Math. & Comput. Sci., 6(4) 2025, 64–73
Online: ISSN 2717-2708

Mathematics and Computational Sciences

Journal Homepage: http://mcs.qut.ac.ir/mcs

A new notion of soft Nano connectedness in soft Nano topological space

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Abstract

A more flexible and nuanced view of space and objects that are inherently ambiguous or inaccurate is made possible by soft nano topology. Connectivity concepts play an major role in topological spaces. The classification and comprehension of the various topological space structures is aided by connectivity. The main aspect of this article is to introduce new types of soft nano connected space known as soft nano *ic*-pre generalized connected space and discuss its properties. Further, we introduce new type of hyper connected space known as soft nano *ic*-pre generalized hyper connected space and look over its characteristics. This idea makes it instinctive to comprehend topological structures and some difficult theorems, and it aids in the development of concepts like compactness and separation axioms in soft nano topology.

Keywords: Soft nano *ic*- pre generalized continuous, Soft nano *ic*-pre generalized irresolute, Soft nano *ic*-pre generalized connected, Soft nano *ic*-pre generalized hyper connected

2020 MSC: 54D05

1. Introduction

The study of space, dimensions and transformation is known as topology in mathematics. In various field, researchers lack both assurance and ambiguity about their data. To address these issues Russian mathematician Molodtsov [3], introduced soft set theory.Nano topological space was inroduced by Lellis thivagar [8], with respect to a subset X of the universe which is described in terms of X's lower and upper approximation. Benchalli S.S et al [2], developed the concept of soft nano topology. He combines the concepts ofsoft sets, which deal with ambiguity, with nano topology, which deals with nano scale structures, to provide a strong foundation for researching nano scale phenomena and their applications. In order to comprehend the structures of soft nano topological space and examine its qualities, he also proposed a various soft nano open sets. In topology, connectedness is one of the prominent ideas. Classifying and comprehending the many topological space structures as well as how they relate to other mathematical concepts applications, is made easier by connectedness. Also Connectivity allows us to gain insight into the qualitative nature of space by focusing on properties that remain constant throughout continuous

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doi: 10.30511/mcs.2025.2075298.1551

Received: 21 October 2025 Accepted: 28 November 2025

deformation rather than exact shapes or sizes. P. G Patil and S.S Benakanawari [4] introduced soft nano compactness and soft nano connectedness in soft nano topological spaces. Recently, the authors [6, 7] of this paper introduced soft nano *ic*-pre generalized open sets and analyzed its characteristics. We have also introduced soft nano *ic* –pre generalized continuous function and its contra version. The main aspect of this article is to introduce new types of soft nano connected space known as soft nano *ic*-pre generalized connected space and discuss its properties. Further, we introduce new type of hyper connected space known as soft nano *ic*-pre generalized hyper connected space and look over its characteristics.

2. Preliminaries

Definition 2.1. [3] Let U be an initial universe and M be a set of parameters. Let $P(U)$ denote the power set of U and A be a non empty subset M. A pair (G, A) is called a soft set over U, where G is a mapping given by $G : A \to P(U)$. In other words, a soft set over U is a parameterized family of subsets of of the universe U. For $m \in A$, $G(m)$ may be considered as the set of $m-$approximate elements in soft set (G, A). Clearly, a soft set need not be a set.

Definition 2.2. [2] Let U be a non- empty finite set of objects called the universe and M be a set of parameters. Let S be a soft equivalence relation on U. The triplet (U, S, M) is said to be the soft approximation space. Let $X \subseteq U$.

(i) The soft lower approximation of X with respect to S and the set of parameters M is the set of all objects, which can be for certain classified as X with respect to S and it is denoted by $(L_{SN}(X), M)$. that is $(L_{SN}(X), M) = \cup\{S(x) : S(x) \subseteq X\}$, where $S(x)$ denotes the equivalence class determined bt $x \in U$.

(ii) The soft upper approximation of X with respect to S and the set of parameters M is the set of all objects, which can be possibly classified as X with respect to S and it denoted by $(U_{SN}(X), M)$. that is $(U_{SN}(X), M) = \cup\{S(x) : S(x) \cap X \neq \varnothing\}$.

(iii) The soft boundary region of X with respect to S and the set of parameters M is the set of all objects, which can be classified neither inside X nor as outside X with respect to S and is denoted by $(B_{SN}(X), M) = (U_{SN}(X), M) - (L_{SN}(X), M)$.

Definition 2.3. [2] Let U be a non-empty universal set and M be a set of parameters. Let S be a soft equivalence relation on U. Let $X \subseteq U$ and $(\tau_{SN}, U, M) = \{U, \varnothing, (L_{SN}(X), M), (U_{SN}(X), M), (B_{SN}(X), M)\}$. Then $(\tau_{SN}(X), U, M)$ is a soft topology on (U, M), called as the soft nano topology with respect to X. Elements of the soft nano topology are known as the soft nano open and $(\tau_{SN}(X), U, M)$ is called soft nano topological space. The complements of soft nano open sets are called as soft nano closed sets in $(\tau_{SN}(X), U, M)$.

Definition 2.4. [4] A soft nano topological spaces $(\tau_{SN}(X), U, M)$ is said to be soft nano connected if $(\tau_{SN}(X), U, M)$ cannot be expressed as union of two disjoint nonempty soft nano open sets in $(\tau_{SN}(X), U, M)$.

Definition 2.5. [5] A soft subset (V, M_1) of soft nano topological space $(\tau_{SN}(X_1), U_1, M_1)$ is said to
(i) SN dense if $Nic - PGcl(V, M_1) = (U_1, M_1)$.
(ii) SN nowhere dense if $Nic - PGcl(Nic - PGint(V, M_1)) = (\varnothing, M_1)$.

Definition 2.6. [1] A topological space (X, τ) is said to be hyper connected if V is dense for every non empty open set V in(X, τ)

Definition 2.7. [6] A soft subset (V, M) of a soft nano topological space $(\tau_{SN}(X), U, M)$ is said to be soft nano ic-pre generalized open set if there is a soft nano closed set $(H, M) \neq (U, M), \varnothing$ such that $[(V, M) \cap (H, M)] \subseteq Ngcl(Nint(Ncl(V, M)))$ and it is denoted by $SN\ ic - PGO(U, M)$.

Theorem 2.8. *[6] Arbitrary union of* $\mathrm{SN\ ic-PG}$ *open sets is also* $\mathrm{SN\ ic-PG}$ *open.*

Theorem 2.9. *[6] In any soft nano topological space* $(\tau_{SN}(X), U, M)$,

(i) *Every* SN *open is* $\mathrm{SN\ ic-PG}$ *open.*
(ii) *Every* SN *pre open is* $\mathrm{SN\ ic-PG}$ *open.*
(iii) *Every* SN *regular open is* $\mathrm{SN\ ic-PG}$ *open.*
(iv) *Every* SN *α-open is* $\mathrm{SN\ ic-PG}$ *open.*
(v) *Every* $\mathrm{SN\ ic-PG}$ *open is* SN β *–open.*

Remark 2.10. [6] The above theorem is also true for $\mathrm{SN\ ic-PG}$ closed.

Theorem 2.11. *[6] Let* (V, M) *be any soft subset of* $(\tau_{SN}(X), U, M)$. *Then,* (V, M) *is the soft nano* ic- *pre generalized closed if and only if* $\mathrm{Nic-PGcl}(V, M) = (V, M)$.

Definition 2.12. [7] Let $(\tau_{SN}(X_1), U_1, M_1)$ and $(\tau_{SN'}(X_2), U_2, M_2)$ be soft nano topological spaces. Then the function $f_{SN} : (\tau_{SN}(X_1), U_1, M_1) \to (\tau_{SN'}(X_2), U_2, M_2)$ is said to be soft nano ic- pre generalized continuous if $f^{-1}_{SN}(V, M)$ is soft nano *ic*- pre generalized open in $(\tau_{SN}(X_1), U_1, M_1)$ for every soft nano open set (V, M) in $(\tau_{SN'}(X_2), U_2, M_2)$.

Definition 2.13. [7] Let $(\tau_{SN}(X_1), U_1, M_1)$ and $(\tau_{SN'}(X_2), U_2, M_2)$ be soft nano topological spaces. Then the function $f_{SN} : (\tau_{SN}(X_1), U_1, M_1) \to (\tau_{SN'}(X_2), U_2, M_2)$ is said to be soft nano ic- pre generalized irresolute if $f^{-1}_{SN}(V, M)$ is soft nano *ic*- pre generalized open in $(\tau_{SN}(X_1), U_1, M_1)$ for every soft nano *ic*- pre generalized open set (V, M) in $(\tau_{SN'}(X_2), U_2, M_2)$.

3. Soft Nano *ic* - Pre Generalized Connected Spaces

Definition 3.1. A soft nano topological spaces $(\tau_{SN}(X), U, M)$ is said to be ***soft nano ic - pre generalized connected*** if $(\tau_{SN}(X), U, M)$ cannot be expressed as union of two disjoint nonempty soft nano *ic* – pre generalized open sets in $(\tau_{SN}(X), U, M)$.

Example 3.2. Let $U = \{\vartheta, , \upsilon\}$, $M = \{\zeta_1, \zeta_2, \zeta_3\}$, $U/S = S(X) = \{\{\vartheta\}, \{, \upsilon\}\}$, $X = \{\vartheta\} \subseteq U$, $(L_{SN}(X), M) = \{(\zeta_1, \{\vartheta\}), (\zeta_2, \{\vartheta\}), (\zeta_3, \{\vartheta\})\} = (U_{SN}(X), M), (B_{SN}(X), M) = \{(\varnothing, M)\}$.

Now, $(\tau_{SN}(X), U, M) = \{U, \varnothing, (L_{SN}(X), M)\}$ is the soft nano topology on U. Here SN open sets are U, $\varnothing$, $(L_{SN}(X), M)$ and SN closed sets are U, $\varnothing$, $(L_{SN}(X), M)'$ where $(L_{SN}(X), M)' = \{(\zeta_1, \{, \upsilon\}), (\zeta_2, \{, \upsilon\}), (\zeta_3, \{, \upsilon\})\}$. Here $\mathrm{SN\ ic-PGO}(U, M) = \{U, \varnothing, (A_1, M), (A_2, M), (A_3, M)\}$ where $(A_1, M) = \{(\zeta_1, \{\vartheta\}), (\zeta_2, \{\vartheta\}), (\zeta_3, \{\vartheta\})\}$, $(A_2, M) = \{(\zeta_1, \{\vartheta, \upsilon\}), (\zeta_2, \{\vartheta, \upsilon\}), (\zeta_3, \{\vartheta, \upsilon\})\}$, $(A_3, M) = \{(\zeta_1, \{\vartheta, \}), (\zeta_2, \{\vartheta, \}), (\zeta_3, \{\vartheta, \})\}$. Hence $(\tau_{SN}(X), U, M)$ is $\mathrm{SN\ ic-PG}$ connected.

Definition 3.3. A subset (V, M) of soft nano topological space $(\tau_{SN}(X), U, M)$ is said to be soft nano *ic* – pre generalized connected set in $(\tau_{SN}(X), U, M)$ if (V, M) cannot be expressed as the union of two disjoint non empty soft nano *ic* – pre generalized open sets in $(\tau_{SN}(X), U, M)$.

Theorem 3.4. *Every* $\mathrm{SN\ ic-PG}$ *connected space is* SN *connected.*

Proof. Let $(\tau_{SN}(X), U, M)$ be $\mathrm{SN\ ic-PG}$ connected. Suppose $(\tau_{SN}(X), U, M)$ is not SN connected, then there exists disjoint non empty SN open sets (V, M) and (G, M) such that $(\tau_{SN}(X), U, M) = (V, M) \cup (G, M)$. By theorem 2.9,$(V, M)$ and (G, M) are $\mathrm{SN\ ic-PG}$ open sets. This is contradiction to $(\tau_{SN}(X), U, M)$ is $\mathrm{SN\ ic-PG}$ connected. Hence $(\tau_{SN}(X), U, M)$ is SN connected. □

Remark 3.5. The above theorem does not imply their reverse part by the following example.

Example 3.6. Let $U = \{\vartheta,,\upsilon,\mu\}$, $M = \{\zeta_1,\zeta_2,\zeta_3\}$,$U/S = S(X) = \{\{\vartheta\},\{\},\{\upsilon,\ \mu\}\}$, $X = \{\vartheta,\mu\} \subseteq U$, $(L_{SN}(X),M) = \{(\zeta_1,\{\vartheta\}),(\zeta_2,\{\vartheta\}),(\zeta_3,\{\vartheta\})\}$, $(U_{SN}(X),M) = \{(\zeta_1,\{\vartheta,\upsilon,\mu\}),(\zeta_2,\{\vartheta,\upsilon,\mu\}),(\zeta_3,\{\vartheta,\upsilon,\mu\})\}$,
$(B_{SN}(X),M) = \{(\zeta_1,\{\upsilon,\mu\}),(\zeta_2,\{\upsilon,\mu\}),(\zeta_3,\{\upsilon,\mu\})\}$.

Now $(\tau_{SN}(X),U,M) = \{U,\varnothing,(L_{SN}(X),M),(U_{SN}(X),M),(B_{SN}(X),M)\}$ is the soft nano topology on U. Here SN open sets are U, $\varnothing$, $(L_{SN}(X),M)$, $(U_{SN}(X),M)$, $(B_{SN}(X),M)$ and SN closed sets are U, $\varnothing$, $(L_{SN}(X),M)'$, $(U_{SN}(X),M)'$, $(B_{SN}(X),M)'$

where $(L_{SN}(X),M)' = \{(\zeta_1,\{,\upsilon,\mu\}),(\zeta_2,\{,\upsilon,\mu\}),(\zeta_3,\{,\upsilon,\mu\})\}$,
$(U_{SN}(X),M)' = \{(\zeta_1,\{\}),(\zeta_2,\{\}),(\zeta_3,\{\})\}$, $(B_{SN}(X),M)' = \{(\zeta_1,\{\vartheta,\}),(\zeta_1,\{\vartheta,\}),(\zeta_3,\{\vartheta,\})\}$.

Here $SN\ ic-PGO(U,M) = \{U,\varnothing,(A_1,M),(A_2,M)\ldots,(A_{13},M)\}$

where $(A_1,M) = \{(\zeta_1,\{\vartheta\}),(\zeta_2,\{\vartheta\}),(\zeta_3,\{\vartheta\})\}$, $(A_2,M) = \{(\zeta_1,\{\}),(\zeta_2,\{\}),(\zeta_3,\{\})\}$,
$(A_3,M) = \{(\zeta_1,\{\mu\}),(\zeta_2,\{\mu\}),(\zeta_3,\{\mu\})\}$ $(A_4,M) = \{(\zeta_1,\{,\upsilon\}),(\zeta_2,\{,\upsilon\}),(\zeta_3,\{,\upsilon\})\}$,
$(A_5,M) = \{(\zeta_1,\{\vartheta,\}),(\zeta_2,\{\vartheta,\}),(\zeta_3,\{\vartheta,\})\}$ $(A_6,M) = \{(\zeta_1,\{\upsilon,\mu\}),(\zeta_2,\{\upsilon,\mu\}),(\zeta_3,\{\upsilon,\mu\})\}$, $(A_7,M) = \{(\zeta_1,\{\vartheta,\upsilon\}),(\zeta_2,\{\vartheta,\upsilon\}),(\zeta_3,\{\vartheta,\upsilon\})\}$, $(A_8,M) = \{(\zeta_1,\{,\mu\}),(\zeta_2,\{,\mu\}),(\zeta_3,\{,\mu\})\}$
$(A_9,M) = \{(\zeta_1,\{\vartheta,\mu\}),(\zeta_2,\{\vartheta,\mu\}),(\zeta_3,\{\vartheta,\mu\})\}$,
$(A_{10},M) = \{(\zeta_1,\{\vartheta,,\upsilon\}),(\zeta_2,\{\vartheta,,\upsilon\}),(\zeta_3,\{\vartheta,,\upsilon\})\}$,
$(A_{11},M) = \{(\zeta_1,\{,\upsilon,\mu\}),(\zeta_2,\{,\upsilon,\mu\}),(\zeta_3,\{,\upsilon,\mu\})\}$, $(A_{12},M) = \{(\zeta_1,\{\vartheta,,\mu\}),(\zeta_2,\{\vartheta,,\mu\},(\zeta_3,\{\vartheta,,\mu\}))\}$, $(A_{13},M) = \{(\zeta_1,\{\vartheta,\upsilon,\mu\}),(\zeta_2,\{\vartheta,\upsilon,\mu\}),(\zeta_3,\{\vartheta,\upsilon,\mu\})\}$ is SN connected, But not SN ic PG connected.

Definition 3.7. The soft sub sets (V,M) and (G,M) in a soft nano topological space $(\tau_{SN}(X),U,M)$ are said to be ***soft nano ic – pre generalized separated*** if $(V,M)\cap Nic-PGcl(G,M) = Nic-PGcl(V,M)\cap(G,M) = \varnothing$.

Theorem 3.8. *For the soft nano topological spaces* $(\tau_{SN}(X_1),U_1,M_1)$ *and* $(\tau_{SN'}(X_2),U_2,M_2)$ *the following statements are equivalent*

i) $(\tau_{SN}(X_1),U_1,M_1)$ *is* $SN\ ic-PG$ *connected.*
ii) $(\tau_{SN}(X_1),U_1,M_1)$ *cannot be expressed as the union of two disjoint nonempty* $SN\ ic-PG$ *closed set in* $(\tau_{SN}(X_1),U_1,M_1)$.
iii) *The only soft subsets which are both* $SN\ ic-PG$ *open and* $SN\ ic-PG$*closed are* U_1 *and* $\varnothing$
iv) *Every* $SN\ ic-PG$*continuous function of* $(\tau_{SN}(X_1),U_1,M_1)$ *into a* SN *discrete space* $(\tau_{SN'}(X_2),U_2,M_2)$ *is constant.*
v) $(\tau_{SN}(X_1),U_1,M_1)$ *cannot be expressed as the union of two non empty* $SN\ ic-PG$ *separated sets.*

Proof. (i) $\Longrightarrow$ (ii) Let $(\tau_{SN}(X_1),U_1,M_1)$ be $SN\ ic-PG$ connected space. Suppose $(\tau_{SN}(X_1),U_1,M_1) = (V,M_1)\cup(G,M_1)$ where (V,M_1) and (G,M_1) are disjoint non empty $SN\ ic-PG$ closed sets. Then $(V,M_1) = (G,M_1)'$ and $(G,M_1) = (V,M_1)'$ are disjoint non empty open sets in $(\tau_{SN}(X_1),U_1,M_1)$. This is a contradiction to $(\tau_{SN}(X_1),U_1,M_1)$ is $SN\ ic-PG$ connected. Hence (ii) holds.

(ii) $\Longrightarrow$ (i) Assume that $(\tau_{SN}(X_1),U_1,M_1)$ cannot be expressed as the union of disjoint non empty $SN\ ic-PG$ closed sets. Suppose $(\tau_{SN}(X_1),U_1,M_1) = (V,M_1)\cup(G,M_1)$ where (V,M_1) and (G,M_1) are disjoint non empty $SN\ ic-PG$ open sets. Then $(V,M_1) = (G,M_1)'$ and $(G,M_1) = (V,M_1)'$ are disjoint non empty $SN\ ic-PG$ closed sets in $(\tau_{SN}(X_1),U_1,M_1)$. This is a contradiction to (ii). Hence $(\tau_{SN}(X_1),U_1,M_1)$ is $SN\ ic-PG$ connected.

(i) $\Longrightarrow$ (iii)Suppose $(\tau_{SN}(X_1),U_1,M_1)$ be $SN\ ic-PG$ connected space. Let (V,M_1) be non empty proper soft subset of $(\tau_{SN}(X_1),U_1,M_1)$ that is both $SN\ ic-PG$ open and $SN\ ic-PG$ closed. Then $(V,M_1)'$ is non empty $SN\ ic-PG$ open and $(\tau_{SN}(X_1),U_1,M_1) = (V,M_1)\cup(V,M_1)'$. This is a contradiction to $(\tau_{SN}(X_1),U_1,M_1)$ is $SN\ ic-PG$ connected.

(iii) $\Longrightarrow$ (i) suppose $(\tau_{SN}(X_1),U_1,M_1) = (V,M_1)\cup(G,M_1)$ where (V,M_1) and (G,M_1) are disjoint non empty $SN\ ic-PG$ open sets. Then $(V,M_1) = (G,M_1)'$ is $SN\ ic-PG$ closed. Thus (V,M_1) is non empty proper subset that is both $SN\ ic-PG$ open and $SN\ ic-PG$ closed. This is a contradiction to (iii) . Hence $(\tau_{SN}(X_1),U_1,M_1)$ is $SN\ ic-PG$ connected.

(iii) $\Longrightarrow$ (iv) let f_{SN} be $SN\ ic-PG$ continuous function from $(\tau_{SN}(X_1),U_1,M_1)$ into a soft nano discrete space $(\tau_{SN}(X_2),U_2,M_2)$. Then for each $(G,M_2)\in(\tau_{SN}(X_2),U_2,M_2)$, $f^{-1}_{SN}(G,M_2)$ is both

SN $ic - PG$ open and SN $ic - PG$ closed. Since by (iii), $f_{SN}^{-1}(G, M_2) = \varnothing$ or U_1. If $f_{SN}^{-1}(G, M_2) = \varnothing$ for all $(G, M_2) \in (\tau_{SN}(X_2), U_2, M_2)$, then f_{SN} fails to be function. Hence $f_{SN}^{-1}(V, M_2) = U_1$ for a unique $(V, M_2) \in (\tau_{SN}(X_2), U_2, M_2)$.This implies $f_{SN}\{(U_1, M_1)\} = (V, M_2)$ and hence f_{SN} is a constant function.

$(iv) \implies (iii)$ Let (A, M_1)be both SN $ic - PG$ open and SN $ic - PG$ closed set in $(\tau_{SN}(X_1), U_1, M_1)$. Suppose $(A, M_1) \neq \varnothing$, we claim that $(A, M_1) = (\tau_{SN}(X_1), U_1, M_1)$. Otherwise, choose two fixed points a and b in U_2. Define $f_{SN} : (\tau_{SN}(X_1), U_1, M_1) \to (\tau_{SN'}(X_2), U_2, M_2)$ by $f_{SN}(x) = \begin{cases} a & x \in (A, M_1) \\ b & otherwise \end{cases}$. Then for any SN open set (V, M_2) in $(\tau_{SN'}(X_2), U_2, M_2)$,$f_{SN}^{-1}(V, M_2)$ equals (A, M_1) if (V, M_1) contains a but not b, equals $(A, M_1)'$ if (V, M_1) contains b but not a, equals$(\tau_{SN}(X_1), U_1, M_1)$ if (V, M_1) contains a and b and equals $\varnothing$ otherwise. In all cases $f_{SN}^{-1}(V, M_2)$ is SN $ic - PG$ open. Hence f_{SN} is non constant SN $ic - PG$ continuous function of $(\tau_{SN}(X_1), U_1, M_1)$ to $(\tau_{SN'}(X_2), U_2, M_2)$. This is a contradiction to our assumption. This proves that the only soft subsets which are both SN $ic - PG$ open and SN $ic -$ PGclosed are U_1 and $\varnothing$.

$(i) \implies (v)$ Suppose $(\tau_{SN}(X_1), U_1, M_1) = (V, M_1) \cup (G, M_1)$ where (V, M_1) and (G, M_1) are disjoint non empty SN $ic - PG$ separated sets in $(\tau_{SN}(X_1), U_1, M_1)$. Since $(V, M_1) \cap Nic - PGcl(G, M_1) = \varnothing$, $Nic - PGcl(G, M_1) \subseteq (V, M_1)' = (G, M_1)$ and hence $Nic - PGcl(G, M_1) = (G, M_1)$ and by Theorem 2.11, (G, M_1) is SN ic PG closed. Hence (V, M_1) is SN ic PG open. Similarly (G, M_1) is SN ic PG open. Hence $(\tau_{SN}(X_1), U_1, M_1)$ is not SN $ic - PG$ connected. This is a contradiction to (i).

$(v) \implies (i)$ Suppsose $(\tau_{SN}(X_1), U_1, M_1)$ is not SN $ic - PG$ connected. Then $(\tau_{SN}(X_1), U_1, M_1)$ can be written as $(\tau_{SN}(X_1), U_1, M_1) = (V, M_1) \cup (G, M_1)$ where (V, M_1) and (G, M_1) are disjoint non empty SN $ic - PG$ open sets. Now $(V, M_1) = (G, M_1)'$ is SN $ic - PG$ closed and hence by theorem 2.11, $Nic - PGcl(V, M_1) = (V, M_1)$ and so $Nic - PGcl(V, M_1) \cap (G, M_1) = \varnothing$. Similarly, $(V, M_1) \cap Nic - PGcl(G, M_1) = \varnothing$.Thus (V, M_1) and (G, M_1) are SN $ic - PG$ separated. This is a contradiction to (v). □

Theorem 3.9. SN ic PG *separated sets are always disjoint.*

Proof. Let (A, M) and (G, M) be two SN $ic - PG$ separated sets. Then $(A, M) \cap Nic - PGcl(G, M) = \varnothing = Nic - PGcl(A, M) \cap (G, M)$.Now $(A, M) \cap (G, M) \subseteq (A, M) \cap Nic - PGcl(G, M) = \varnothing$. Then$(A, M) \cap (G, M) = \varnothing$, Hence (A, M) and (G, M) are disjoint. □

Theorem 3.10. *If* $(V, M) \subseteq (A, M) \cup (G, M)$ *where* (V, M) *is* SN $ic - PG$ *connected set and* $(A, M), (G, M)$ *are* SN $ic - PG$ *separated set, then either* $(V, M) \subseteq (A, M)$ *or*$(V, M) \subseteq (G, M)$.

Proof. Suppose $(V, M) \nsubseteq (A, M)$ and $(V, M) \nsubseteq (G, M)$. Let $(V_1, M) = (A, M) \cap (V, M)$ and $(V_2, M) = (G, M) \cap (V, M)$. Since $(V, M) \subseteq (A, M) \cup (G, M)$,$(V_1, M)$ and (V_2, M) are non empty sets and $(V_1, M) \cup (V_2, M) = [(A, M) \cap (V, M)] \cup [(G, M) \cap (V, M)] = [(A, M) \cup (G, M)] \cap (V, M) = (V, M)$. Since $(V_1, M) \subseteq (A, M)$ and $(V_2, M) \subseteq (G, M)$ and $(A, M), (G, M)$ are SN $ic - PG$ separated sets, $Nic - PGcl(V_1, M) \cap (V_2, M) \subseteq Nic - PGcl(A, M) \cap (G, M) = \varnothing$ and

$(V_1, M) \cap Nic - PGcl(V_2, M) \subseteq (A, M) \cap Nic - PGcl(G, M) = \varnothing$. Then $(V_1, M), (V_2, M)$ are SN $ic - PG$ separated sets such that $(V, M) = (V_1, M) \cup (V_2, M)$. Hence by theorem 3.8(v), (V, M) is not SN $ic - PG$ connected which is a contradiction to (V, M) is SN $ic - PG$ connected. Hence either $(V, M) \subseteq (A, M)$ or$(V, M) \subseteq (G, M)$. □

Theorem 3.11. *If* (V, M) *is* SN $ic - PG$ *connected, then* $Nic - PGcl(V, M)$ *is also* SN $ic - PG$ *connected set.*

Proof. Let (V, M) be SN $ic - PG$ connected set. Suppose $Nic - PGcl(V, M)$ not SN $ic - PG$ connected set, then by theorem 3.8(v), there exist a SN $ic - PG$ separated sets (A, M) and (G, M) such that $Nic - PGcl(V, M) = (A, M) \cup (G, M)$. Since (V, M) is SN $ic - PG$ connected set and $(V, M) \subseteq Nic - PGcl(V, M) = (A, M) \cup (G, M)$. Then by theorem 3.10, either $(V, M) \subseteq (A, M)$ or$(V, M) \subseteq (G, M)$. If $(V, M) \subseteq (A, M)$, then $Nic - PGcl(V, M) \subseteq Nic - PGcl(A, M)$. Since (A, M) and (G, M) are separated sets, by theorem 3.9, $(A, M) \neq \varnothing$, $(G, M) \neq \varnothing$ and $Nic - PGcl(V, M) \cap (A, M) \subseteq Nic - PGcl(G, M) \cap (A, M) = \varnothing$ and hence $(A, M) \subseteq (Nic - PGcl(V, M))'$. Also $(A, M) \subseteq (A, M) \cup (G, M) = Nic - PGcl(V, M)$, $(A, M) \subseteq$

$(\mathrm{Nic}-\mathrm{PGcl}(V,M))' \cap (\mathrm{Nic}-\mathrm{PGcl}(V,M)) = \varnothing$. This is contradiction to$(A,M) \neq \varnothing$. Similarly, if $(V,M) \subseteq (G,M)$ we get contradiction to $(G,M) \neq \varnothing$. Hence $\mathrm{Nic}-\mathrm{PGcl}(V,M)$ is also SN $\mathrm{ic}-\mathrm{PG}$ connected. □

Theorem 3.12. *Let* (V,M) *be a* SN $\mathrm{ic}-\mathrm{PG}$ *connected subset of* $(\tau_{SN}(X),U,M)$.*If* (G,M) *is a subset of* $(\tau_{SN}(X),U,M)$ *such that* $(V,M) \subseteq (G,M) \subseteq \mathrm{Nic}-\mathrm{PGcl}(V,M)$, *then* (G,M) *is* SN $\mathrm{ic}-\mathrm{PG}$ *connected.*

Proof. Suppose $(G,M)$is not SN $\mathrm{ic}-\mathrm{PG}$ conneced. By theorem 3.8(v), there exists two non empty SN ic PG separated sets (A,M) and (B,M) such that $(G,M) = (A,M) \cup (B,M)$. Since $(V,M) \subseteq (G,M) = (A,M) \cup (B,M)$ and by theorem 3.10, $(V,M) \subseteq (A,M)$ or $(V,M) \subseteq (B,M)$. If $(V,M) \subseteq (A,M)$ implies that $\mathrm{Nic}-\mathrm{PGcl}(V,M) \subseteq \mathrm{Nic}-\mathrm{PGcl}(A,M)$. Now $\mathrm{Nic}-\mathrm{PGcl}(V,M) \cap (B,M) \subseteq \mathrm{Nic}-\mathrm{PGcl}(A,M) \cap (B,M) = \varnothing$ which implies $\mathrm{Nic}-\mathrm{PGcl}(V,M) \cap (B,M) = \varnothing$. Also $(A,M) \cup (B,M) = (G,M) \subseteq \mathrm{Nic}-\mathrm{PGcl}(V,M)$, $(B,M) \subseteq (G,M) \subseteq \mathrm{Nic}-\mathrm{PGcl}(V,M)$. Hence $\mathrm{Nic}-\mathrm{PGcl}(V,M) \cap (B,M) = (B,M)$. Then $(B,M) = \varnothing$, which is contradiction to (B,M) is non empty. Similarly, $(V,M) \subseteq (B,M)$, we get contradiction to (A,M) is non empty. Hence (G,M) is SN $\mathrm{ic}-\mathrm{PG}$ connected. □

Theorem 3.13. *If* (V,M) *and* (G,M) *are* SN $\mathrm{ic}-\mathrm{PG}$ *connected subset of* $(\tau_{SN}(X),U,M)$ *such that* $(V,M) \cap (G,M) \neq \varnothing$ *then* $(V,M) \cup (G,M)$ *is a* SN $\mathrm{ic}-\mathrm{PG}$ *connected subset of* $(\tau_{SN}(X),U,M)$.

Proof. Suppose that $(V,M) \cup (G,M)$is not SN $\mathrm{ic}-\mathrm{PG}$ connected. Then by theorem 3.8(v), there exist a two SN $\mathrm{ic}-\mathrm{PG}$ separated sets (A,M) and(B,M) such that $(V,M) \cup (G,M) = (A,M) \cup (B,M)$. Since (A,M) and (B,M) are SN $\mathrm{ic}-\mathrm{PG}$ separated, (A,M) and (B,M) are non empty sets and $(A,M) \cap (B,M) \subseteq \mathrm{Nic}-\mathrm{PGcl}\ (A,M) \cap\ (B,M) = \varnothing$. Since $(V,M) \subseteq (V,M) \cup (G,M) = (A,M) \cup (B,M)$, $(G,M) \subseteq (V,M) \cup (G,M) = (A,M) \cup (B,M)$ and (V,M) and (G,M) are SN $\mathrm{ic}-\mathrm{PG}$ connected by theorem 3.10, $(V,M) \subseteq (A,M)$ or $(V,M) \subseteq (B,M)$ and$(G,M_1) \subseteq (A,M_1)$ or $(G,M_1) \subseteq (B,M_1)$.

Case (i) If $(V,M) \subseteq (A,M)$ and $(G,M) \subseteq (A,M)$ then $(V,M) \cup (G,M) \subseteq (A,M)$ and so$(V,M) \cup (G,M) = (A,M)$. Since (A,M) and (B,M) are disjoint, $(B,M) = \varnothing$ which contradiction to (B,M) is non empty. Similarly if $(V,M) \subseteq (B,M)$ and $(G,M) \subseteq (B,M)$ we get the contradiction.

Case (ii) If $(V,M) \subseteq (A,M)$ and $(G,M) \subseteq (B,M)$ then $(V,M) \cap (G,M) \subseteq (A,M) \cap (B,M) = \varnothing$. Then $(V,M) \cap (G,M) = \varnothing$ which is contradiction to $(V,M) \cap (G,M) \neq \varnothing$. Similarly if $(V,M) \subseteq (B,M)$ and $(G,M) \subseteq (A,M)$, we get the contradiction. Hence $(V,M) \cup (G,M)$ is a SN $\mathrm{ic}-\mathrm{PG}$ connected. □

Theorem 3.14. *Let* $f_{SN} : (\tau_{SN}(X_1), U_1, M_1) \to (\tau_{SN'}(X_2), U_2, M_2)$ *be a* SN $\mathrm{ic}-\mathrm{PG}$ *continuous surjection and* $(\tau_{SN}(X_1), U_1, M_1)$ *be* SN $\mathrm{ic}-\mathrm{PG}$ *connected. Then* $(\tau_{SN'}(X_2), U_2, M_2)$ *is* SN *connected.*

Proof. Suppose that $(\tau_{SN'}(X_2), U_2, M_2) = (V,M_2) \cup (G,M_2)$ where (V,M_2) and (G,M_2) are disjoint non empty SN open sets of $(\tau_{SN'}(X_2), U_2, M_2)$. Since f_{SN} is SN $\mathrm{ic}-\mathrm{PG}$ continuous surjection,

$f_{SN}(\tau_{SN}(X_1), U_1, M_1) = (\tau_{SN'}(X_2), U_2, M_2)$ implies $(\tau_{SN}(X_1), U_1, M_1) = f_{SN}^{-1}(\tau_{SN'}(X_2), U_2, M_2) = f_{SN}^{-1}((V,M_2) \cup (G,M_2)) = f_{SN}^{-1}(V,M_2) \cup f_{SN}^{-1}(G,M_2)$ where $f_{SN}^{-1}(V,M_2)$ and $f_{SN}^{-1}(G,M_2)$ are disjoint non empty SN $\mathrm{ic}-\mathrm{PG}$ open sets in $(\tau_{SN}(X_1), U_1, M_1)$. This is a contradiction to $(\tau_{SN}(X_1), U_1, M_1)$ is SN $\mathrm{ic}-\mathrm{PG}$ connected so $(\tau_{SN'}(X_2), U_2, M_2)$ is SN connected. □

Theorem 3.15. *Let* $f_{SN} : (\tau_{SN}(X_1), U_1, M_1) \to (\tau_{SN'}(X_2), U_2, M_2)$ *be a* SN $\mathrm{ic}-\mathrm{PG}$ *irresolute surjection. If* $(\tau_{SN}(X_1), U_1, M_1)$ *is* SN $\mathrm{ic}-\mathrm{PG}$ *connected so* $(\tau_{SN'}(X_2), U_2, M_2)$.

Proof. Suppose that $(\tau_{SN'}(X_2), U_2, M_2) = (V,M_2) \cup (G,M_2)$ where (V,M_2) and (G,M_2) are disjoint non empty SN $\mathrm{ic}-\mathrm{PG}$ open sets of $(\tau_{SN'}(X_2), U_2, M_2)$. Since f_{SN} is SN $\mathrm{ic}-\mathrm{PG}$ irresolute surjection,

$f_{SN}(\tau_{SN}(X_1), U_1, M_1) = (\tau_{SN'}(X_2), U_2, M_2)$ implies $(\tau_{SN}(X_1), U_1, M_1) = f_{SN}^{-1}(\tau_{SN'}(X_2), U_2, M_2) = f_{SN}^{-1}((V,M_2) \cup (G,M_2)) = f_{SN}^{-1}(V,M_2) \cup f_{SN}^{-1}(G,M_2)$ where $f_{SN}^{-1}(V,M_2)$ and $f_{SN}^{-1}(G,M_2)$ are disjoint non empty SN $\mathrm{ic}-\mathrm{PG}$ open sets in $(\tau_{SN}(X_1), U_1, M_1)$. This is a contradiction to $(\tau_{SN}(X_1), U_1, M_1)$ is SN $\mathrm{ic}-\mathrm{PG}$ connected and so $(\tau_{SN'}(X_2), U_2, M_2)$ is SN $\mathrm{ic}-\mathrm{PG}$ connected. □

4. Soft Nano *ic* - Pre Generalized Hyper Connected Spaces

Definition 4.1. A soft subset (V,M) of soft nano topological space $(\tau_{SN}(X),U,M)$ is said to
(i) **SN ic − PG *dense*** if $Nic-PGcl(V,M)=(U,M)$.
(ii) **SN ic − PG *nowhere dense*** if $Nic-PGint\,(Nic-PGcl(V,M))=(\varnothing,M)$.

Definition 4.2. A soft nano topological space $(\tau_{SN}(X),U,M\ )$ is said to be ***soft nano ic – pre generalized hyper connected*** if (V,M) is SN ic − PG dense for every non empty SN ic − PG open set (V,M) of $(\tau_{SN}(X),U,M\ )$.

Example 4.3. Let $U=\{\vartheta,,\upsilon,\mu\}$, $M=\{\zeta_1,\zeta_2,\zeta_3\}$,$U/S=S(X)=\{\{\vartheta\},\{\},\{\upsilon,\ \mu\}\}$, $X=\{\}\subseteq U$, $(L_{SN}(X),M)=\{(\zeta_1,\{\}),(\zeta_2,\{\}),(\zeta_3,\{\})\}=(U_{SN}(X),M),(B_{SN}(X),M)=\{(\varnothing,M)\}$.

Now, $(\tau_{SN}(X),U,M)=\{U,\varnothing,(L_{SN}(X),M)\}$ is the soft nano topology on U. Here SN open sets are U, $\varnothing$, $(L_{SN}(X),M)$ and SN closed sets are U, $\varnothing$, $(L_{SN}(X),M)'$ where $(L_{SN}(X),M)'=\{(\zeta_1,\{\vartheta,\upsilon,\mu\}),(\zeta_2,\{\vartheta,\upsilon,\mu\}),(\zeta_3,\{\vartheta,\upsilon,\mu\})\}$. Here $SN\ ic-PGO(U,M)=\{U,\varnothing,(A_1,M),(A_2,M)\ldots,(A_7,M)\}$ where $(A_1,M)=\{(\zeta_1,\{\}),(\zeta_2,\{\}),(\zeta_3,\{\})\}$, $(A_2,M)=\{(\zeta_1,\{,\upsilon\}),(\zeta_2,\{,\upsilon\}),(\zeta_3,\{,\upsilon\})\}$, $(A_3,M)=\{(\zeta_1,\{\vartheta,\}),(\zeta_2,\{\vartheta,\}),$
$(A_4,M)=\{(\zeta_1,\{,\mu\}),(\zeta_2,\{,\mu\}),(\zeta_3,\{,\mu\})\}$,
$(A_5,M)=\{(\zeta_1,\{\vartheta,,\upsilon\}),(\zeta_2,\{\vartheta,,\upsilon\}),(\zeta_3,\{\vartheta,,\upsilon\})\}$,
$(A_6,M)=\{(\zeta_1,\{,\upsilon,\mu\}),(\zeta_2,\{,\upsilon,\mu\}),(\zeta_3,\{,\upsilon,\mu\})\}$, $(A_7,M)=\{(\zeta_1,\{\vartheta,,\mu\}),(\zeta_2,\{\vartheta,,\mu\},(\zeta_3,\{\vartheta,,\mu\}))\}$.

Here every nonempty SN ic − PG open set in $(\tau_{SN}(X),U,M\ )$ is SN ic − PG dense in $(\tau_{SN}(X),U,M\ )$.

Theorem 4.4. *Let $(\tau_{SN}(X),U,M\ )$ be soft nano topological space. Then the following are equivalent*

(i) $(\tau_{SN}(X),U,M\ )$ *is* SN ic − PG *hyper connected.*
(ii) (V,M) *is* SN ic − PG *dense or* SN ic − PG *nowhere dense for every soft subset* (V,M) *of* $(\tau_{SN}(X),U,M\ )$.
(iii) $(V,M)\cap(G,M)\neq(\varnothing,M)$ *for every non empty* SN ic − PG *open subsets* (V,M) *and* (G,M) *of* $(\tau_{SN}(X),U,M\ )$.

Proof. (i) $\Longrightarrow$ (ii) Let $(\tau_{SN}(X),U,M\ )$ be SN ic − PG hyper connected and (V,M) be soft subset of $(\tau_{SN}(X),U,M\ )$. Assume (V,M) is not nowhere dense.

Then $Nic-PGcl\,(Nic-PGcl(V,M))'=\ (Nic-PGint\,(Nic-PGcl(V,M)))'\neq(U,M)$. Since $Nic-PGint\,(Nic-PGcl(V,M))\neq(\varnothing,M)$, so by (i) ($Nic-PGcl\,(Nic-PGint\,(Nic-PGcl(V,M)))=(U,M)$. Since $Nic-PGcl(\ Nic-PGint\,(Nic-PGcl(V,M))=(U,M)\subset\ Nic-PGcl(V,M)$.

Then $Nic-PGcl(V,M)=(U,M)$. Hence (V,M) is SN ic − PG dense.

(ii) $\Longrightarrow$ (iii) Suppose that $(V,M)\cap(G,M)=\varnothing$ for some nonempty SN ic − PG open sets (V,M) and (G,M) of $(\tau_{SN}(X),U,M\ )$. Then $Nic-PGcl(V,M)\cap(G,M)=\varnothing$ and (V,M) is not SN ic − PG dense. Since (V,M) is SN ic − PG open and non empty, $(V,M)\subset Nic-PGint\,(Nic-PGcl(V,M))$ which implies $Nic-PGint\,(Nic-PGcl(V,M))\neq(\varnothing,M)$. Hence (V,M) is not SN ic − PG nowhere dense. This is a contradiction.

(iii) $\Longrightarrow$ (i) Let $(V,M)\cap(G,M)\neq(\varnothing,M)$ for every non empty SN ic − PG open sets (V,M) and (G,M) in $(\tau_{SN}(X),U,M\ )$. Suppose that $(\tau_{SN}(X),U,M\ )$ is not SN ic − PG hyper connected, then there is non empty SN ic − PG open sets (V,M) of $(\tau_{SN}(X),U,M\ )$ such that (V,M) is not SN ic − PG dense, thus $Nic-PGcl(V,M)\neq(U,M)$. Hence $(Nic-PGcl(V,M))'$ and (V,M) are disjoint nonempty SN ic − PG open sets in $(\tau_{SN}(X),U,M\ )$. This is a contradiction. Hence $(\tau_{SN}(X),U,M\ )$ is SN ic − PG hyper connected. □

Theorem 4.5. *Every* SN ic − PG *hyper connected space is* SN ic − PG *connected.*

Proof. Let $(\tau_{SN}(X),U,M\ )$ be a SN ic − PG hyper connected space. Suppose SN ic − PG disconnected. Then, there exist SN ic − PG open sets (V,M) and (G,M) in $(\tau_{SN}(X),U,M\ )$ such that $(V,M)\cap(G,M)=\varnothing$ and $(V,M)\cup(G,M)=(\tau_{SN}(X),U,M\ )$. This is a contradiction to above theorem. Hence $(\tau_{SN}(X),U,M\ )$ is SN ic − PG connected. □

Remark 4.6. The above theorem does not imply their reverse part by the following example.

Example 4.7. Let $U = \{\vartheta, , \upsilon, \mu, \rho\}$, $M = \{\zeta_1, \zeta_2, \zeta_3\}$, $U/S = \{\{\mu\}, \{\vartheta, \}, \{\upsilon, \rho\}\}$,
$X = \{\vartheta, \mu\} \subseteq U$, $(L_{SN}(X), M) = \{(\zeta_1, \{\vartheta\}), (\zeta_2, \{\vartheta\}), (\mu_3, \{\vartheta\})\}$,
$(U_{SN}(X), M) = \{(\zeta_1, \{\vartheta, \upsilon, \rho\}), (\zeta_2, \{\vartheta, \upsilon, \rho\}), (\zeta_3, \{\vartheta, \upsilon, \rho\})\}$,
$(B_{SN}(X), M) = \{(\zeta_1, \{\upsilon, \rho\}), (\zeta_2, \{\upsilon, \rho\}), (\zeta_3, \{\upsilon, \rho\})\}$.

Now, $(\tau_{SN}(X), U, M) = \{U, \varnothing, (L_{SN}(X), M), (U_{SN}(X), M), (B_{SN}(X), M)\}$ are soft nano topology on U. Here SN open sets are U, $\varnothing$, $(L_{SN}(X), M), (U_{SN}(X), M), (B_{SN}(X), M)$.

SN closed set are U, $\varnothing$, $(L_{SN}(X), M)'$, $(U_{SN}(X), M)'$, $(B_{SN}(X), M)'$ where
$(L_{SN}(X), M)' = \{(\zeta_1, \{, \upsilon, \mu, \rho\}), (\zeta_2, \{, \upsilon, \mu, \rho\}), (\zeta_3, \{, \upsilon, \mu, \rho\})\}$,
$(U_{SN}(X), M)' = \{(\zeta_1, \{, \mu\}), (\zeta_2, \{, \mu\}), (\zeta_3, \{, \mu\})\}$,
$(B_{SN}(X), M)' = \{(\zeta_1, \{\vartheta, , \mu\}), (\zeta_2, \{\vartheta, , \mu\}), (\zeta_3, \{\vartheta, , \mu\})\}$.

Here SN $ic-PGO(U, M) = \{U, \varnothing, (A_1, M), (A_2, M), , \ldots\ldots\ldots\ldots\ldots, (A_{16}, M)\}$
where $(A_1, M) = \{(\zeta_1, \{\vartheta\}), (\zeta_2, \{\vartheta\}), (\zeta_3, \{\vartheta\})\}$, $(A_2, M) = \{(\zeta_1, \{\upsilon\}), (\zeta_2, \{\upsilon\}), (\zeta_3, \{\upsilon\})\}$,
$(A_3, M) = \{(\zeta_1, \{\rho\}), (\zeta_2, \{\rho\}), (\zeta_3, \{\rho\})\}$, $(A_4, M) = \{(\zeta_1, \{\vartheta, \upsilon\}), (\zeta_2, \{\vartheta, \upsilon\}), (\zeta_3, \{\vartheta, \upsilon\})\}$,
$(A_5, M) = \{(\zeta_1, \{\upsilon, \rho\}), (\zeta_2, \{\upsilon, \rho\}), (\zeta_3, \{\upsilon, \rho\})\}$,
$(A_6, M) = \{(\zeta_1, \{\vartheta, \rho\}), (\zeta_2, \{\vartheta, \rho\}), (\zeta_3, \{\vartheta, \rho\})\}$
$(A_7, M) = \{(\zeta_1, \{\vartheta, , \upsilon\}), (\zeta_2, \{\vartheta, , \upsilon\}), (\zeta_3, \{\vartheta, , \upsilon\})\}$,
$(A_8, M) = \{(\zeta_1, \{\vartheta, \upsilon, \mu\}), (\zeta_2, \{\vartheta, \upsilon, \mu\}), (\zeta_3, \{\vartheta, \upsilon, \mu\})\}$,
$(A_9, M) = \{(\zeta_1, \{\vartheta, \mu, \rho\}), (\zeta_2, \{\vartheta, \mu, \rho\}), (\zeta_3, \{\vartheta, \mu, \rho\})\}$,
$(A_{10}, M) = \{(\zeta_1, \{\vartheta, , \rho\}), (\zeta_2, \{\vartheta, , \rho\}), (\zeta_3, \{\vartheta, , \rho\})\}$,
$(A_{11}, M) = \{(\zeta_1, \{\vartheta, \upsilon, \rho\}), (\zeta_2, \{\vartheta, \upsilon, \rho\}), (\zeta_3, \{\vartheta, \upsilon, \rho\})\}$,
$(A_{12}, M) = \{(\zeta_1, \{\vartheta, , \upsilon, \mu\}), (\zeta_2, \{\vartheta, , \upsilon, \mu\}), (\zeta_3, \{\vartheta, , \upsilon, \mu\})\}$,
$(A_{13}, M) = \{(\zeta_1, \{\vartheta, , \mu, \rho\}), (\zeta_2, \{\vartheta, , \mu, \rho\}), (\zeta_3, \{\vartheta, , \mu, \rho\})\}$,
$(A_{14}, M) = \{(\zeta_1, \{\vartheta, , \upsilon, \rho\}), (\zeta_2, \{\vartheta, , \upsilon, \rho\}), (\zeta_3, \{\vartheta, , \upsilon, \rho\})\}$,
$(A_{15}, M) = \{(\zeta_1, \{\vartheta, \upsilon, \mu, \rho\}), (\zeta_2, \{\vartheta, \upsilon, \mu, \rho\}), (\zeta_3, \{\vartheta, \upsilon, \mu, \rho\})\}$ is SN $ic-PG$ connected. But not SN $ic-PG$ hyper connected.

Theorem 4.8. *A soft nano topological space* $(\tau_{SN}(X), U, M)$ *is* SN $ic-PG$ *hyper connected then the intersection of any two* SN $ic-PG$ *open sets is also* SN $ic-PG$ *open* .

Proof. Let $(\tau_{SN}(X), U, M)$ be SN $ic-PG$ hyper connected space and (V, M) and (G, M) be SN $ic-PG$ open sets. Then by theorem 4.4,$(V, M) \cap (G, M) \neq (\varnothing, M)$ for every non empty SN $ic-PG$ open sets (V, M) and (G, M) in $(\tau_{SN}(X), U, M)$. Since (V, M) is SN $ic-PG$ open set, $[(V, M) \cap (H, M)] \subseteq Ngcl(Nint(Ncl(V, M)))$ for any soft nano closed set $(H, M) \neq (U, M), \varnothing$ and also since (G, M) is SN $ic-PG$ open set, $[(G, M) \cap (H, M)] \subseteq Ngcl(Nint(Ncl(G, M)))$ for any soft nano closed set$(H, M) \neq (U, M), \varnothing$. Then $[(V, M) \cap (H, M)] \cap [(G, M) \cap (H, M)] \subseteq$

$$Ngcl(Nint(Ncl(V, M))) \cap Ngcl(Nint(Ncl(G, M)))$$

$$\begin{aligned}[(V, M) \cap (G, M)] \cap (H, M) &\subseteq Ngcl(Nint(Ncl(V, M)) \cap Nint(Ncl(G, M))) \\ &\subseteq Ngcl(Nint(Ncl(V, M) \cap Ncl(G, M))) \\ &\subseteq Ngcl(Nint(Ncl((V, M) \cap (G, M))))\end{aligned}$$

which implies $(V, M) \cap (G, M)$ is also SN$ic-PG$ open sets. □

Theorem 4.9. *Let* $(\tau_{SN}(X), U, M)$ *be soft nano topological space. Then the following are equivalent*

(i) $(\tau_{SN}(X), U, M)$ *is* SN $ic-PG$ *hyper connected.*
(ii) $(\tau_{SN}(X), U, M)$ *has no proper disjoint* SN $ic-PG$ *open sets* (V, M) *and* (G, M)*such that* $(U, M) = Nic-PGcl(V, M) \cup (G, M) = (V, M) \cup Nic-PGcl(G, M)$

(iii) $(\tau_{SN}(X), \mathcal{U}, M)$ *does not have proper* $SN\ ic-PG$ *closed sets* (A, M) *and* (B, M) *such that* $(\mathcal{U}, M) = (A, M) \cup (B, M)$ *and* $Nic-PGint(A, M) \cap (B, M) = (A, M) \cap Nic-PGint(B, M) = (\emptyset, M)$.

Proof. (i) $\Longrightarrow$ (ii) Assume that there exists two proper disjoint $SN\ ic-PG$ open sets (V, M) and (G, M)such that $(\mathcal{U}, M) = Nic-PGcl(V, M) \cup (G, M) = (V, M) \cup Nic-PGcl(G, M)$. $Nic-PGcl(V, M)$ is non empty $SN\ ic-PG$ closed set. Since $(V, M) \cap (G, M) = \emptyset$, $Nic-PGcl(V, M) \cap (G, M) = \emptyset$. This implies $Nic-PGcl(V, M) \neq (\mathcal{U}, M)$. This is contradiction to $(\tau_{SN}(X), \mathcal{U}, M)$ is $SN\ ic-PG$ hyper connected.

(ii) $\Longrightarrow$ (iii) Suppose there exists two proper non empty $SN\ ic-PG$ closed sets (A, M) and (B, M)such that $(\mathcal{U}, M) = (A, M) \cup (B, M)$, $Nic-PGint(A, M) \cap (B, M) = (A, M) \cap Nic-PGint(B, M) = (\emptyset, M)$. Then $(V, M) = (A, M)'$, $(G, M) = (B, M)'$ are non empty $SN\ ic-PG$ open sets Then $Nic\ PGcl(V, M) \cup (G, M) = Nic-PGcl(A, M)' \cup (G, M) = (Nic-PGint(A, M))' \cup (G, M) = (Nic-PGint(A, M))' \cup (B, M)' = (\mathcal{U}, M)$, which implies $Nic-PGcl(V, M) \cup (G, M) = (V, M) \cup Nic-PGcl(G, M) = (\mathcal{U}, M)$. This is a contradiction to (ii).

(iii) $\Longrightarrow$ (i) Suppose $(\tau_{SN}(X), \mathcal{U}, M)$ is not $SN\ ic-PG$ hyper connected. Then there exist proper non empty $SN\ ic-PG$ open sets (V, M) of $(\tau_{SN}(X), \mathcal{U}, M)$ such that $Nic-PGcl(V, M) \neq (\mathcal{U}, M)$. Then $Nic-PGint\ (Nic-PGcl(V, M)) \neq (\mathcal{U}, M)$. Take $Nic-PGcl(V, M) = (A, M)$ and

$(Nic-PGint\ (Nic-PGcl(V, M)))' = (B, M)$. This implies $(A, M) \cup (B, M) = (Nic-PGcl(V, M)) \cup (Nic-PGint\ (Nic-PGcl(V, M)))' = (Nic-PGcl(V, M)) \cup Nic-PGcl\left((Nic-PGcl(V, M))'\right) =$

$(Nic-PGcl(V, M)) \cup Nic-PGcl(A, M)' = (A, M) \cup (A, M)' = (\mathcal{U}, M)$. Since (A, M) $SN\ ic-PG$ closed set, then $Nic-PGint\,(Nic-PGcl(V, M)) \cap (Nic-PGint\,(Nic-PGcl(V, M)))' = (\emptyset, M)$.

This implies $Nic-PGcl(V, M) \cap Nic-PGint\,(Nic-PGint\,(Nic-PGcl(V, M)))' = Nic-PGcl(V, M) \cap Nic-PGint\left(Nic-PGcl\,(Nic-PGcl(V, M))'\right) = Nic-PGcl(V, M) \cap Nic-PGint\,(Nic-PGcl(A, M)') = (A, M) \cap (A, M)' = (\emptyset, M)$. Since $(A, M)'$ is $SN\ ic-PG$ open set. Thus $(\tau_{SN}(X), \mathcal{U}, M)$ has two proper $SN\ ic-PG$ closed set.(A, M),(B, M) such that $(\mathcal{U}, M) = (A, M) \cup (B, M)$ and $Nic-PG\ int\ (A, M) \cap (B, M) = (A, M) \cap Nic-PGint\ (B, M) = (\emptyset, M)$. This is contradiction to (iii). □

5. Conclusion

Connectivity makes it easier to categorize and understand the many topological space structures and how they relate to other mathematical concepts and applications. In this work, we examined the features of $SN\ ic-PG$ connected space in soft nano topological spaces and also studied its properties. Also, we introduced the idea of $SN\ ic-PG$ hyper connected space and used relevant examples of its characteristics to further support our findings. It may be feasible to develop this work and look into other concepts like separation axioms and compactness.

Acknowledgment

I would be grateful to thank my institution for their support and my guide for her precious guidance

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