Math. & Comput. Sci., 6(3) 2025, 143–152

Online: ISSN 2717-2708

Mathematics and Computational Sciences

Journal Homepage: http://mcs.qut.ac.ir/mcs

Qom University of Technology

Solving nonlinear fractional autonomous dynamical systems using a Laplace-Daftardar-Jafari method

Abubker Ahmed[a,b,*]

[a]Department of General Sciences, College of Engineering, AlMughtaribeen University, Sudan

[b]Program of Information Technology, Ibn Khaldoon College, Khartoum, Sudan

Abstract

In this manuscript, we present a novel analytical method, namely the Laplace Daftardar Jafari method (LDJM), for solving nonlinear fractional-order autonomous dynamical systems using the Caputo fractional derivative. The proposed method is demonstrated to be highly accurate and efficient through various examples, graphical illustrations, and numerical comparisons with the Laplace residual power series method (LRPS). The LDJM has the potential to be applied to a wide range of complex dynamical systems, including biological and physical systems.

Keywords: Nonlinear fractional-order systems, Autonomous dynamical systems, Laplace Daftardar-Jafari method, Caputo fractional derivative.

2020 MSC: 37XX, 34AXX

©2025 All rights reserved.

1. Introduction

Autonomous nonlinear dynamical systems are complex systems that appear in various scientific and engineering fields, such as physics, engineering, and economics [28, 6, 10]. These systems are characterized by the absence of external energy sources, and therefore their behavior is solely determined by the initial conditions and the governing laws. Therefore, the fractional derivatives are particularly suitable for such cases, as they can capture the long-term effects of these systems [11, 7, 29]. As a result, numerous researchers investigate these systems to understand nonlinear behavior, analyze stability and instability, develop strategies and methods for controlling their behavior, and comprehend the complex phenomena that arise in them [8, 17, 12, 18, 32, 14]. However, solving these equations can be difficult, especially when they are nonlinear. To resolve this challenge, numerous analytical and numerical methods have been developed to solve differential equations; for instance, the Adomian decomposition method (ADM) [23], the Hussein–Jassim method (HJM) [25], the Chebyshev spectral collocation method [37], the Sawi decomposition method [22], the integral transforms methods [5], the Sumudu decomposition method (SDM) [15], the Elzaki homotopy analysis method [34], the modification homotopy perturbation method [13], the Laplace variational iteration method [24], the Sumudu variational iteration method [27], and other

*Corresponding author

Email address: edpma4m23@utq.edu.iq (Abubker Ahmed)

doi: 10.30511/mcs.2025.2056237.1321

Received: 18 March 2025 Accepted: 02 October 2025

methods [26, 36, 2, 19, 1, 20, 21, 33]. Among these methods, the Daftardar-Jafari method (DJM) is one of the elective methods developed in 2006 [16]; it is a modified version of Adomians decomposition method (ADM) [3], which eliminates the need for calculating Adomian polynomials in any of the iterations. The DJM has been extensively studied by many researchers; see [30, 35, 4, 16].

This study aims to solve autonomous nonlinear dynamical systems of fractional order in the Caputo sense, using a novel hybrid approach that reduces computational time, effort, and error. This hybrid method combines the Laplace transform method (LTM) with the Daftardar-Jafari method (DJM), known as the Laplace-Daftardar-Jafari method (LDJM). Furthermore, numerical results will be compared with those obtained using the Laplace residual power series method (LRPS) to provide a comprehensive and effective analysis for solving these complex systems. This comparative analysis will highlight the advantages and limitations of each method, thereby offering insights into the most efficient techniques for tackling autonomous nonlinear dynamical systems of fractional order. By demonstrating the effectiveness of the LDJM, we aim to enhance the understanding and application of fractional calculus in real-world scenarios.

This paper is organized as follows. In Section 2, we present some key definitions of fractional calculus. In Section 3, the LDJM is scrutinized. Section 4 provides illustrative applications of the proposed method. The paper concludes with section 5.

2. Preliminaries and Basic Definitions

This section provides an overview of the definitions of fractional calculus.

Definition 2.1. The Laplace transform of a function $y(t)$ is defined as: [7, 20]

$$L[y(t)] = \int_0^\infty y(t)e^{-pt}dt = Y(P), \quad t \geqslant 0 \tag{2.1}$$

Definition 2.2. The Caputo fractional derivative of a function $y(t)$ is defined as: [7, 20]

$$D_t^\gamma y(t) = \frac{1}{\Gamma(m-\gamma)} \int_0^t \frac{y^{(m)}(x)}{(t-x)^{\gamma-m+1}} dx, \tag{2.2}$$

for $m-1 < \gamma < m$, $m \in \mathbb{N}$.

Definition 2.3. The Laplace transform of the Caputo fractional derivative is defined as:[7, 9]

$$L[D_t^\gamma y(t)] = p^\gamma Y(p) - \sum_{i=0}^{m-1} p^{\gamma-i-1} y^{(i)}(0), \tag{2.3}$$

for $m-1 < \gamma < m$, $m \in \mathbb{N}$.

3. Analysis of the Method

Consider the following fractional differential equation:

$$D_t^\gamma y(t) = \mathcal{R}[y(t)] + \mathcal{M}[y(t)], \quad 0 < \gamma < 1, \quad t > 0 \tag{3.1}$$

with initial condition

$$y(0) = \varphi, \tag{3.2}$$

where D_t^γ is the Caputo fractional derivative, $\mathcal{R}$ and $\mathcal{M}$ are linear and nonlinear operators acting on $y(t)$ respectively.

Applying the Laplace transform to both sides of equation (4) and using the initial condition (5), we obtain

$$y(t) = \varphi + \mathcal{L}^{-1}\left[\frac{1}{p^\gamma}\mathcal{L}\left[\mathcal{R}[y(t)] + \mathcal{M}[y(t)]\right]\right]. \tag{3.3}$$

3.1. The Adomian Decomposition Method (ADM)

Suppose the solution of equation (6) takes the form

$$y(t) = \sum_{i=0}^{\infty} y_i(t), \tag{3.4}$$

and the nonlinear function $M[y(t)]$ is formulated as

$$M\left[\sum_{i=0}^{\infty} y_i(t)\right] = \sum_{i=0}^{\infty} A_i(t), \tag{3.5}$$

where the Adomian polynomials $A_i(t)$ are obtained through the relation

$$A_i(t) = \frac{1}{i!}\frac{d^i}{d\lambda^i} M\left(\sum_{n=0}^{i} \lambda^n y_n(t)\right)\Bigg|_{\lambda=0}, \quad i \geqslant 0. \tag{3.6}$$

As a result, the terms of the series are given by:

$$\begin{cases} y_0(t) = \varphi, \\ y_i(t) = \mathcal{L}^{-1}\left[\frac{1}{p^\gamma}\mathcal{L}\left[\mathcal{R}[y_{i-1}(t)] + A_{i-1}(t)\right]\right], \quad i \geqslant 1. \end{cases} \tag{3.7}$$

Refer to [14] for a discussion on the convergence of this series.

3.2. The Daftardar-Jafari Method (DJM)

In recent years, Daftardar-Gejji and Jafari [24] employed a different decomposition approach for the nonlinear term $M[y(t)]$, decomposing it as follows:

$$M\left[\sum_{i=0}^{\infty} y_i(t)\right] = M[y_0(t)] + \sum_{i=1}^{\infty}\left(M\left[\sum_{n=0}^{i} y_n(t)\right] - M\left[\sum_{n=0}^{i-1} y_n(t)\right]\right). \tag{3.8}$$

The recurrence formula is formulated as:

$$\begin{cases} y_0(t) = \varphi, \\ y_1(t) = \mathcal{L}^{-1}\left[\frac{1}{p^\gamma}\mathcal{L}\left[\mathcal{R}[y_0(t)] + M[y_0(t)]\right]\right], \\ y_i(t) = \mathcal{L}^{-1}\left[\frac{1}{p^\gamma}\mathcal{L}\left[\mathcal{R}[y_{i-1}(t)] + M\left[\sum_{n=0}^{i-1} y_n(t)\right] - M\left[\sum_{n=0}^{i-2} y_n(t)\right]\right]\right], \quad i \geqslant 2. \end{cases} \tag{3.9}$$

4. Application and Numerical results

In this section, we illustrate two examples of fractional nonlinear autonomous systems of varying orders. The implementation steps of the LDJM are elucidated, and the effectiveness of the proposed approach is validated through a comprehensive analysis, including graphical representations and tabular data.

Example 4.1. Firstly, consider the following autonomous 2-dimensional fractional nonlinear systems [1]:

$$\begin{cases} D_t^\gamma x(t) = 0.5x(t), \\ D_t^\gamma y(t) = x^2(t) + y(t), \end{cases} \qquad 0 < \gamma < 1, \tag{4.1}$$

subject to initial conditions

$$x(0) = 1, \quad y(0) = 0. \tag{4.2}$$

Operating Laplace transform on both sides of (13) and using (14), we get

$$\begin{cases} x(t) = 1 + 0.5\mathcal{L}^{-1}\left[\frac{1}{p^{\gamma}}\mathcal{L}\left[x(t)\right]\right], \\ y(t) = \mathcal{L}^{-1}\left[\frac{1}{p^{\gamma}}\mathcal{L}\left[\sum_{i=0}^{\infty} A_i(t) + y(t)\right]\right], \end{cases} \tag{4.3}$$

where $A_i(t)$ denote the Daftardar-Jafari polynomials corresponding to the nonlinear operator $x^2(t)$.

In view of LDJM,

$$\begin{aligned}
&x_0(t) = 1, \quad y_0(t) = 0,\\
&x_1(t) = \frac{0.5t^{\gamma}}{\Gamma(\gamma+1)}, \quad y_1(t) = \frac{t^{\gamma}}{\Gamma(\gamma+1)},\\
&x_2(t) = \frac{0.25t^{2\gamma}}{\Gamma(2\gamma+1)}, \quad y_2(t) = \frac{2t^{2\gamma}}{\Gamma(2\gamma+1)} + \frac{0.25\Gamma(2\gamma+1)}{\Gamma^2(\gamma+1)\Gamma(3\gamma+1)}t^{3\gamma},\\
&x_3(t) = \frac{0.125t^{3\gamma}}{\Gamma(3\gamma+1)},\\
&y_3(t) = \frac{2.5}{\Gamma(3\gamma+1)}t^{3\gamma} + \frac{0.25\Gamma(2\gamma+1)}{\Gamma^2(\gamma+1)\Gamma(4\gamma+1)}\left[1 + \frac{\Gamma(\gamma+1)\Gamma(3\gamma+1)}{\Gamma^2(2\gamma+1)}\right]t^{4\gamma}\\
&\qquad + \frac{0.0625\Gamma(4\gamma+1)}{\Gamma^2(2\gamma+1)\Gamma(5\gamma+1)}t^{5\gamma}.
\end{aligned}$$

Thus, the approximate solution of system (13) is

$$\begin{aligned}
x(t) &= 1 + \frac{0.5t^{\gamma}}{\Gamma(\gamma+1)} + \frac{0.25t^{2\gamma}}{\Gamma(2\gamma+1)} + \frac{0.125t^{3\gamma}}{\Gamma(3\gamma+1)} + \cdots,\\
y(t) &= \frac{t^{\gamma}}{\Gamma(\gamma+1)} + \frac{2t^{2\gamma}}{\Gamma(2\gamma+1)} + \frac{0.25\Gamma(2\gamma+1)}{\Gamma^2(\gamma+1)\Gamma(3\gamma+1)}t^{3\gamma} + \frac{2.5}{\Gamma(3\gamma+1)}t^{3\gamma}\\
&\quad + \frac{0.25\Gamma(2\gamma+1)}{\Gamma^2(\gamma+1)\Gamma(4\gamma+1)}\left[1 + \frac{\Gamma(\gamma+1)\Gamma(3\gamma+1)}{\Gamma^2(2\gamma+1)}\right]t^{4\gamma}\\
&\quad + \frac{0.0625\Gamma(4\gamma+1)}{\Gamma^2(2\gamma+1)\Gamma(5\gamma+1)}t^{5\gamma} + \cdots.
\end{aligned}$$

In particular, if we put $\gamma = 1$, we have the exact solution in closed form

$$\begin{cases} x(t) = 1 + \frac{1}{2}t + \frac{(\frac{1}{2}t)^2}{2!} + \frac{(\frac{1}{2}t)^3}{3!} + \cdots \cong e^{\frac{1}{2}t}, \\ y(t) = t + t^2 + \frac{1}{12}t^3 + \frac{5}{128}t^4 + \frac{1}{320}t^5 + \cdots \cong te^{t}. \end{cases} \tag{4.4}$$

Table 2: A Comparison of absolute errors obtained for example 1 using LDJM and LRPS when $\gamma = 1$.

t	LDJM solution		LRPS solution	
	$x(t)$	$y(t)$	$x(t)$	$y(t)$
0	0.000000	0.000000	0.000000	0.000000
0.1	0.000000	0.000012	0.000000	0.000017
0.2	0.000004	0.000196	0.000004	0.000281
0.3	0.000022	0.001028	0.000022	0.001458
0.4	0.000069	0.003365	0.000069	0.004730
0.5	0.000171	0.008508	0.000171	0.011861

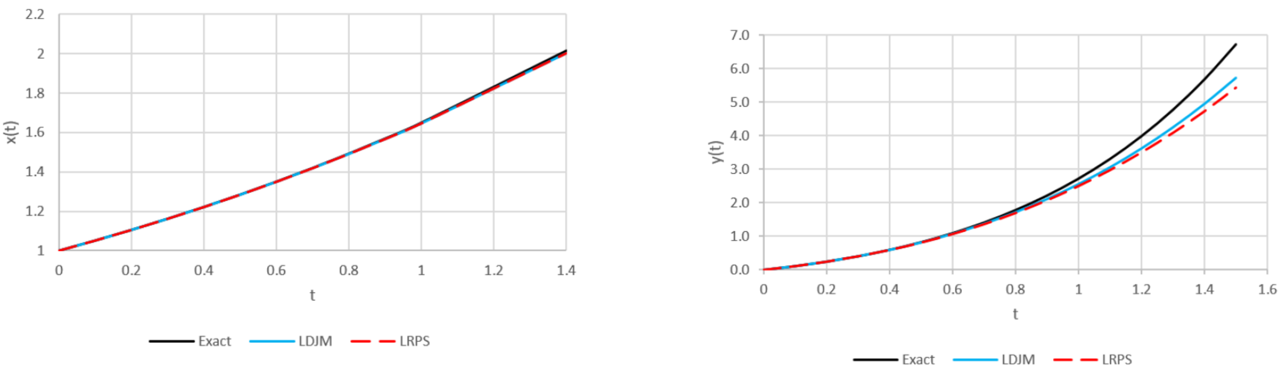
Table 1: A Comparison of numerical results obtained for example 1 using LDJM and LRPS when $\gamma = 1$.

t	Exact solution		LDJM solution		LRPS solution	
	$x(t)$	$y(t)$	$x(t)$	$y(t)$	$x(t)$	$y(t)$
0	1.000000	0.000000	1.000000	0.000000	1.000000	0.000000
0.1	1.051271	0.110517	1.051271	0.110505	1.051271	0.110500
0.2	1.105171	0.244281	1.105167	0.244084	1.105167	0.244000
0.3	1.161834	0.404958	1.161813	0.403929	1.161813	0.403500
0.4	1.221403	0.596730	1.221333	0.593365	1.221333	0.592000
0.5	1.284025	0.824361	1.283854	0.815853	1.283854	0.812500

Tab .1 presents the numerical results for the approximate solution of example 1 (4-term solution) at $\gamma = 1$ and compares them with the exact solution and LRPS method [1]. Tab .2 shows a comparison of absolute errors. Furthermore, Tab .3 displays the approximate solutions obtained by LDJM, for Example .4.1 at different values of γ. Fig. 1 illustrates the comparison between the LDJM and LRPS methods for $x(t)$ and $y(t)$ at $\gamma = 1$.

Table 3: Numerical results obtained for example 1 using LDJM for different value of γ.

t	$\gamma = 0.9$		$\gamma = 0.7$		$\gamma = 0.5$	
	$x(t)$	$y(t)$	$x(t)$	$y(t)$	$x(t)$	$y(t)$
0	1.000000	0.000000	1.000000	0.000000	1.000000	0.000000
0.1	1.066818	0.142279	1.111243	0.150740	1.179709	0.367889
0.2	1.127716	0.292428	1.184315	0.310995	1.257689	0.553165
0.3	1.188725	0.466284	1.250654	0.498776	1.321539	0.736956
0.4	1.251105	0.670329	1.314838	0.722049	1.379834	0.942428
0.5	1.315519	0.910049	1.378931	0.987577	1.436069	1.182673



(a) **(b)**

Figure 1: A comparison of $x(t)$ and $y(t)$ solutions between exact, LDJM, and LRPS methods for Example 4.1 when $\gamma = 1$.

Example 4.2. Secondly, consider the following autonomous 3-dimensional fractional nonlinear systems [1]:

$$\begin{cases} D_t^{\gamma} x(t) = x(t), \\ D_t^{\gamma} y(t) = 2x^2(t), \qquad 0 < \gamma < 1, \\ D_t^{\gamma} z(t) = 3x(t)y(t), \end{cases} \tag{4.5}$$

subject to initial conditions

$$x(0) = 1, \quad y(0) = 1, \quad z(0) = 0. \tag{4.6}$$

Operating Laplace transform on both sides of (17) and using (18), we get

$$\begin{cases} x(t) = 1 + \mathcal{L}^{-1}\left[\frac{1}{p^{\gamma}}\mathcal{L}[x(t)]\right], \\ y(t) = \mathcal{L}^{-1}\left[\frac{1}{p^{\gamma}}\mathcal{L}\left[\sum_{n=0}^{\infty} A_n(t)\right]\right], \\ z(t) = \mathcal{L}^{-1}\left[\frac{1}{p^{\gamma}}\mathcal{L}\left[\sum_{n=0}^{\infty} B_n(t)\right]\right], \end{cases} \tag{4.7}$$

where the nonlinear terms $2x^2(t)$ and $3x(t)y(t)$ in the system are expressed via Daftardar-Jafari polynomials $A_n(t)$ and $B_n(t)$, respectively.

In view of LDJM,

$$\begin{aligned}
&x_0(t) = 1, \quad y_0(t) = 1, \quad z_0(t) = 0,\\
&x_1(t) = \frac{t^{\gamma}}{\Gamma(\gamma+1)}, \quad y_1(t) = \frac{2t^{\gamma}}{\Gamma(\gamma+1)}, \quad z_1(t) = \frac{3t^{\gamma}}{\Gamma(\gamma+1)},\\
&x_2(t) = \frac{t^{2\gamma}}{\Gamma(2\gamma+1)}, \quad y_2(t) = \frac{4t^{2\gamma}}{\Gamma(2\gamma+1)} + \frac{2\Gamma(2\gamma+1)t^{3\gamma}}{\Gamma^2(\gamma+1)\Gamma(3\gamma+1)},\\
&z_2(t) = \frac{9t^{2\gamma}}{\Gamma(2\gamma+1)} + \frac{6\Gamma(2\gamma+1)t^{3\gamma}}{\Gamma^2(\gamma+1)\Gamma(3\gamma+1)},\\
&x_3(t) = \frac{t^{3\gamma}}{\Gamma(3\gamma+1)},\\
&y_3(t) = \frac{4t^{3\gamma}}{\Gamma(3\gamma+1)} + \frac{4\Gamma(3\gamma+1)t^{4\gamma}}{\Gamma(\gamma+1)\Gamma(2\gamma+1)\Gamma(4\gamma+1)} + \frac{2\Gamma(4\gamma+1)t^{5\gamma}}{\Gamma^2(\gamma+1)\Gamma(5\gamma+1)},\\
&z_3(t) = \frac{15t^{3\gamma}}{\Gamma(3\gamma+1)} + \frac{1}{\Gamma(\gamma+1)\Gamma(4\gamma+1)}\left(\frac{2\Gamma(3\gamma+1)}{\Gamma(\gamma+1)} + \frac{18\Gamma(3\gamma+1)}{\Gamma(2\gamma+1)}\right)t^{4\gamma}\\
&\qquad + \frac{6\Gamma(2\gamma+1)\Gamma(4\gamma+1)t^{5\gamma}}{\Gamma^3(\gamma+1)\Gamma(3\gamma+1)\Gamma(5\gamma+1)}.
\end{aligned}$$

Thus, the approximate solution of system (17) is

$$\begin{aligned}
x(t) &= 1 + \frac{t^{\gamma}}{\Gamma(\gamma+1)} + \frac{t^{2\gamma}}{\Gamma(2\gamma+1)} + \frac{t^{3\gamma}}{\Gamma(3\gamma+1)} + \cdots,\\
y(t) &= 1 + \frac{2t^{\gamma}}{\Gamma(\gamma+1)} + \frac{4t^{2\gamma}}{\Gamma(2\gamma+1)} + \left(4 + \frac{2\Gamma(2\gamma+1)}{\Gamma^2(\gamma+1)}\right)\frac{t^{3\gamma}}{\Gamma(3\gamma+1)}\\
&\quad + \frac{4\Gamma(3\gamma+1)t^{4\gamma}}{\Gamma(\gamma+1)\Gamma(2\gamma+1)\Gamma(4\gamma+1)} + \frac{2\Gamma(4\gamma+1)t^{5\gamma}}{\Gamma^2(2\gamma+1)\Gamma(5\gamma+1)} + \cdots,\\
z(t) &= \frac{3t^{\gamma}}{\Gamma(\gamma+1)} + \frac{9t^{2\gamma}}{\Gamma(2\gamma+1)} + \left(\frac{6\Gamma(2\gamma+1)}{\Gamma^2(\gamma+1)} + 15\right)\frac{t^{3\gamma}}{\Gamma(3\gamma+1)}\\
&\quad + \frac{1}{\Gamma(\gamma+1)\Gamma(4\gamma+1)}\left(\frac{2\Gamma(3\gamma+1)}{\Gamma(\gamma+1)} + \frac{18\Gamma(3\gamma+1)}{\Gamma(2\gamma+1)}\right)t^{4\gamma}\\
&\quad + \frac{6\Gamma(2\gamma+1)\Gamma(4\gamma+1)t^{5\gamma}}{\Gamma^3(\gamma+1)\Gamma(3\gamma+1)\Gamma(5\gamma+1)} + \cdots.
\end{aligned}$$

In particular, if we put $\gamma = 1$, we have the exact solution in closed form

$$\begin{cases} x(t) = 1 + t + \frac{t^2}{2!} + \frac{t^3}{3!} + \cdots \cong e^t, \\ y(t) = 1 + 2t + 2t^2 + \frac{4}{3}t^3 + \frac{1}{2}t^4 + \frac{2}{5}t^5 + \cdots \cong e^{2t}, \\ z(t) = 3t + \frac{9}{2}t^2 + \frac{23}{6}t^3 + \frac{11}{4}t^4 + \frac{2}{5}t^5 + \cdots \cong e^{3t} - 1. \end{cases} \tag{4.8}$$

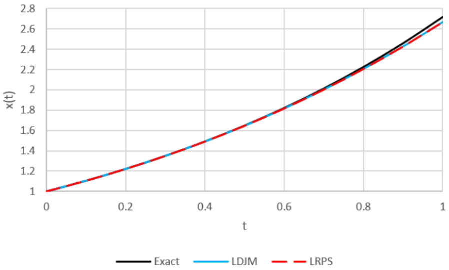
Tabl .4 presents the numerical results for the approximate solution of Example .4.2 (4-term solution) at $\gamma = 1$ and compares them with the exact solution and LRPS method [1]. Tab .5 shows a comparison of absolute errors. Furthermore, Tab .6 displays the approximate solutions obtained by LDJM, for Example 4.2 at different values of γ. Fig1b . illustrates the comparison between the LDJM and LRPS methods for $y(t)$ and $z(t)$ at $\gamma = 1$.

Table 4: A Comparison of numerical results obtained for Example 2 using LDJM and LRPS when $\gamma = 1$.

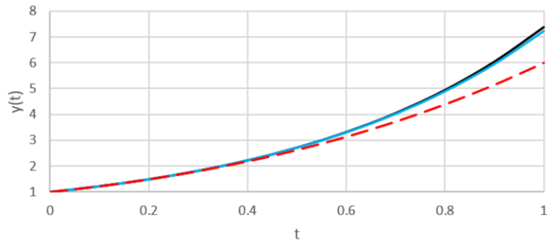
t	Exact solution			LDJM solution			LRPS solution		
	$x(t)$	$y(t)$	$z(t)$	$x(t)$	$y(t)$	$z(t)$	$x(t)$	$y(t)$	$z(t)$
0	1.000000	1.000000	0.000000	1.000000	1.000000	0.000000	1.000000	1.000000	0.000000
0.1	1.105171	1.221403	0.349859	1.105167	1.221387	0.349112	1.105167	1.221000	0.348833
0.2	1.221403	1.491825	0.822119	1.221333	1.491595	0.815195	1.221333	1.488000	0.810667
0.3	1.349859	1.822119	1.459603	1.349500	1.821022	1.431747	1.349500	1.807000	1.408500
0.4	1.491825	2.225541	2.320117	1.490667	2.222229	2.239829	1.490667	2.184000	2.165333
0.5	1.648721	2.718282	3.481689	1.645833	2.710417	3.288542	1.645833	2.625000	3.104167

Table 5: A Comparison of absolute errors obtained for Example 2 using LDJM and LRPS when $\gamma = 1$.

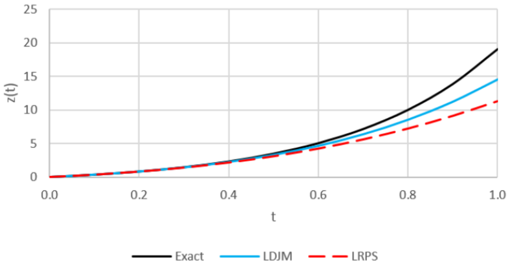
t	LDJM solution			LRPS solution		
	$x(t)$	$y(t)$	$z(t)$	$x(t)$	$y(t)$	$z(t)$
0	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.1	0.000004	0.000015	0.000080	0.000004	0.000403	0.001025
0.2	0.000069	0.000230	0.001591	0.000069	0.003825	0.011452
0.3	0.000359	0.001097	0.009856	0.000359	0.015119	0.051103
0.4	0.001158	0.003312	0.037621	0.001158	0.041541	0.154784
0.5	0.002888	0.007865	0.109814	0.002888	0.093282	0.377522



(a) $x(t)$ solutions



(b) $y(t)$ solutions



(c) $z(t)$ solutions

Figure 2: Comparison of $x(t)$, $y(t)$, and $z(t)$ solutions between exact, LDJM, and LRPS methods for Example .4.2

Table 6: Numerical results obtained for Example 2 using LDJM for different values of γ.

t	$\gamma = 0.9$			$\gamma = 0.7$			$\gamma = 0.5$		
	$x(t)$	$y(t)$	$z(t)$	$x(t)$	$y(t)$	$z(t)$	$x(t)$	$y(t)$	$z(t)$
0	1.000000	1.000000	0.000000	1.000000	1.000000	0.000000	1.000000	1.000000	0.000000
0.1	1.136481	1.284988	0.446971	1.225542	1.464073	0.717750	1.362201	1.736270	1.124769
0.2	1.267463	1.588855	0.976899	1.381811	1.823546	1.344304	1.527636	2.111597	1.774780
0.3	1.405977	1.948203	1.672894	1.533116	2.221817	2.136360	1.673194	2.496572	2.555300
0.4	1.555515	2.383411	2.599905	1.690047	2.699275	3.206975	1.817722	2.951133	3.620373
0.5	1.718374	2.915930	3.830516	1.858165	3.291506	4.673168	1.969900	3.521252	5.118808

Comment: Based on the above tables, it is evident that the proposed method exhibits a clear superiority in terms of result accuracy and convergence speed, particularly in nonlinear equations characterized by computational complexities, where the proposed method demonstrated stability and consistency in results.

5. Conclusion

This study has developed a novel analytical solution for nonlinear fractional autonomous dynamical systems using the Caputo fractional derivative. The proposed method, based on a combination of the Laplace transform and the Daftardar-Jafari method, has demonstrated high accuracy and efficiency in solving these systems. The results of this study confirm the effectiveness of the proposed method in solving similar problems and its potential applications in various scientific and engineering fields. Future research directions will focus on extending the application of this technique to more complex systems and exploring its potential in solving real-world problems.

References

[1] Aboodh, K. S., & Ahmed, A. (2021). On the Application of Homotopy Analysis Method to Fractional Differential Equations. Journal of The Faculty of Science and Technology, 7, 1–18. 1, 4.1, 4, 4.2, 4

[2] Abdoon, M. A., Elbadri, M., Alzahrani, A. B. M., Berir, M., & Ahmed, A. (2024). Analyzing the inverted fractional Rössler system through two approaches: numerical scheme and LHAM. Physica Scripta, 99(11), 115220. 1

[3] Adomian, G. (1994). Solving Frontier Problems of Physics: The Decomposition Method. Springer Netherlands. 1

[4] Ahmed, A. (2024). On the Approximate Solutions for Fractional Differential Equations with Caputo-Fabrizio Fractional Derivative. 1

[5] Aggarwal, S., & Bhatnagar, K. (2019). Dualities between Laplace Transform and Some Useful Integral Transforms. International Journal of Engineering and Advanced Technology, 9(1), 936–941. 1

[6] Alabedalhadi, M., Al-Smadi, M., Al-Omari, S., Baleanu, D., & Momani, S. (2020). Structure of optical soliton solution for nonlinear resonant space-time Schrödinger equation in conformable sense with full nonlinearity term. Physica Scripta, 95(10), 105215. 1

[7] Alaroud, M. (2022). Application of Laplace residual power series method for approximate solutions of fractional IVP's. Alexandria Engineering Journal, 61(2), 1585–1595. 1, 2.1, 2.2, 2.3

[8] Ali, A., Shah, K., Alrabaiah, H., Shah, Z., Rahman, G. U., & Islam, S. (2021). Computational modeling and theoretical analysis of nonlinear fractional order prey–predator system. Fractals, 29(01), 2150001. 1

[9] Alquran, M., Alsukhour, M., Ali, M., & Jaradat, I. (2021). Combination of Laplace transform and residual power series techniques to solve autonomous n-dimensional fractional nonlinear systems. Nonlinear Engineering, 10(1), 282–292. 2.3

[10] Al-Smadi, M., & Arqub, O. A. (2019). Computational algorithm for solving Fredholm time-fractional partial integrodifferential equations of Dirichlet functions type with error estimates. Applied Mathematics and Computation, 342, 280–294. 1

[11] Al-Smadi, M. H., & Gumah, G. N. (2014). On the Homotopy Analysis Method for Fractional SEIR Epidemic Model. Research Journal of Applied Sciences, Engineering and Technology, 7(18), 3809–3820. 1

[12] Balcı, E., Öztürk, İ., & Kartal, S. (2019). Dynamical behaviour of fractional order tumor model with Caputo and conformable fractional derivative. Chaos, Solitons & Fractals, 123, 43–51. 1

[13] Baleanu, D., & Jassim, H. K. (2019). A Modification Fractional Homotopy Perturbation Method for Solving Helmholtz and Coupled Helmholtz Equations on Cantor Sets. Fractal and Fractional, 3(2), 30. 1

[14] Cherruault, Y. (1989). Convergence of Adomian's Method. Kybernetes, 18(2), 31–38. 1, 3.1

[15] Cui, P., & Jassim, H. K. (2024). Local fractional Sumudu decomposition method to solve fractal PDEs arising in mathematical physics. Fractals, 32(04). 1

[16] Daftardar-Gejji, V., & Jafari, H. (2006). An iterative method for solving nonlinear functional equations. Journal of Mathematical Analysis and Applications, 316(2), 753–763. 1

[17] Da Graça Marcos, M., Duarte, F. B. M., & Tenreiro Machado, J. A. (2008). Fractional dynamics in the trajectory control of redundant manipulators. Communications in Nonlinear Science and Numerical Simulation, 13(9), 1836–1844. 1

[18] Dokuyucu, M. A., & Dutta, H. (2020). A fractional order model for Ebola Virus with the new Caputo fractional derivative without singular kernel. Chaos, Solitons & Fractals, 134, 109717. 1

[19] Elbadri, M. (2018). Comparison between the Homotopy Perturbation Method and Homotopy Perturbation Transform Method. Applied Mathematics, 09(02), 130–137. 1

[20] Elbadri, M. (2023). An approximate solution of a time fractional Burgers' equation involving the Caputo-Katugampola fractional derivative. Partial Differential Equations in Applied Mathematics, 8, 100560. 1, 2.1, 2.2

[21] Elbadri, M. (2022). Initial Value Problems with Generalized Fractional Derivatives and Their Solutions via Generalized Laplace Decomposition Method. Advances in Mathematical Physics, 2022, 1–7. 1

[22] Higazy, M., Aggarwal, S., & Nofal, T. A. (2020). Sawi Decomposition Method for Volterra Integral Equation with Application. Journal of Mathematics, 2020, 1–13. 1

[23] Jafari, H., & Daftardar-Gejji, V. (2006). Solving a system of nonlinear fractional differential equations using Adomian decomposition. Journal of Computational and Applied Mathematics, 196(2), 644–651. 1

[24] Jafari, H., & Jassim, H. (2014). Local Fractional Laplace Variational Iteration Method for Solving Nonlinear Partial Differential Equations on Cantor Sets within Local Fractional Operators. Journal of Zankoy Sulaimani - Part A, 16(4), 49–57. 1, 3.2

[25] Jassim, H. K., & Hussein, M. A. (2023). A New Approach for Solving Nonlinear Fractional Ordinary Differential Equations. Mathematics, 11(7), 1565. 1

[26] Kamran, Ahmad, S., Shah, K., Abdeljawad, T., & Abdalla, B. (2023). On the Approximation of Fractal-Fractional Differential Equations Using Numerical Inverse Laplace Transform Methods. Computer Modeling in Engineering & Sciences, 135(3), 2743–2765. 1

[27] Khafif, S. A. H., Jassim, H. K., & Mohammed, M. G. (2021). SVIM for Solving Burger's and Coupled Burger's Equations of Fractional Order. Progress in Fractional Differentiation and Applications, 7(1), 73–78. 1

[28] Kilbas, A. A., Srivastava, H. M., & Trujillo, J. J. (2006). Preface. Theory and Applications of Fractional Differential Equations, vii–x. 1

[29] Kiryakova, V., & Luchko, Y. (2019). Multiple Erdélyi–Kober integrals and derivatives as operators of generalized fractional calculus. Basic Theory, 127–158. 1

[30] Noor, K. I., & Noor, M. A. (2007). Iterative methods with fourth-order convergence for nonlinear equations. Applied Mathematics and Computation, 189(1), 221–227. 1

[31] Öztürk, I., & Özköse, F. (2020). Stability analysis of fractional order mathematical model of tumor-immune system interaction. Chaos, Solitons & Fractals, 133, 109614. 1

[32] Qureshi, S. (2020). Real life application of Caputo fractional derivative for measles epidemiological autonomous dynamical system. Chaos, Solitons & Fractals, 134, 109744. 1

[33] Sachit, S. A., & Jassim, H. K. (2023). Solving fractional PDEs by Elzaki homotopy analysis method. International Conference of Computational Methods in Sciences and Engineering (ICCMSE 2021), 2611, 040074. 1

[34] Sari, M., Gunay, A., & Gurarslan, G. (2011). Approximate solutions of linear and non-linear diffusion equations by using Daftardar-Gejji-Jafari's method. International Journal of Mathematical Modelling and Numerical Optimisation, 2(4), 376. 1

[35] Shah, K., Abdeljawad, T., Jarad, F., & Al-Mdallal, Q. (2023). On Nonlinear Conformable Fractional Order Dynamical System via Differential Transform Method. Computer Modeling in Engineering & Sciences, 136(2), 1457–1472. 1

[36] Tripathi, V. M., Srivastava, H. M., Singh, H., Swarup, C., & Aggarwal, S. (2021). Mathematical Analysis of Non-Isothermal Reaction–Diffusion Models Arising in Spherical Catalyst and Spherical Biocatalyst. Applied Sciences, 11(21), 10423. 1