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Double ARA-Sumudu decomposition method for the solution of linear fractional Partial integro-differential equations

Jagdish A Nanaware[a], Akshaykumar Lawu Dongardive[b,*]

[a]Department of PG Studies and Research in Mathematics, Shrikrishna Mahavidyalaya, Gunjoti, Dharashiv, 413606, Maharashtra, India.
[b]Department of Mathematics, Shankarrao Patil Mahavidyalaya, Bhoom, Dharashiv, 413504, Maharashtra, India.

Abstract

This paper focuses on deriving the exact solution to the linear fractional partial integro-differential equation through the application of the double ARA-Sumudu transform. By leveraging the double ARA-Sumudu transform, a decomposition method is constructed using the definition of the Caputo fractional derivative. Surface graphs of solutions are provided for visualization purposes.

Keywords: Fractional partial integro-differential equation, Double ARA-Sumudu transform, Decomposition method

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1. Introduction

Fractional calculus is a branch of mathematical analysis that generalizes differentiation and integration to non-integer orders. It has many applications in various disciplines ([2],[3],[8],[9],[13],[14],[23]). Fractional integro-differential equations are used to model various processes in science and engineering. For example, in([25]), the author formulated a fractional integro-differential equation (FI-DEs) from electromagnetic waves in a dielectric medium. Many authors have studied the existence and uniqueness of solutions to fractional integro-differential equations (FIDEs) ([7],[12],[20],[21],[26],[17]). Recently, several numerical and analytical techniques have been explored to solve fractional integro-differential equations.One of these methods is the Adomian decomposition method, which was introduced by George Adomian ([4]). Authors in ([10]), applied the least squares method using a compact combination of shifted Chebyshev polynomials of the first kind to solve the fractional integro-differential equation.In([15]), applied C^3-spline method for solution of fractional integro-differental equation.Authors in ([24]) employed Haar & Legendre wavelet technique for linear system of fractional integro-differential equations. In ([6]), CaputoKatugampola fractional VolterraFredholm integro-differential equation is solved by the modified Adomian decomposition method. The least square collocation Chebyshev technique is used to solve system of linear fractional integro-differential equation in ([22]).The collocation method was used for the numerical solution of nonlinear Volterra integro- differential equations of Fractional order in ([5]). In ([19]),

*Corresponding author
Email addresses: jag_skmg91@rediffmail.com (Jagdish A Nanaware), akshay.dongardive26892@gmail.com (Akshaykumar Lawu Dongardive)

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the Toeplitz matrix and Nyström method were applied to solve a linear fractional integro-differential equation. In ([1]), an efficient spline technique is used to solve time-fractional integro-differential equations. The decomposition method for approximating the solution of the FI-DEs system was utilized in ([16]), while in ([11]), The system of linear Fredholm fractional integro-differential equations is solved by using the least squares method and Lauguerre Polynomials.

In this paper we obtained the exact solution of the following linear fractional partial integro-differential equation (LFPI-DE) [19]

$$\frac{\partial^{\alpha}u(x,t)}{\partial t^{\alpha}} = f(x,t) + \lambda \int_{a}^{b} k(x,y)u(y,t)dy, 0 < \alpha < 1, \tag{1}$$

with the initial condition

$$u(x,0) = u_0(x), \tag{2}$$

where $\frac{\partial^{\alpha}u(x,t)}{\partial t^{\alpha}}$ is the Caputo derivative of order α, λ is constant and the functions $f(x,t)$, $k(x,y)$ are predefined.

2. Double ARA-Sumudu Transform(DARA-ST)

The double ARA-Sumudu transform (DARA-ST) [18] of the continuous function $f(x,t)$ of two variables $x > 0, t > 0$ is given by

$$G_x S_t[f(x,t)] = F(s,v) = \frac{s}{v}\int_0^{\infty}\int_0^{\infty} e^{-sx-\frac{1}{v}} f(x,t)dx\,dt, s>0, v>0$$

provided the integral exists.
The inverse DARA-ST is given by

$$G_x^{-1}S_t^{-1}[F(s,v)] = f(x,t) = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty}\frac{e^{sx}}{s}\,ds\frac{1}{2\pi i}\int_{\omega-i\infty}^{\omega+i\infty}\frac{e^{\frac{t}{v}}}{v}F(s,v)\,dv$$

2.1. *Linearity property [18]*

The DARA-ST is linear , since

$$G_x S_t[af(x,t) + b\,g(x,t)] = a\,G_x S_t[f(x,t)] + b\,G_x S_t[g(x,t)]$$

2.2.

The Caputo derivative of order α and β of the function $f(x,t)$ with respect to x and t respectively are given by [18]

$$\frac{\partial^{\alpha}f(x,t)}{\partial x^{\alpha}} = \begin{cases} \frac{1}{\Gamma(n-\alpha)}\int_0^x (x-\tau)^{n-\alpha-1}\frac{\partial^n f(\tau,t)}{\partial \tau^n}\,d\tau, & n-1<\alpha<n, n\in\mathbb{N} \\ \frac{\partial^n f}{\partial x^n}, & n=\alpha \end{cases}$$

and

$$\frac{\partial^{\beta}f(x,t)}{\partial t^{\beta}} = \begin{cases} \frac{1}{\Gamma(m-\beta)}\int_0^t (t-\tau)^{m-\beta-1}\frac{\partial^m f(x,\tau)}{\partial \tau^m}\,d\tau, & m-1<\beta<m, m\in\mathbb{N} \\ \frac{\partial^m f}{\partial x^m}, & m=\beta \end{cases}$$

2.3. *DARA-ST for some basic functions [18]*

1. $G_xS_t[1] = 1$
2. $G_xS_t[x^a\, t^b] = s^{-a}\,\Gamma(a+1)v^b\,\Gamma(b+1)$
3. $G_xS_t[e^{ax+bt}] = \frac{s}{(s-a)(1-bv)}$

2.4. *The DARA-ST for Caputo fractional derivatives [18]*

1. $G_xS_t[\frac{\partial^\alpha f(x,t)}{\partial x^\alpha}] = s^\alpha F(s,v) - \sum_{i=0}^{n-1} s^{\alpha-i} S_t[\frac{\partial^i f(0,t)}{\partial x^i}], n-1 < \alpha \leqslant n$
2. $G_xS_t[\frac{\partial^\beta f(x,t)}{\partial t^\beta}] = \frac{F(s,v)}{v^\beta} - \sum_{j=0}^{m-1} v^{j-\beta} G_x[\frac{\partial^j f(x,0)}{\partial t^j}], m-1 < \beta \leqslant m$

3. Double ARA-Sumudu Decomposition method

In this section, we develop the method of finding the exact solution for the initial value linear fractional partial integro-differential equation given by equation (1-2).

Taking the DARA-ST on both sides of equation (1) subject to the initial conditions in (2), we get:

$$G_xS_t\left[\frac{\partial^\alpha u(x,t)}{\partial t^\alpha}\right] = G_xS_t\,[f(x,t)] + G_xS_t\left[\lambda\int_a^b k(x,y)u(y,t)\,dy\right]$$

Using property of Double ARA-Sumudu transform ,we get

$$\frac{F(s,v)}{v^\alpha} - \sum_{j=0}^{m-1} v^{j-\alpha}G_x\left[\frac{\partial^j u(x,0)}{\partial t^j}\right] = G_xS_t\,[f(x,t)] + G_xS_t\left[\lambda\int_a^b k(x,y)u(y,t)\,dy\right]$$

and

$$\frac{F(s,v)}{v^\alpha} = v^{-\alpha}G_x\,[u_0(x)] + G_xS_t\,[f(x,t)] + G_xS_t\left[\lambda\int_a^b k(x,y)u(y,t)\,dy\right]$$

$$F(s,v) = G_x\,[u_0(x)] + v^\alpha G_xS_t\,[f(x,t)] + v^\alpha G_xS_t\left[\lambda\int_a^b k(x,y)u(y,t)\,dy\right]$$

Taking inverse DARA-ST on both side we get

$$u(x,t) = G_x^{-1}S_t^{-1}\,[G_x\,[u_0(x)] + v^\alpha G_xS_t\,[f(x,t)]] + G_x^{-1}S_t^{-1}\left[v^\alpha G_xS_t\left[\lambda\int_a^b k(x,y)u(y,t)\,dy\right]\right]$$

$$u(x,t) = f_1(x,t) + f_2(x,t) + G_x^{-1}S_t^{-1}\left[v^\alpha G_xS_t\left[\lambda\int_a^b k(x,y)u(y,t)\,dy\right]\right], \tag{3}$$

where

$$f_1(x,t) + f_2(x,t) = G_x^{-1}S_t^{-1}\,[G_x\,[u_0(x)] + v^\alpha G_xS_t\,[f(x,t)]]$$

Now we assume that the function u(x,t) have the series solution

$$u(x,t)] = \sum_{m=0}^{\infty} u_m(x,t)$$

Thus the equation (3) becomes

$$\sum_{m=0}^{\infty} u_m(x,t) = f_1(x,t) + f_2(x,t) + G_x^{-1}S_t^{-1}\left[v^\alpha G_xS_t\left[\lambda\int_a^b k(x,y)\sum_{m=0}^{\infty} u_m(y,t)\,dy\right]\right]$$

Thus our recursive relation is as follows

$$u_0(x,t) = f_1,(x,t) \tag{4}$$

$$u_1(x,t) = f_2(x,t) + G_x^{-1}S_t^{-1}\left[\nu^\alpha G_x S_t\left[\lambda\int_a^b k(x,y)u_0(y,t)\,dy\right]\right] \tag{5}$$

$$u_{m+1}(x,t) = G_x^{-1}S_t^{-1}\left[\nu^\alpha G_x S_t\left[\lambda\int_a^b k(x,y)u_m(y,t)\,dy\right]\right], m \geqslant 1 \tag{6}$$

The solution through DARA-ST decomposition method depends upon the choice of $f_1(x,t)$ and $f_2(x,t)$ where $f_1(x,t)$ and $f_2(x,t)$ represent the term arising from source term and prescribrd initial conditions.

4. Application of DARA-ST Decomposition method

In this section we give examples to illustrate this method for linear fractional partial integro-differential equations and surface graph of solution for each example.

Example 4.1. [19] Consider the LFPI-DE

$$\frac{\partial^\alpha u(x,t)}{\partial t^\alpha} = \frac{xt^{1-\alpha}}{\Gamma(2-\alpha)} - \frac{2t^{2-\alpha}}{\Gamma(3-\alpha)} - \lambda x(t - t^2 e + t^2) + \lambda\int_0^1 xe^y u(y,t)dy \tag{7}$$

with initial condition

$$u_0(x) = 0 \tag{8}$$

Applying DARA-ST on both side of (7) we get

$$G_x S_t\left[\frac{\partial^\alpha u(x,t)}{\partial t^\alpha}\right] = G_x S_t\left[\frac{xt^{1-\alpha}}{\Gamma(2-\alpha)} - \frac{2t^{2-\alpha}}{\Gamma(3-\alpha)} - \lambda x(t-t^2e+t^2)\right] + G_x S_t\left[\lambda\int_0^1 xe^y u(y,t)dy\right]$$

Using property of Double ARA-Sumudu transform and initial condition (8) we get

$$\frac{F(s,\nu)}{\nu^\alpha} - \sum_{j=0}^{m-1}\nu^{j-\alpha}G_x\left[\frac{\partial^j u(x,0)}{\partial t^j}\right] = \frac{1}{s}\Gamma(2)\nu^{1-\alpha} - 2\nu^{2-\alpha} - \frac{\lambda}{s}(\nu\Gamma(2) - \nu^2 e\Gamma(3) + \nu^2\Gamma(3))$$
$$+ G_x S_t\left[\lambda\int_0^1 xe^y u(y,t)dy\right]$$

and

$$\frac{F(s,\nu)}{\nu^\alpha} = \frac{1}{s}\nu^{1-\alpha} - 2\nu^{2-\alpha} - \frac{\lambda}{s}(\nu - 2\nu^2 e + 2\nu^2) + G_x S_t\left[\lambda\int_0^1 xe^y u(y,t)dy\right]$$

$$F(s,\nu) = \frac{1}{s}\nu - 2\nu^2 - \frac{\lambda}{s}(\nu^{\alpha+1} - 2\nu^{\alpha+2}e + 2\nu^{\alpha+2}) + \nu^\alpha G_x S_t\left[\lambda\int_0^1 xe^y u(y,t)dy\right] \tag{9}$$

Taking inverse DARA-ST on both side of (9) we get

$$u(x,t) = xt - t^2 - \lambda x\left(\frac{t^{\alpha+1}}{\Gamma(\alpha+2)} - 2e\frac{t^{\alpha+2}}{\Gamma(\alpha+3)} + 2\frac{t^{\alpha+2}}{\Gamma(\alpha+3)}\right)$$
$$+ G_x^{-1}S_t^{-1}\left[\nu^\alpha G_x S_t\left[\lambda\int_0^1 xe^y u(y,t)\,dy\right]\right] \tag{10}$$

Now we assume that the function $u(x,t)$ have the series solution

$$u(x,t) = \sum_{m=0}^{\infty} u_m(x,t)$$

Thus equation (10) becomes

$$\begin{aligned}\sum_{m=0}^{\infty} u_m(x,t) &= xt - t^2 - \lambda x\left(xt - t^2 - \lambda x\left(\frac{t^{\alpha+1}}{\Gamma(\alpha+2)} - 2e\frac{t^{\alpha+2}}{\Gamma(\alpha+3)} + 2\frac{t^{\alpha+2}}{\Gamma(\alpha+3)}\right)\right)\\ &\quad + G_x^{-1}S_t^{-1}\left[v^{\alpha}G_xS_t\left[\lambda\int_0^1 xe^y \sum_{m=0}^{\infty} u_m(y,t)\,dy\right]\right]\end{aligned}$$

Thus using recursive relation we have

$$u_0(x,t) = xt - t^2$$

$$\begin{aligned}u_1(x,t) &= -\lambda x\left(xt - t^2 - \lambda x\left(\frac{t^{\alpha+1}}{\Gamma(\alpha+2)} - 2e\frac{t^{\alpha+2}}{\Gamma(\alpha+3)} + 2\frac{t^{\alpha+2}}{\Gamma(\alpha+3)}\right)\right)\\ &\quad + G_x^{-1}S_t^{-1}\left[v^{\alpha}G_xS_t\left[\lambda\int_0^1 xe^y u_0(y,t)\,dy\right]\right]\\ &= 0\end{aligned}$$

thus

$$u_{m+1}(x,t) = G_x^{-1}S_t^{-1}\left[v^{\alpha}G_xS_t\left[\lambda\int_0^1 k(x,y)u_m(y,t)\,dy\right]\right] = 0 \quad \forall m \geqslant 1$$

By adding all the iterations we get

$$u(x,t) = xt - t^2$$

which is the exact solution.

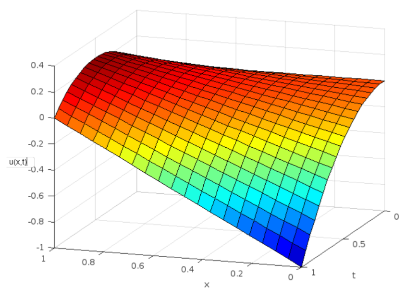

Figure 1: Surface graph for example 4.1.

Example 4.2. [19] Consider the LFPI-DE

$$\frac{\partial^{\alpha}u(x,t)}{\partial t^{\alpha}} = \frac{2x^2t^{2-\alpha}}{\Gamma(3-\alpha)} - \lambda x^2\left(\frac{x}{4} + \frac{x^2}{5}\right) + \lambda\int_0^1 xe^y u(y,t)dy \tag{11}$$

with initial condition

$$u_0(x) = 0 \tag{12}$$

Applying DARA-ST on both side of (11) we get

$$G_x S_t\left[\frac{\partial^\alpha u(x,t)}{\partial t^\alpha}\right] = G_x S_t\left[\frac{2x^2t^{2-\alpha}}{\Gamma(3-\alpha)} - \lambda x^2(\frac{x}{4}+\frac{x^2}{5})\right] + G_x S_t\left[\lambda\int_0^1 xe^y u(y,t)dy\right]$$

Using property of double ARA-Sumudu transform and initial condition (12) we get

$$\frac{F(s,u)}{v^\alpha} = 2s^{-2}\Gamma(3)v^{2-\alpha} - \lambda v^2\Gamma(3)\left(\frac{s^{-1}}{4}+\frac{s^{-2}\Gamma(3)}{5}\right) + G_x S_t\left[\lambda\int_0^1 (xy+x^2y^2)u(y,t)dy\right]$$

and

$$F(s,u) = 4s^{-2}v^2 - 2\lambda v^{\alpha+2}\left(\frac{s^{-1}}{4}+\frac{s^{-2}\Gamma(3)}{5}\right) + \left[v^\alpha G_x S_t\left[\lambda\int_0^1 (xy+x^2y^2)u(y,t)dy\right]\right] \tag{13}$$

Taking inverse DARA-ST on both sides of (13) we get

$$u(x,t) = \frac{4x^2t^2}{\Gamma(3)\Gamma(3)} - \frac{2\lambda t^{\alpha+2}}{\Gamma(\alpha+3)}\left(\frac{x}{4}+\frac{2x^2}{5\Gamma(3)}\right) + G_x^{-1}S_t^{-1}\left[v^\alpha G_x S_t\left[\lambda\int_0^1 (xy+x^2y^2)u(y,t)dy\right]\right] \tag{14}$$

Now we assume that the function $u(x,t)$ have the series solution

$$u(x,t) = \sum_{m=0}^{\infty} u_m(x,t)$$

thus equation (14) becomes

$$\sum_{m=0}^{\infty} u_m(x,t) = x^2t^2 - \frac{2\lambda t^{\alpha+2}}{\Gamma(\alpha+3)}\left(\frac{x}{4}+\frac{x^2}{5}\right) + G_x^{-1}S_t^{-1}\left[v^\alpha G_x S_t\left[\lambda\int_0^1 (xy+x^2y^2)\sum_{m=0}^{\infty} u_m(y,t)dy\right]\right]$$

Using recursive relation we have

$$u_0(x,t) = x^2t^2$$

$$\begin{aligned}
u_1(x,t) &= -\frac{2\lambda t^{\alpha+2}}{\Gamma(\alpha+3)}\left(\frac{x}{4}+\frac{x^2}{5}\right) + G_x^{-1}S_t^{-1}\left[v^\alpha G_x S_t\left[\lambda\int_0^1 (xy+x^2y^2)u_0(y,t)dy\right]\right] \\
&= -\frac{2\lambda t^{\alpha+2}}{\Gamma(\alpha+3)}\left(\frac{x}{4}+\frac{x^2}{5}\right) + G_x^{-1}S_t^{-1}\left[v^\alpha G_x S_t\left[\lambda\int_0^1 (xy+x^2y^2)y^2t^2dy\right]\right] \\
&= -\frac{2\lambda t^{\alpha+2}}{\Gamma(\alpha+3)}\left(\frac{x}{4}+\frac{x^2}{5}\right) + G_x^{-1}S_t^{-1}\left[v^\alpha G_x S_t\left[\frac{\lambda x t^2}{4}+\frac{\lambda x^2t^2}{5}\right]\right] \\
&= -\frac{2\lambda t^{\alpha+2}}{\Gamma(\alpha+3)}\left(\frac{x}{4}+\frac{x^2}{5}\right) + G_x^{-1}S_t^{-1}\left[\frac{\lambda}{2}s^{-1}v^{\alpha+2}+\frac{4}{5}\lambda s^{-2}v^{\alpha+2}\right] \\
&= -\frac{2\lambda t^{\alpha+2}}{\Gamma(\alpha+3)}\left(\frac{x}{4}+\frac{x^2}{5}\right) + \frac{2\lambda t^{\alpha+2}}{\Gamma(\alpha+3)}\left(\frac{x}{4}+\frac{x^2}{5}\right) \\
&= 0
\end{aligned}$$

and

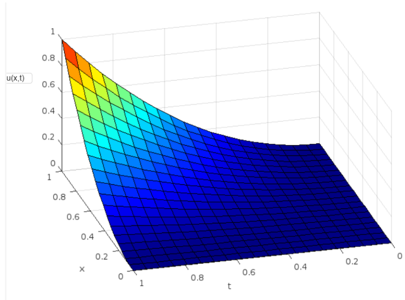

Figure 2: Surface graph for example 4.2.

$$u_{m+1}(x,t) = G_x^{-1}S_t^{-1}\left[v^{\alpha}G_xS_t\left[\lambda\int_0^1(xy+x^2y^2)u_m(y,t)dy\right]\right] = 0 \quad \forall m \geqslant 1$$

By adding all the iterations we get

$$u(x,t) = x^2t^2$$

which is the exact solution .

Example 4.3. Consider the LFPI-DE

$$\frac{\partial^{\alpha}u(x,t)}{\partial t^{\alpha}} = \frac{-xt^{1-\alpha}}{\Gamma(2-\alpha)} + \frac{1}{6}(3x+2t-3xt-2) - \int_0^1(x-y)u(y,t)dy \tag{15}$$

with initial condition

$$u_0(x) = x \tag{16}$$

Applying DARA-ST on both side of (15) we get

$$G_xS_t\left[\frac{\partial^{\alpha}u(x,t)}{\partial t^{\alpha}}\right] = G_xS_t\left[\frac{-xt^{1-\alpha}}{\Gamma(2-\alpha)} + \frac{1}{6}(3x+2t-3xt-2)\right] - G_xS_t\left[\int_0^1(x-y)u(y,t)dy\right]$$

Using property of double ARA-Sumudu transform and initial condition (16) we get

$$\frac{F(s,v)}{v^{\alpha}} - \sum_{j=0}^{m-1}v^{j-\alpha}G_x\left[\frac{\partial^j u(x,0)}{\partial t^j}\right] = -s^{-1}v^{1-\alpha} + \frac{1}{6}\left[3s^{-1}+2v-3s^{-1}v-2\right] - G_xS_t\left[\int_0^1(x-y)u(y,t)dy\right]$$

and

$$\frac{F(s,v)}{v^{\alpha}} = v^{-\alpha}s^{-1} - s^{-1}v^{1-\alpha} + \frac{1}{6}\left[3s^{-1}+2v-3s^{-1}v-2\right] - G_xS_t\left[\int_0^1(x-y)u(y,t)dy\right]$$

and

$$F(s,v) = s^{-1} - s^{-1}v + \frac{1}{6}\left[3s^{-1}v^{\alpha}+2v^{\alpha+1}-3s^{-1}v^{\alpha+1}-2v^{\alpha}\right] - v^{\alpha}G_xS_t\left[\int_0^1(x-y)u(y,t)dy\right] \tag{17}$$

Taking inverse DARA-ST on both sides of (17) we get

$$
\begin{aligned}
u(x,t) = x - xt + \frac{1}{6}\left[\frac{3xt^{\alpha}}{\Gamma(\alpha+1)} + \frac{2t^{\alpha+1}}{\Gamma(\alpha+2)} - \frac{3xt^{\alpha+1}}{\Gamma(\alpha+2)} - \frac{2t^{\alpha}}{\Gamma(\alpha+1)}\right] \\
- G_x^{-1}S_t^{-1}\left[v^{\alpha}G_xS_t\left[\int_0^1 (x-y)u(y,t)dy\right]\right]
\end{aligned} \tag{18}
$$

Now we assume that the function $u(x,t)$ have the series solution

$$
u(x,t) = \sum_{m=0}^{\infty} u_m(x,t)
$$

Thus equation (18) becomes

$$
\begin{aligned}
\sum_{m=0}^{\infty} u_m(x,t) = x - xt + \frac{1}{6}\left[\frac{3xt^{\alpha}}{\Gamma(\alpha+1)} + \frac{2t^{\alpha+1}}{\Gamma(\alpha+2)} - \frac{3xt^{\alpha+1}}{\Gamma(\alpha+2)} - \frac{2t^{\alpha}}{\Gamma(\alpha+1)}\right] \\
- G_x^{-1}S_t^{-1}\left[v^{\alpha}G_xS_t\left[\int_0^1 (x-y)\sum_{m=0}^{\infty} u_m(y,t)dy\right]\right]
\end{aligned}
$$

Using recursive relation we have

$$
u_0(x,t) = x - xt
$$

$$
\begin{aligned}
u_1(x,t) &= \frac{1}{6}\left[\frac{3xt^{\alpha}}{\Gamma(\alpha+1)} + \frac{2t^{\alpha+1}}{\Gamma(\alpha+2)} - \frac{3xt^{\alpha+1}}{\Gamma(\alpha+2)} - \frac{2t^{\alpha}}{\Gamma(\alpha+1)}\right] \\
&\quad - G_x^{-1}S_t^{-1}\left[v^{\alpha}G_xS_t\left[\int_0^1 (x-y)u_0(y,t)dy\right]\right] \\
&= 0
\end{aligned}
$$

and

$$
u_{m+1}(x,t) = G_x^{-1}S_t^{-1}\left[v^{\alpha}G_xS_t\left[\lambda\int_0^1 (x-y)u_m(y,t)dy\right]\right] = 0 \quad \forall m \geqslant 1
$$

By adding all the iterations we get

$$
u(x,t) = x - xt
$$

which is the exact solution.

5. Conclusion

The double ARA-Sumudu transform Decomposition method for finding the exact solution of linear fractional partial integro-differential equation by applying the proposed integral transform, with the help of the double ARA-Sumudu transform and Adomian decomposition method is developed in the paper. Using the definition of Caputo fractional derivative we conclude the double ARA-Sumudu transform decomposition method. also, the method is applied to some examples for finding the exact solutions in a minimum number of iterations without any discretization. Thus the proposed method can also be applied to solve various fractional models more efficiently.

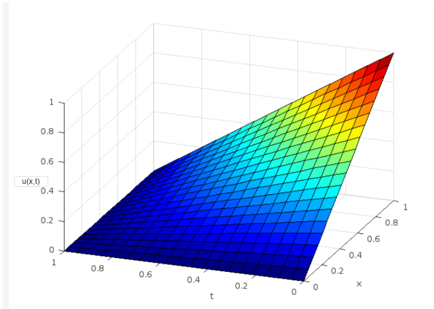

Figure 3: Surface graph for example 4.3.

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