corresponds to the distributor $\tilde{A}_1, at 0.6070$.

5.1 Validation test

In this section, the validation of the example solved in the previous section is performed using the evaluation tests provided by Wang and Triantaphyllou [**?**] for the TOPSIS method.

Validity Test 1

In a multi-criteria group decision-making problem, if we choose an arbitrary alternative except for the optimal option from one of the decision-makers and change the values corresponding to it in each criterion in such a way that they are still between the best and worst alternatives, there should be no change in the final results. Create an alternative. For example, if decision maker #1 changes the value corresponding to the first criterion for the second candidate from M to MH and the value corresponding to the third criterion from MH to M, After solving the general problem, the following values are obtained:

$$CC(\tilde{A}_1) = 0.6014, CC(\tilde{A}_2) = 0.4588, CC(\tilde{A}_3) = 0.4326, CC(\tilde{A}_4) = 0.5175$$

Therefore, there is no change in the order of the alternatives.

Validity test 2

Any effective multi-criteria group decision-making strategy must have transitivity property.

Validity test 3

If the multi-criteria group decision-making problem with trapezoidal intuitionistic fuzzy numbers is broken down into smaller group multi-criteria decision-making problems (with fewer options), and these are solved using the same method as the main problem, there is no difference in how the main problem options are ranked. In other words, the ranking of the alternatives under the problems should be the same as the overall ranking of all the alternatives. To check tests 2 and 3, consider three sub-problems from the main problem as follows:

Sub-problem 1: including three alternatives $\{\tilde{A}_1, \tilde{A}_2, \tilde{A}_3\}$ results in the following ranking:

$$CC(\tilde{A}_1) = 0.6281, CC(\tilde{A}_2) = 0.4342, CC(\tilde{A}_3) = 0.4220 \tilde{A}_1 \succ \tilde{A}_2 \succ \tilde{A}_3$$

Sub-problem 2: including three alternatives $\{\tilde{A}_2, \tilde{A}_3, \tilde{A}_4\}$ results in the following ranking:

$$CC(\tilde{A}_2) = 0.4339, CC(\tilde{A}_3) = 0.4271, CC(\tilde{A}_4) = 0.5060 \tilde{A}_4 \succ \tilde{A}_2 \succ \tilde{A}_3$$

Sub-problem 3: including three alternatives $\{\tilde{A}_3, \tilde{A}_4, \tilde{A}_1\}$ results in the following ranking:

$$CC(\tilde{A}_1) = 0.6095, CC(\tilde{A}_3) = 0.4134, CC(\tilde{A}_4) = 0.5180 \tilde{A}_1 \succ \tilde{A}_4 \succ \tilde{A}_3$$

The results of each sub-problem confirm the findings of the main problem. Additionally, the ranking of alternatives in sub-problems 1 and 2 leads to the ranking in sub-problem 3.

5.2 Sensitivity analysis

In the parametric distance measure used in the multi-criteria decision problem, two parameters α and β', are effective in the distance calculations. Hence, changes in these values can affect the distance result and the decision to choose the best alternative. In the presented example, $\alpha = 0.1$ and $\beta' = 0.9$ are considered. Now, we want to follow three scenarios to check the results with parameter changes α and β'.

In the first scenario, α increases and β' decreases simultaneously. Findings illustrate no changes in ordering alternatives (Table 8).

In the second scenario, $\alpha = 0.1$ is constant and β' decreases from 0.9 to 0.1. Findings illustrate no changes in ordering alternatives except in cases H and I (Table 9).

In the third scenario, α decreases from 0.9 to 0.1 and $\beta' = 0.9$ is constant. Findings illustrate no changes in ordering alternatives except in cases A and B (Table 10).

In the fourth scenario, α and β' increase simultaneously. Findings illustrate no changes in ordering alternatives except in cases H and I (Table 11).

5.3 Comparative analysis

Here, to compare the results of the novel method based on the defined measure with previous measures or methods, we will solve the example presented in the previous section. The summary of solution results is listed in Table 12.

As it can be deduced from the data in Table 12, the desirable candidate in all methods is $\tilde{A}_1$.

Table 8: Ordering results based on $\alpha = 0.1; 0.9$ and $\beta' = 0.9 : 0.1$.

α, β'	$CC(\tilde{A}_1)$	$CC(\tilde{A}_2)$	$CC(\tilde{A}_3)$	$CC(\tilde{A}_4)$	Ranking results	Best choice
$A \begin{cases} \alpha = 0.1 \\ \beta' = 0.9 \end{cases}$	0.6070	0.4439	0.4095	0.5210	$\tilde{A}_1 \succ \tilde{A}_4 \succ \tilde{A}_2 \succ \tilde{A}_3$	$\tilde{A}_1$
$B \begin{cases} \alpha = 0.2 \\ \beta' = 0.8 \end{cases}$	0.6041	0.4426	0.4166	0.5125	$\tilde{A}_1 \succ \tilde{A}_4 \succ \tilde{A}_2 \succ \tilde{A}_3$	$\tilde{A}_1$
$C \begin{cases} \alpha = 0.3 \\ \beta' = 0.7 \end{cases}$	0.6021	0.4417	0.4215	0.5064	$\tilde{A}_1 \succ \tilde{A}_4 \succ \tilde{A}_2 \succ \tilde{A}_3$	$\tilde{A}_1$
$D \begin{cases} \alpha = 0.4 \\ \beta' = 0.6 \end{cases}$	0.6010	0.4412	0.4240	0.5027	$\tilde{A}_1 \succ \tilde{A}_4 \succ \tilde{A}_2 \succ \tilde{A}_3$	$\tilde{A}_1$
$E \begin{cases} \alpha = 0.5 \\ \beta' = 0.5 \end{cases}$	0.6010	0.4411	0.4242	0.5017	$\tilde{A}_1 \succ \tilde{A}_4 \succ \tilde{A}_2 \succ \tilde{A}_3$	$\tilde{A}_1$
$F \begin{cases} \alpha = 0.6 \\ \beta' = 0.4 \end{cases}$	0.6020	0.4415	0.4219	0.5035	$\tilde{A}_1 \succ \tilde{A}_4 \succ \tilde{A}_2 \succ \tilde{A}_3$	$\tilde{A}_1$
$G \begin{cases} \alpha = 0.7 \\ \beta' = 0.3 \end{cases}$	0.6042	0.4423	0.4170	0.5079	$\tilde{A}_1 \succ \tilde{A}_4 \succ \tilde{A}_2 \succ \tilde{A}_3$	$\tilde{A}_1$
$H \begin{cases} \alpha = 0.8 \\ \beta' = 0.2 \end{cases}$	0.6074	0.4435	0.4097	0.5150	$\tilde{A}_1 \succ \tilde{A}_4 \succ \tilde{A}_2 \succ \tilde{A}_3$	$\tilde{A}_1$
$I \begin{cases} \alpha = 0.9 \\ \beta' = 0.1 \end{cases}$	0.6115	0.4451	0.4001	0.5245	$\tilde{A}_1 \succ \tilde{A}_4 \succ \tilde{A}_2 \succ \tilde{A}_3$	$\tilde{A}_1$

Table 9: Ordering results based on $\alpha = 0.1$ and $\beta' = 0.9 : 0.1$.

α, β'	$CC(\tilde{A}_1)$	$CC(\tilde{A}_2)$	$CC(\tilde{A}_3)$	$CC(\tilde{A}_4)$	Ranking results	Best choice
$A \begin{cases} \alpha = 0.1 \\ \beta' = 0.9 \end{cases}$	0.6070	0.4439	0.4095	0.5210	$\tilde{A}_1 \succ \tilde{A}_4 \succ \tilde{A}_2 \succ \tilde{A}_3$	$\tilde{A}_1$
$B \begin{cases} \alpha = 0.1 \\ \beta' = 0.8 \end{cases}$	0.6059	0.4430	0.4137	0.5169	$\tilde{A}_1 \succ \tilde{A}_4 \succ \tilde{A}_2 \succ \tilde{A}_3$	$\tilde{A}_1$
$C \begin{cases} \alpha = 0.1 \\ \beta' = 0.7 \end{cases}$	0.6055	0.4426	0.4156	0.5148	$\tilde{A}_1 \succ \tilde{A}_4 \succ \tilde{A}_2 \succ \tilde{A}_3$	$\tilde{A}_1$
$D \begin{cases} \alpha = 0.1 \\ \beta' = 0.6 \end{cases}$	0.6055	0.4425	0.4157	0.5145	$\tilde{A}_1 \succ \tilde{A}_4 \succ \tilde{A}_2 \succ \tilde{A}_3$	$\tilde{A}_1$
$E \begin{cases} \alpha = 0.1 \\ \beta' = 0.5 \end{cases}$	0.6061	0.4427	0.4144	0.5154	$\tilde{A}_1 \succ \tilde{A}_4 \succ \tilde{A}_2 \succ \tilde{A}_3$	$\tilde{A}_1$
$F \begin{cases} \alpha = 0.1 \\ \beta' = 0.4 \end{cases}$	0.6070	0.4431	0.4120	0.5174	$\tilde{A}_1 \succ \tilde{A}_4 \succ \tilde{A}_2 \succ \tilde{A}_3$	$\tilde{A}_1$
$G \begin{cases} \alpha = 0.1 \\ \beta' = 0.3 \end{cases}$	0.6083	0.4370	0.4087	0.5202	$\tilde{A}_1 \succ \tilde{A}_4 \succ \tilde{A}_2 \succ \tilde{A}_3$	$\tilde{A}_1$
$H \begin{cases} \alpha = 0.1 \\ \beta' = 0.2 \end{cases}$	0.6098	0.4445	0.4046	0.5237	$\tilde{A}_1 \succ \tilde{A}_4 \succ \tilde{A}_3 \succ \tilde{A}_2$	$\tilde{A}_1$
$I \begin{cases} \alpha = 0.1 \\ \beta' = 0.1 \end{cases}$	0.6115	0.4453	0.4001	0.5277	$\tilde{A}_1 \succ \tilde{A}_4 \succ \tilde{A}_3 \succ \tilde{A}_2$	$\tilde{A}_1$

Table 10: Ordering results based on $\alpha = 0.9 : 0.1$ and $\beta' = 0.1$.

α, β'	$CC(\tilde{A}_1)$	$CC(\tilde{A}_2)$	$CC(\tilde{A}_3)$	$CC(\tilde{A}_4)$	Ranking results	Best choice
$A \begin{cases} \alpha = 0.9 \\ \beta' = 0.9 \end{cases}$	0.5873	0.4365	0.4516	0.4733	$\tilde{A}_1 \succ \tilde{A}_4 \succ \tilde{A}_3 \succ \tilde{A}_2$	$\tilde{A}_1$
$B \begin{cases} \alpha = 0.8 \\ \beta' = 0.9 \end{cases}$	0.5916	0.4392	0.4396	0.4841	$\tilde{A}_1 \succ \tilde{A}_4 \succ \tilde{A}_3 \succ \tilde{A}_2$	$\tilde{A}_1$
$C \begin{cases} \alpha = 0.7 \\ \beta' = 0.9 \end{cases}$	0.5946	0.4406	0.4322	0.4915	$\tilde{A}_1 \succ \tilde{A}_4 \succ \tilde{A}_2 \succ \tilde{A}_3$	$\tilde{A}_1$
$D \begin{cases} \alpha = 0.6 \\ \beta' = 0.9 \end{cases}$	0.5971	0.4415	0.4267	0.4975	$\tilde{A}_1 \succ \tilde{A}_4 \succ \tilde{A}_2 \succ \tilde{A}_3$	$\tilde{A}_1$
$E \begin{cases} \alpha = 0.5 \\ \beta' = 0.9 \end{cases}$	0.5993	0.4422	0.4221	0.5028	$\tilde{A}_1 \succ \tilde{A}_4 \succ \tilde{A}_2 \succ \tilde{A}_3$	$\tilde{A}_1$
$F \begin{cases} \alpha = 0.4 \\ \beta' = 0.9 \end{cases}$	0.6014	0.4427	0.4183	0.5077	$\tilde{A}_1 \succ \tilde{A}_4 \succ \tilde{A}_2 \succ \tilde{A}_3$	$\tilde{A}_1$
$G \begin{cases} \alpha = 0.3 \\ \beta' = 0.9 \end{cases}$	0.6033	0.4432	0.4151	0.5123	$\tilde{A}_1 \succ \tilde{A}_4 \succ \tilde{A}_2 \succ \tilde{A}_3$	$\tilde{A}_1$
$H \begin{cases} \alpha = 0.2 \\ \beta' = 0.9 \end{cases}$	0.6052	0.4436	0.4122	0.5168	$\tilde{A}_1 \succ \tilde{A}_4 \succ \tilde{A}_2 \succ \tilde{A}_3$	$\tilde{A}_1$
$I \begin{cases} \alpha = 0.1 \\ \beta' = 0.9 \end{cases}$	0.6070	0.4439	0.4095	0.5210	$\tilde{A}_1 \succ \tilde{A}_4 \succ \tilde{A}_2 \succ \tilde{A}_3$	$\tilde{A}_1$

Table 11: Ordering results based on $\alpha = 0.1 : 0.9$ and $\beta' = 0.1 : 0.9$.

α, β'	$CC(\tilde{A}_1)$	$CC(\tilde{A}_2)$	$CC(\tilde{A}_3)$	$CC(\tilde{A}_4)$	Ranking results	Best choice
$A \begin{cases} \alpha = 0.9 \\ \beta' = 0.9 \end{cases}$	0.6070	0.4439	0.4095	0.5210	$\tilde{A}_1 \succ \tilde{A}_4 \succ \tilde{A}_2 \succ \tilde{A}_3$	$\tilde{A}_1$
$B \begin{cases} \alpha = 0.8 \\ \beta' = 0.9 \end{cases}$	0.6089	0.4443	0.4058	0.5215	$\tilde{A}_1 \succ \tilde{A}_4 \succ \tilde{A}_2 \succ \tilde{A}_3$	$\tilde{A}_1$
$C \begin{cases} \alpha = 0.7 \\ \beta' = 0.9 \end{cases}$	0.6063	0.4433	0.4118	0.5150	$\tilde{A}_1 \succ \tilde{A}_4 \succ \tilde{A}_2 \succ \tilde{A}_3$	$\tilde{A}_1$
$D \begin{cases} \alpha = 0.6 \\ \beta' = 0.9 \end{cases}$	0.6037	0.4422	0.4178	0.5085	$\tilde{A}_1 \succ \tilde{A}_4 \succ \tilde{A}_2 \succ \tilde{A}_3$	$\tilde{A}_1$
$E \begin{cases} \alpha = 0.5 \\ \beta' = 0.9 \end{cases}$	0.6010	0.4411	0.4242	0.5017	$\tilde{A}_1 \succ \tilde{A}_4 \succ \tilde{A}_2 \succ \tilde{A}_3$	$\tilde{A}_1$
$F \begin{cases} \alpha = 0.4 \\ \beta' = 0.9 \end{cases}$	0.5982	0.4400	0.4308	0.4948	$\tilde{A}_1 \succ \tilde{A}_4 \succ \tilde{A}_2 \succ \tilde{A}_3$	$\tilde{A}_1$
$G \begin{cases} \alpha = 0.3 \\ \beta' = 0.9 \end{cases}$	0.5953	0.4389	0.4376	0.4877	$\tilde{A}_1 \succ \tilde{A}_4 \succ \tilde{A}_2 \succ \tilde{A}_3$	$\tilde{A}_1$
$H \begin{cases} \alpha = 0.2 \\ \beta' = 0.9 \end{cases}$	0.5918	0.4377	0.4446	0.4805	$\tilde{A}_1 \succ \tilde{A}_4 \succ \tilde{A}_3 \succ \tilde{A}_2$	$\tilde{A}_1$
$I \begin{cases} \alpha = 0.1 \\ \beta' = 0.9 \end{cases}$	0.5873	0.4365	0.4516	0.4733	$\tilde{A}_1 \succ \tilde{A}_4 \succ \tilde{A}_3 \succ \tilde{A}_2$	$\tilde{A}_1$

Table 12: Results of multi-criteria decision-making based on different measures.

Solution Method	$CC(\tilde{A}_1)$	$CC(\tilde{A}_2)$	$CC(\tilde{A}_3)$	$CC(\tilde{A}_4)$	Ordering	Best choice
Euclidean distance [68]	0.6116	0.4458	0.3958	0.5330	$\tilde{A}_1 \succ \tilde{A}_4 \succ \tilde{A}_2 \succ \tilde{A}_3$	$\tilde{A}_1$
The method presented in [30]	0.552	0.391	0.482	0.462	$\tilde{A}_1 \succ \tilde{A}_3 \succ \tilde{A}_4 \succ \tilde{A}_2$	$\tilde{A}_1$
New parametric distance per $\alpha = 0.1, \beta' = 0.9$	0.6070	0.4439	0.4095	0.5210	$\tilde{A}_1 \succ \tilde{A}_4 \succ \tilde{A}_2 \succ \tilde{A}_3$	$\tilde{A}_1$

6 Conclusion

This study presents a conceptual framework for comparing trapezoidal intuitionistic fuzzy numbers, introducing a novel type of distance measure. As demonstrated in Theorem 3.2, the new parametric distance measure adheres to axiomatic principles. When combined with measure-dependence processes, the proposed measures demonstrate effective performance. It is important to note that the weight coefficients in the defined measure offer significant flexibility in responding to the decision-maker's preferences. By applying this distance measure in multi-criteria group decision-making, an example from the literature was selected and solved at different cut levels. The results obtained from this process provide valuable insights into the effectiveness and realistic performance of the proposed method, particularly when compared to existing methods in the literature. For future research, extending this approach and integrating it with other distance measures, as well as similar algorithms in clustering and classification, would be of interest. Additionally, by exploring the relationship between distance and similarity measures, new similarity measures could be defined based on the proposed distance measure.

Compliance with ethical standards

Conflict of interest The authors declare that they have no conflict of interest.

Informed consent was not required as no humans or animals were involved.

Human and animal rights This article does not contain any studies with human or animal subjects performed by any of the authors.

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