Math. & Comput. Sci., 7(1) 2026, 45–55

Online: ISSN 2717-2708

Mathematics and Computational Sciences

Journal Homepage: http://mcs.qut.ac.ir/mcs

Characterizations of Neutrosophic nano β-Continuous functions in Neutrosophic nano topological spaces

Panimayam Anbarasi Rodrigo[a], Johnson Charlet Jenisha[a,*]

[a]*Department of Mathematics, St, Mary's College(Autonomous), (Affliated to Manonmaniam Sundaranar University, Abishekapatti, Tirunelveli), Thoothukudi-628001, TamilNadu, India.*

Abstract

This paper introduces neutrosophic nano β-continuous functions and explores their defining characteristics in neutrosophic nano topological spaces. The study establishes their role in extending continuity concepts for handling uncertainty and indeterminacy.

Keywords: Neutrosophic Nano β-continuous mapping, Neutrosophic Nano β-open, Neutrosophic continuous function, Nano continuous function, Neutrosophic Nano open sets, Neutrosophic open sets, Nano open sets, Neutrosophic β-open sets, Nano β-open sets

2020 MSC: 54A40, 03E72, 54C08.

1. Introduction

The study of broadened forms of continuity has played a central role in advancing modern topology and its applications. In recent years, neutrosophic theory has emerged as a strong instrument for handling ambiguity, vagueness, and insufficient information within mathematical structures. When combined with nano topological concepts, it provides a refined framework for analyzing spaces that arise in uncertain or granular settings. The concept of neutrosophic nano β-continuous functions is presented as a logical extension of continuity that incorporates both the neutrosophic approach to indeterminacy and the nano perspective of refinement. Such functions preserve the structure of β-open sets under inverse mappings, thereby creating opportunities to explore new relationships between classical continuity, nano continuity, and other generalized forms. This investigation not only broadens the theoretical foundation of neutrosophic topology but also sets the stage for potential significance for systems, analysing data, as well as decision-making characterized by uncertainty.

*Corresponding author

Email addresses: anbu.n.u@gmail.com (Panimayam Anbarasi Rodrigo), jenishacharlet@gmail.com (Johnson Charlet Jenisha)

doi: 10.30511/mcs.2025.2075275.1546

Received: 21 October 2025 Accepted: 24 November 2025

2. Preliminaries

Definition 2.1. [3] Let X be a non-empty set. A Neutrosophic (Neut.). set $A \subseteq X$ is specified as $A = \{< x, \mu_A(x), \sigma_A(x), \gamma_A(x) > x \in X\}$ such that $\mu_A(x)$, $\sigma_A(x)$, $\gamma_A(x)$ denote respectively the degree of membership, indeterminacy & non-membership of each object x in A.

Definition 2.2. [5]
Let $\mathfrak{U}$ be the universe, $\mathfrak{R}$ an equivalence relation on $\mathfrak{U}$ & $\tau_{\mathcal{R}}(X) = \{\mathfrak{U}, \phi, L_{\mathcal{R}}(X), U_{\mathcal{R}}(X), B_{\mathcal{R}}(X)\}$, $X \subseteq \mathfrak{U}$. The collection $\tau_{\mathcal{R}}(X)$ holds the below properties:

(i) $\mathfrak{U}$ and $\phi \in \tau_{\mathcal{R}}(X)$
(ii) The union of any sub-collection of $\tau_{\mathcal{R}}(X)$ is also in $\tau_{\mathcal{R}}(X)$.
(iii) The intersection of any finite subcollection $\tau_{\mathcal{R}}(X)$ is in $\tau_{\mathcal{R}}(X)$.

Hence $\tau_{\mathcal{R}}(X)$ defines a nano topology on $\mathfrak{U}$ on X. The pair $(\mathfrak{U}, \tau_{\mathcal{R}}(X))$ as the nano topological space and the members of $\tau_{\mathcal{R}}(X)$ are referred to as nano-open sets.

Definition 2.3. [4]
The non-empty set $\mathfrak{U}$ and the equivalence relation $\mathfrak{R}$ on $\mathfrak{U}$ are assumed to exist. With the membership function μ_H, the indeterminacy function σ_H, and the non-membership function γ_H, let H be a neut. set in $\mathfrak{U}$. The neutrosophic nano border of H in the approximation $(\mathfrak{U}, \mathfrak{R})$, the neutrosophic nano lower approximation, and the neutrosophic nano higher approximation are indicated by $\underline{\mathfrak{N}_{\mathfrak{N}}}(H)$, $\overline{\mathfrak{N}_{\mathfrak{N}}}(H)$, $B_{\mathfrak{N}_{\mathfrak{N}}}(H)$ are respectively classified as follows:

$\underline{\mathfrak{N}_{\mathfrak{N}}}(H) = \{\langle x, \mu_{\underline{\mathfrak{R}}(\zeta)}(x), \sigma_{\underline{\mathfrak{R}}(\zeta)}(x), \gamma_{\underline{\mathfrak{R}}(\zeta)}(x)\rangle / y \epsilon [x]_{\mathfrak{R}}, x \in U\}$
$\overline{\mathfrak{N}_{\mathfrak{N}}}(H) = \{\left\langle x, \mu_{\overline{\mathfrak{R}}(\zeta)}(x), \sigma_{\overline{\mathfrak{R}}(\zeta)}(x), \gamma_{\overline{\mathfrak{R}}(\zeta)}(x)\right\rangle / y \epsilon [x]_{\mathfrak{R}}, x \in U\}$
$B_{\mathfrak{N}_{\mathfrak{N}}}(H) = \overline{\mathfrak{N}_{\mathfrak{N}}}(H) - \underline{\mathfrak{N}_{\mathfrak{N}}}(H)$
where $\mu_{\underline{\mathfrak{R}}(\zeta)}(x) = \bigwedge_{y \epsilon [x]_{\mathfrak{R}}} \mu_{\zeta}(x)$, $\sigma_{\underline{\mathfrak{R}}(\zeta)}(x) = \bigwedge_{y \epsilon [x]_{\mathfrak{R}}} \sigma_{\zeta}(x)$, $\gamma_{\underline{\mathfrak{R}}(\zeta)}(x) = \bigvee_{y \epsilon [x]_{\mathfrak{R}}} \gamma_{\zeta}(x)$
$\mu_{\overline{\mathfrak{R}}(\zeta)}(x) = \bigvee_{y \epsilon [x]_{\mathfrak{R}}} \mu_{\zeta}(x)$, $\sigma_{\overline{\mathfrak{R}}(\zeta)}(x) = \bigvee_{y \epsilon [x]_{\mathfrak{R}}} \sigma_{\zeta}(x)$, $\gamma_{\overline{\mathfrak{R}}(\zeta)}(x) = \bigwedge_{y \epsilon [x]_{\mathfrak{R}}} \gamma_{\zeta}(x)$

Definition 2.4. [4] Let $\mathfrak{U}$ & $\mathfrak{R}$ denotes universe & equivalence relation on $\mathfrak{U}$ and H refers a neut. set in $\mathfrak{U}$ and if the class $\tau_N(H) = \{0_N, 1_N, \underline{\mathfrak{N}_{\mathfrak{N}}}(H), \overline{\mathfrak{N}_{\mathfrak{N}}}(H), B_{\mathfrak{N}_{\mathfrak{N}}}(H)\}$ creates a topology, it is referred to as a $\mathfrak{N}_{\mathfrak{N}}$ topology. We call $(\mathfrak{U}, \tau_N(H))$ as the $\mathfrak{N}_{\mathfrak{N}}$ TS. The objects of $\tau_N(H)$ are symbolized $\mathfrak{N}_{\mathfrak{N}}$ open sets.

Definition 2.5. [4]
Let $(\mathfrak{U}, \tau_N(H))$ be a $\mathfrak{N}_{\mathfrak{N}}$ TS and $S = < x, \mu_S(x), \sigma_S(x), \gamma_S(x) >$ be a Neut. set in X. Then the $\mathfrak{N}_{\mathfrak{N}}$ closure and $\mathfrak{N}_{\mathfrak{N}}$ interior of S are specified by
$\mathfrak{N}_{\mathfrak{N}}cl\ (S) = \cap \{\mathcal{K} : \mathcal{K}$ is a $\mathfrak{N}_{\mathfrak{N}}$ CSs in X and $S \subseteq \mathcal{K}\}$
$\mathfrak{N}_{\mathfrak{N}}\ int\ (S) = \cup \{\mathcal{G} : \mathcal{G}$ is a $\mathfrak{N}_{\mathfrak{N}}$ OSs in X and $\mathcal{G} \subseteq A\}$

Definition 2.6. [3]
Let $\mathfrak{U} \neq \phi$ and the neutrosophic subsets (briefly, Neut subs's) P & Q in the form $\mathcal{P} = \{x : \mu_{\mathcal{P}}(x), \sigma_{\mathcal{P}}(x), \gamma_{\mathcal{P}}(x), x \in \mathfrak{U}\}$, $\mathcal{Q} = \{x : \mu_{\mathcal{Q}}(x), \sigma_{\mathcal{Q}}(x), \gamma_{\mathcal{Q}}(x), x \in \mathfrak{U}\}$. Thus the statements are hold:
(i) $0_N = \{x, 0, 0, 1 : x \in \mathfrak{U}\}$.
(ii) $1_N = \{x, 1, 1, 0 : x \in \mathfrak{U}\}$.
(iii) $\mathcal{P} \subseteq \mathcal{Q} \Leftrightarrow \mu_{\mathcal{P}}(x) \leqslant \mu_{\mathcal{Q}}(x), \sigma_{\mathcal{P}}(x), \leqslant \sigma_{\mathcal{Q}}(x), \gamma_{\mathcal{P}}(x) \geqslant \gamma_{\mathcal{Q}}(x)\ \forall\ x \in \mathfrak{U}\}$.
(iv) $\mathcal{P} = \mathcal{Q} \Leftrightarrow \mathcal{P} \subseteq \mathcal{Q}$ and $\mathcal{Q} \subseteq \mathcal{P}$
(v) $\mathcal{P}^c = \{x, \gamma_{\mathcal{P}}(x), 1\text{-}\sigma_{\mathcal{P}}(x), \gamma_{\mathcal{P}}(x) : x \in \mathfrak{U}\}$
(vi) $\mathcal{P} \cap \mathcal{Q} = \{x, \mu_{\mathcal{P}}(x) \wedge \mu_{\mathcal{Q}}(x), \sigma_{\mathcal{P}}(x) \wedge \sigma_{\mathcal{Q}}(x), \gamma_{\mathcal{P}}(x) \vee \gamma_{\mathcal{Q}}(x)\ \forall\ x \in \mathfrak{U}\}$.
(vii) $\mathcal{P} \cup \mathcal{Q} = \{x, \mu_{\mathcal{P}}(x) \vee \mu_{\mathcal{Q}}(x), \sigma_{\mathcal{P}}(x) \vee \sigma_{\mathcal{Q}}(x), \gamma_{\mathcal{P}}(x) \wedge \gamma_{\mathcal{Q}}(x)\ \forall\ x \in \mathfrak{U}\}$.

Definition 2.7. [1]
Let $(\mathfrak{U}, \tau_{\mathfrak{N}}(S))$ symbolise an $\mathfrak{N}_{\mathfrak{N}}$ Topological Space (TS). The $\mathfrak{N}_{\mathfrak{N}}$ subset K of U is called the Neutrosophic Nano β - open(notated by $\mathfrak{N}_{\mathfrak{N}}$ β-open) if $K \subseteq \mathfrak{N}_{\mathfrak{N}}cl(\mathfrak{N}_{\mathfrak{N}}int(\mathfrak{N}_{\mathfrak{N}}cl(K)))$.

Definition 2.8. [1]
(i) The $\mathfrak{N}_{\mathfrak{N}}\beta$-interior is the culmination of all $\mathfrak{N}_{\mathfrak{N}}\beta$-OSs included in A & is represented by $\mathfrak{N}_{\mathfrak{N}}\beta$ -int.
(ii) The intersection of all $\mathfrak{N}_{\mathfrak{N}}\beta$-CSs containing A is called the $\mathfrak{N}_{\mathfrak{N}}\beta$-closure and it is notated by $\mathfrak{N}_{\mathfrak{N}}\beta$-cl.

Definition 2.9. [2]

(i) If $P = \{\langle z, \mu_P(z), \sigma_P(z), \gamma_P(z)\rangle : z \in Z\}$ is a neut. set in Z, then the pre-image of P under f , notated by $f^{-1}(P)$, is the neut. set in Y specified by

$$f^{-1}(P) = \{\langle y,\ f^{-1}(\mu_P)(y), f^{-1}(\sigma_P)(y), f^{-1}(\gamma_P)(y)\rangle : y \in Y\}.$$

(ii) If $D = \{\langle x, \mu_D(x), \sigma_D(x), \gamma_D(x)\rangle : x \in X\}$ is a neut. set in X, then image of D under f, notated by f(D) is the neut. set in Y specified by $f(D) = \langle y, f(\mu_D)(y), f(\sigma_D)(y), (1 - f(1-\gamma_D)(y)\rangle : y \in Y\}$, where

$$f(\mu_D)(y) = \begin{cases} \sup_{x\in f^{-1}(y)} \mu_D(x), & \text{if } f^{-1}(y) \neq \varnothing \\ 0, & \text{otherwise} \end{cases},$$

$$f(\sigma_D)(y) = \begin{cases} \sup_{x\in f^{-1}(y)} \sigma_D(x), & \text{if } f^{-1}(y) \neq \varnothing \\ 0, & \text{otherwise} \end{cases},$$

$$(1 - f(1-\gamma_D)(y) = \begin{cases} \inf_{x\in f^{-1}(y)} \gamma_D(x), & \text{if } f^{-1}(y) \neq \varnothing \\ 1, & \text{otherwise} \end{cases}$$

3. Neutrosophic Nano β-Continuous function

Definition 3.1. A function $k:(\mathfrak{U}, \tau_{\mathfrak{N}_{\mathfrak{N}}}(\mathfrak{U})) \to (\mathfrak{B}, \tau_{\mathfrak{N}_{\mathfrak{N}}}(\mathfrak{B}))$ is called Neutrosophic Nano β-continuous($\mathfrak{N}_{\mathfrak{N}}$ β - cts) function if $k^{-1}(R)$ is $\mathfrak{N}_{\mathfrak{N}}$ β-open in $\mathfrak{U}$ for each $\mathfrak{N}_{\mathfrak{N}}$ OS R in $\mathfrak{B}$.

Example 3.2. Consider $\mathfrak{U} = \{c_1, c_2, c_3, c_4\}$ and $\mathfrak{U} \setminus \mathfrak{R} = \{\{c_1, c_2\}, c_3, c_4\}$. Define an Neut. subset $H \subseteq \mathfrak{U} \ni$ $H = \left\{\left\langle \frac{c_1}{0.2,0.4,0.7}\right\rangle, \left\langle \frac{c_2}{0.3,0.4,0.6}\right\rangle, \left\langle \frac{c_3}{0.2,0.5,0.8}\right\rangle, \left\langle \frac{c_4}{0.4,0.4,0.7}\right\rangle\right\}$

Then $\overline{\mathfrak{N}_{\mathfrak{N}}(H)} = \left\{\left\langle \frac{c_1,c_2}{0.3,0.4,0.6}\right\rangle, \left\langle \frac{c_3}{0.2,0.5,0.8}\right\rangle, \left\langle \frac{c_4}{0.4,0.4,0.7}\right\rangle\right\}$, $\underline{\mathfrak{N}_{\mathfrak{N}}(H)} = \left\{\left\langle \frac{c_1,c_2}{0.2,0.4,0.7}\right\rangle, \left\langle \frac{c_3}{0.2,0.5,0.8}\right\rangle, \left\langle \frac{c_4}{0.4,0.4,0.7}\right\rangle\right\}$
and $B_{\mathfrak{N}_{\mathfrak{N}}}(H) = \left\{\left\langle \frac{c_1,c_2}{0.3,0.4,0.6}\right\rangle, \left\langle \frac{c_3}{0.2,0.5,0.8}\right\rangle, \left\langle \frac{c_4}{0.4,0.4,0.7}\right\rangle\right\}$.
Thus $\tau_{\mathfrak{N}_{\mathfrak{N}}}(\mathfrak{U}) = \{0_N, 1_N, \overline{\mathfrak{N}_{\mathfrak{N}}(H)}, \underline{\mathfrak{N}_{\mathfrak{N}}(H)}, B_{\mathfrak{N}_{\mathfrak{N}}}(H)\}$.
Also $\mathfrak{B} = \{d_1, d_2, d_3, d_4\}$ and $\mathfrak{B} \setminus \mathfrak{R} = \{\{d_1, d_4\}, d_2, d_3\}$
Define an Neut. subset $T \subseteq \mathfrak{B} \ni T = \left\{\left\langle \frac{d_1}{0.4,0.7,0.3}\right\rangle, \left\langle \frac{d_2}{0.5,0.7,0.4}\right\rangle, \left\langle \frac{d_3}{0.3,0.7,0.5}\right\rangle, \left\langle \frac{d_4}{0.3,0.6,0.4}\right\rangle\right\}$
Then $\overline{\mathfrak{N}_{\mathfrak{N}}(T)} = \left\{\left\langle \frac{d_1,d_4}{0.4,0.7,0.3}\right\rangle, \left\langle \frac{d_2}{0.5,0.7,0.4}\right\rangle, \left\langle \frac{d_3}{0.3,0.7,0.5}\right\rangle\right\}$, $\underline{\mathfrak{N}_{\mathfrak{N}}(T)} = \left\{\left\langle \frac{d_1,d_4}{0.3,0.6,0.4}\right\rangle, \left\langle \frac{d_2}{0.5,0.7,0.4}\right\rangle, \left\langle \frac{d_3}{0.3,0.7,0.5}\right\rangle\right\}$
and $B_{\mathfrak{N}_{\mathfrak{N}}}(T) = \left\{\left\langle \frac{d_1,d_4}{0.4,0.7,0.3}\right\rangle, \left\langle \frac{d_2}{0.5,0.7,0.4}\right\rangle, \left\langle \frac{d_3}{0.3,0.7,0.5}\right\rangle\right\}$.
Thus $\tau_{\mathfrak{N}_{\mathfrak{N}}}(\mathfrak{B}) = \{0_N, 1_N, \overline{\mathfrak{N}_{\mathfrak{N}}(T)}, \underline{\mathfrak{N}_{\mathfrak{N}}(T)}, B_{\mathfrak{N}_{\mathfrak{N}}}(T)\}$.
Then $k:(\mathfrak{U}, \tau_{\mathfrak{N}_{\mathfrak{N}}}(\mathfrak{U})) \to (\mathfrak{B}, \tau_{\mathfrak{N}_{\mathfrak{N}}}(\mathfrak{B}))$ is a function specified by $k(c_i) = d_i$, i = 1, 2, 3, 4.
Consider the set $B = \left\{\left\langle \frac{d_1,d_4}{0.4,0.7,0.3}\right\rangle, \left\langle \frac{d_2}{0.5,0.7,0.4}\right\rangle, \left\langle \frac{d_3}{0.3,0.7,0.5}\right\rangle\right\}$,
then $k^{-1}(B) = \left\{\left\langle \frac{c_1,c_4}{0.4,0.7,0.3}\right\rangle, \left\langle \frac{c_2}{0.5,0.7,0.4}\right\rangle, \left\langle \frac{c_3}{0.3,0.7,0.5}\right\rangle\right\}$ is a $\mathfrak{N}_{\mathfrak{N}}$ β - open set.
Hence $k:(\mathfrak{U}, \tau_{\mathfrak{N}_{\mathfrak{N}}}(\mathfrak{U})) \to (\mathfrak{B}, \tau_{\mathfrak{N}_{\mathfrak{N}}}(\mathfrak{B}))$ is a $\mathfrak{N}_{\mathfrak{N}}$ β - cts function.

Theorem 3.3.

i. *Every* $\mathfrak{N}_{\mathfrak{N}}$- cts *function is* $\mathfrak{N}_{\mathfrak{N}}$ β - cts *function* .
ii. *Every* $\mathfrak{N}_{\mathfrak{N}}$ *- pre* cts *function is* $\mathfrak{N}_{\mathfrak{N}}$ β - cts *function.*
iii. *Every* $\mathfrak{N}_{\mathfrak{N}}$ *- semi* cts *function is* $\mathfrak{N}_{\mathfrak{N}}$ β - cts *function.*
iv. *Every* $\mathfrak{N}_{\mathfrak{N}}$ - α cts *function is* $\mathfrak{N}_{\mathfrak{N}}$ β - cts *function.*

v. *Every* $\mathfrak{N}_{\mathfrak{N}}$ *- regular* cts *function is* $\mathfrak{N}_{\mathfrak{N}}$ β - cts *function.*
vi. *Every* $\mathfrak{N}_{\mathfrak{N}}$ δ - cts *function is* $\mathfrak{N}_{\mathfrak{N}}$ β - cts *function.*
vii. *Every* $\mathfrak{N}_{\mathfrak{N}}$ *g* - cts *function is* $\mathfrak{N}_{\mathfrak{N}}$ β - cts *function.*

Proof:

The proof is clear.

Proposition 3.4. *The afterwards examples shows that the previous theorem's converse doesn't have to be accurate.*

Example 3.5. Take $\mathfrak{U} = \{c_1, c_2, c_3, c_4\}$ and $\mathfrak{U} \setminus \mathfrak{R} = \{c_1, \{c_2, c_3\}, c_4\}$.
Define an Neut. subset $H \subseteq \mathfrak{U} \ni H = \left\{\left\langle\frac{c_1}{0.1,0.4,0.8}\right\rangle, \left\langle\frac{c_2}{0.2,0.5,0.7}\right\rangle, \left\langle\frac{c_3}{0.1,0.5,0.8}\right\rangle, \left\langle\frac{c_4}{0.2,0.5,0.9}\right\rangle\right\}$
Then $\overline{\mathfrak{N}_{\mathfrak{N}}(H)} = \left\{\left\langle\frac{c_1}{0.1,0.4,0.8}\right\rangle, \left\langle\frac{c_2,c_3}{0.2,0.5,0.7}\right\rangle, \left\langle\frac{c_4}{0.2,0.5,0.9}\right\rangle\right\}$, $\underline{\mathfrak{N}_{\mathfrak{N}}(H)} = \left\{\left\langle\frac{c_1}{0.1,0.4,0.8}\right\rangle, \left\langle\frac{c_2,c_3}{0.1,0.5,0.8}\right\rangle, \left\langle\frac{c_4}{0.2,0.5,0.9}\right\rangle\right\}$
and $B_{\mathfrak{N}_{\mathfrak{N}}}(H) = \left\{\left\langle\frac{c_1}{0.1,0.4,0.8}\right\rangle, \left\langle\frac{c_2,c_3}{0.2,0.5,0.7}\right\rangle, \left\langle\frac{c_4}{0.2,0.5,0.9}\right\rangle\right\}$.
Thus $\tau_{\mathfrak{N}_{\mathfrak{N}}}(\mathfrak{U}) = \{0_N, 1_N, \overline{\mathfrak{N}_{\mathfrak{N}}(H)}, \underline{\mathfrak{N}_{\mathfrak{N}}(H)}, B_{\mathfrak{N}_{\mathfrak{N}}}(H)\}$.
Also $\mathfrak{B} = \{d_1, d_2, d_3, d_4\}$ and $\mathfrak{B} \setminus \mathfrak{R} = \{d_1, \{d_2, d_3\}, d_4\}\}$. Define an Neut. subset $T \subseteq \mathfrak{B} \ni$
$T = \left\{\left\langle\frac{d_1}{0.1,0.5,0.7}\right\rangle, \left\langle\frac{d_2}{0.2,0.6,0.7}\right\rangle, \left\langle\frac{d_3}{0.3,0.7,0.4}\right\rangle, \left\langle\frac{d_4}{0.2,0.4,0.6}\right\rangle\right\}$
Then $\overline{\mathfrak{N}_{\mathfrak{N}}(T)} = \left\{\left\langle\frac{d_1}{0.1,0.5,0.7}\right\rangle, \left\langle\frac{d_2,d_3}{0.3,0.7,0.4}\right\rangle, \left\langle\frac{d_4}{0.2,0.4,0.6}\right\rangle\right\}$, $\underline{\mathfrak{N}_{\mathfrak{N}}(T)} = \left\{\left\langle\frac{d_1}{0.1,0.5,0.7}\right\rangle, \left\langle\frac{d_2,d_3}{0.2,0.6,0.7}\right\rangle, \left\langle\frac{d_4}{0.2,0.4,0.6}\right\rangle\right\}$
and $B_{\mathfrak{N}_{\mathfrak{N}}}(T) = \left\{\left\langle\frac{d_1}{0.1,0.5,0.7}\right\rangle, \left\langle\frac{d_2,d_3}{0.3,0.7,0.4}\right\rangle, \left\langle\frac{d_4}{0.2,0.4,0.6}\right\rangle\right\}$.
Thus $\tau_{\mathfrak{N}_{\mathfrak{N}}}(\mathfrak{B}) = \{0_N, 1_N, \overline{\mathfrak{N}_{\mathfrak{N}}(T)}, \underline{\mathfrak{N}_{\mathfrak{N}}(T)}, B_{\mathfrak{N}_{\mathfrak{N}}}(T)\}$.
Then $k : (\mathfrak{U}, \tau_{\mathfrak{N}_{\mathfrak{N}}}(\mathfrak{U})) \to (\mathfrak{B}, \tau_{\mathfrak{N}_{\mathfrak{N}}}(\mathfrak{B}))$ is a function specified by $k(c_i) = d_i$, i = 1, 2, 3, 4. Consider the set $B = \left\{\left\langle\frac{d_1}{0.1,0.5,0.8}\right\rangle, \left\langle\frac{d_2}{0.3,0.6,0.4}\right\rangle, \left\langle\frac{d_3}{0.2,0.6,0.4}\right\rangle \left\langle\frac{d_4}{0.1,0.4,0.6}\right\rangle\right\}$
, then $k^{-1}(B) = \left\{\left\langle\frac{c_1}{0.1,0.5,0.8}\right\rangle, \left\langle\frac{c_2}{0.3,0.6,0.4}\right\rangle, \left\langle\frac{c_3}{0.2,0.6,0.4}\right\rangle \left\langle\frac{c_4}{0.1,0.4,0.6}\right\rangle\right\}$ is a $\mathfrak{N}_{\mathfrak{N}}$ β - OS but not $\mathfrak{N}_{\mathfrak{N}}$ OS.
Hence $k:(\mathfrak{U}, \tau_{\mathfrak{N}_{\mathfrak{N}}}(\mathfrak{U})) \to (\mathfrak{B}, \tau_{\mathfrak{N}_{\mathfrak{N}}}(\mathfrak{B}))$ is a $\mathfrak{N}_{\mathfrak{N}}$ β - cts function but not $\mathfrak{N}_{\mathfrak{N}}$ - cts function.

Example 3.6. Consider $\mathfrak{U} = \{c_1, c_2, c_3, c_4\}$ and $\mathfrak{U} \setminus \mathfrak{R} = \{\{c_1, c_2\}, c_3, c_4\}$. Define an Neut. subset H
$\subseteq \mathfrak{U} \ni H = \left\{\left\langle\frac{c_1}{0.2,0.4,0.5}\right\rangle, \left\langle\frac{c_2}{0.1,0.5,0.7}\right\rangle, \left\langle\frac{c_3}{0.3,0.4,0.5}\right\rangle, \left\langle\frac{c_4}{0.1,0.5,0.8}\right\rangle\right\}$
Then $\overline{\mathfrak{N}_{\mathfrak{N}}(H)} = \left\{\left\langle\frac{c_1,c_3}{0.3,0.4,0.5}\right\rangle, \left\langle\frac{c_2}{0.1,0.5,0.7}\right\rangle, \left\langle\frac{c_4}{0.1,0.5,0.8}\right\rangle\right\}$, $\underline{\mathfrak{N}_{\mathfrak{N}}(H)} = \left\{\left\langle\frac{c_1,c_3}{0.2,0.4,0.5}\right\rangle, \left\langle\frac{c_2}{0.1,0.5,0.7}\right\rangle, \left\langle\frac{c_4}{0.1,0.5,0.8}\right\rangle\right\}$
and $B_{\mathfrak{N}_{\mathfrak{N}}}(H) = \left\{\left\langle\frac{c_1,c_3}{0.3,0.4,0.5}\right\rangle, \left\langle\frac{c_2}{0.1,0.5,0.7}\right\rangle, \left\langle\frac{c_4}{0.1,0.5,0.8}\right\rangle\right\}$.
Thus $\tau_{\mathfrak{N}_{\mathfrak{N}}}(\mathfrak{U}) = \{0_N, 1_N, \overline{\mathfrak{N}_{\mathfrak{N}}(H)}, \underline{\mathfrak{N}_{\mathfrak{N}}(H)}, B_{\mathfrak{N}_{\mathfrak{N}}}(H)\}$.
Also $\mathfrak{B} = \{d_1, d_2, d_3, d_4\}$ and $\mathfrak{B} \setminus \mathfrak{R} = \{\{d_1, d_4\}, d_2, d_3\}$ Define an Neut. subset $T \subseteq \mathfrak{B} \ni$
$T = \left\{\left\langle\frac{d_1}{0.4,0.7,0.3}\right\rangle, \left\langle\frac{d_2}{0.5,0.7,0.4}\right\rangle, \left\langle\frac{d_3}{0.3,0.7,0.5}\right\rangle, \left\langle\frac{d_4}{0.3,0.6,0.4}\right\rangle\right\}$
Then $\overline{\mathfrak{N}_{\mathfrak{N}}(T)} = \left\{\left\langle\frac{d_1,d_4}{0.4,0.7,0.3}\right\rangle, \left\langle\frac{d_2}{0.5,0.7,0.4}\right\rangle, \left\langle\frac{d_3}{0.3,0.7,0.5}\right\rangle\right\}$, $\underline{\mathfrak{N}_{\mathfrak{N}}(T)} = \left\{\left\langle\frac{d_1,d_4}{0.3,0.6,0.4}\right\rangle, \left\langle\frac{d_2}{0.5,0.7,0.4}\right\rangle, \left\langle\frac{d_3}{0.3,0.7,0.5}\right\rangle\right\}$
and $B_{\mathfrak{N}_{\mathfrak{N}}}(T) = \left\{\left\langle\frac{d_1,d_4}{0.4,0.7,0.3}\right\rangle, \left\langle\frac{d_2}{0.5,0.7,0.4}\right\rangle, \left\langle\frac{d_3}{0.3,0.7,0.5}\right\rangle\right\}$.
Thus $\tau_{\mathfrak{N}_{\mathfrak{N}}}(\mathfrak{B}) = \{0_N, 1_N, \overline{\mathfrak{N}_{\mathfrak{N}}(T)}, \underline{\mathfrak{N}_{\mathfrak{N}}(T)}, B_{\mathfrak{N}_{\mathfrak{N}}}(T)\}$.
Then $k:(\mathfrak{U}, \tau_{\mathfrak{N}_{\mathfrak{N}}}(\mathfrak{U})) \to (\mathfrak{B}, \tau_{\mathfrak{N}_{\mathfrak{N}}}(\mathfrak{B}))$ is a function specified by $k(c_i) = d_i$, i = 1, 2, 3, 4.
Consider the set $B = \left\{\left\langle\frac{d_1,d_4}{0.3,0.3,0.4}\right\rangle, \left\langle\frac{d_2}{0.4,0.3,0.5}\right\rangle, \left\langle\frac{d_3}{0.5,0.3,0.3}\right\rangle\right\}$,
then $k^{-1}(B) = \left\{\left\langle\frac{c_1,c_4}{0.3,0.3,0.4}\right\rangle, \left\langle\frac{c_2}{0.4,0.3,0.5}\right\rangle, \left\langle\frac{c_3}{0.5,0.3,0.3}\right\rangle\right\}$ is a $\mathfrak{N}_{\mathfrak{N}}$ β - OS but not pre- OS.
Hence $k:(\mathfrak{U}, \tau_{\mathfrak{N}_{\mathfrak{N}}}(\mathfrak{U})) \to (\mathfrak{B}, \tau_{\mathfrak{N}_{\mathfrak{N}}}(\mathfrak{B}))$ is a $\mathfrak{N}_{\mathfrak{N}}$ β - cts function but not pre - cts function.

Example 3.7. Consider $\mathfrak{U} = \{c_1, c_2, c_3\}$ and $\mathfrak{U} \setminus \mathfrak{R} = \{\{c_1, c_2\}, c_3\}$. Define an Neut. subset H
$\subseteq \mathfrak{U} \ni H = \left\{\left\langle\frac{c_1}{0.2,0.5,0.7}\right\rangle, \left\langle\frac{c_2}{0.1,0.4,0.8}\right\rangle, \left\langle\frac{c_3}{0.4,0.5,0.6}\right\rangle\right\}$
Then $\overline{\mathfrak{N}_{\mathfrak{N}}(H)} = \left\{\left\langle\frac{c_1,c_2}{0.2,0.5,0.7}\right\rangle, \left\langle\frac{c_3}{0.4,0.5,0.6}\right\rangle\right\}$, $\underline{\mathfrak{N}_{\mathfrak{N}}(H)} = \left\{\left\langle\frac{c_1,c_2}{0.1,0.4,0.8}\right\rangle, \left\langle\frac{c_3}{0.4,0.5,0.6}\right\rangle\right\}$

and $B_{\mathfrak{N}_\mathfrak{N}}(H) = \left\{\left\langle \frac{c_1,c_2}{0.2,0.5,0.7}\right\rangle, \left\langle \frac{c_3}{0.4,0.5,0.6}\right\rangle\right\}$.
Thus $\tau_{\mathfrak{N}_\mathfrak{N}}(\mathfrak{U}) = \{0_N, 1_N, \overline{\mathfrak{N}_\mathfrak{N}(H)}, \underline{\mathfrak{N}_\mathfrak{N}(H)}, B_{\mathfrak{N}_\mathfrak{N}}(H)\}$.
Also $\mathfrak{B} = \{d_1, d_2, d_3\}$ and $\mathfrak{B} \setminus \mathfrak{R} = \{ d_1, \{d_2, d_3\}\}$. Define an Neut. subset $T \subseteq \mathfrak{B} \ni$
$T = \left\{\left\langle \frac{d_1}{0.1,0.2,0.8}\right\rangle, \left\langle \frac{d_2}{0.4,0.5,0.6}\right\rangle, \left\langle \frac{d_3}{0.3,0.5,0.7}\right\rangle, \right\}$
Then $\overline{\mathfrak{N}_\mathfrak{N}(T)} = \left\{\left\langle \frac{d_1}{0.1,0.2,0.8}\right\rangle, \left\langle \frac{d_2,d_3}{0.4,0.5,0.6}\right\rangle, \right\}$, $\underline{\mathfrak{N}_\mathfrak{N}(T)} = \left\{\left\langle \frac{d_1}{0.1,0.2,0.8}\right\rangle, \left\langle \frac{d_2,d_3}{0.3,0.5,0.7}\right\rangle\right\}$
and $B_{\mathfrak{N}_\mathfrak{N}}(T) = \left\{\left\langle \frac{d_1}{0.1,0.2,0.8}\right\rangle, \left\langle \frac{d_2,d_3}{0.4,0.5,0.6}\right\rangle\right\}$.
Thus $\tau_{\mathfrak{N}_\mathfrak{N}}(\mathfrak{B}) = \{0_N, 1_N, \overline{\mathfrak{N}_\mathfrak{N}(T)}, \underline{\mathfrak{N}_\mathfrak{N}(T)}, B_{\mathfrak{N}_\mathfrak{N}}(T)\}$.
Then $k : (\mathfrak{U}, \tau_{\mathfrak{N}_\mathfrak{N}}(\mathfrak{U})) \to (\mathfrak{B}, \tau_{\mathfrak{N}_\mathfrak{N}}(\mathfrak{B}))$ is a function specified by $k(c_i) = d_i$, $i = 1, 2, 3, 4$.
Consider the set $B = \left\{\left\langle \frac{d_1}{0.1,0.2,0.8}\right\rangle, \left\langle \frac{d_2,d_3}{0.4,0.5,0.6}\right\rangle\right\}$,
then $k^{-1}(B) = \left\langle \frac{c_1}{0.1,0.2,0.8}\right\rangle, \left\langle \frac{c_2,c_3}{0.4,0.5,0.6}\right\rangle$ is a $\mathfrak{N}_\mathfrak{N}$ β - OS but not $\mathfrak{N}_\mathfrak{N}$ semi -OS .
Hence $k:(\mathfrak{U}, \tau_{\mathfrak{N}_\mathfrak{N}}(\mathfrak{U})) \to (\mathfrak{B}, \tau_{\mathfrak{N}_\mathfrak{N}}(\mathfrak{B}))$ is a $\mathfrak{N}_\mathfrak{N}$ β -cts function but not $\mathfrak{N}_\mathfrak{N}$ semi - cts function.

Example 3.8. Consider $\mathfrak{U} = \{c_1, c_2, c_3\}$ and $\mathfrak{U} \setminus \mathfrak{R} = \{\{c_1, c_2\}, c_3\}$. Define an Neut. subset $H \subseteq \mathfrak{U}$ $\ni H = \left\{\left\langle \frac{c_1}{0.2,0.5,0.8}\right\rangle, \left\langle \frac{c_2}{0.3,0.5,0.6}\right\rangle, \left\langle \frac{c_3}{0.4,0.5,0.9}\right\rangle\right\}$
Then $\overline{\mathfrak{N}_\mathfrak{N}(H)} = \left\{\left\langle \frac{c_1,c_2}{0.3,0.5,0.6}\right\rangle, \left\langle \frac{c_3}{0.4,0.5,0.9}\right\rangle\right\}$, $\underline{\mathfrak{N}_\mathfrak{N}(H)} = \left\{\left\langle \frac{c_1,c_2}{0.2,0.5,0.8}\right\rangle, \left\langle \frac{c_3}{0.4,0.5,0.9}\right\rangle\right\}$
and $B_{\mathfrak{N}_\mathfrak{N}}(H) = \left\{\left\langle \frac{c_1,c_2}{0.3,0.5,0.6}\right\rangle, \left\langle \frac{c_3}{0.4,0.5,0.9}\right\rangle\right\}$.
Thus $\tau_{\mathfrak{N}_\mathfrak{N}}(\mathfrak{U}) = \{0_N, 1_N, \overline{\mathfrak{N}_\mathfrak{N}(H)}, \underline{\mathfrak{N}_\mathfrak{N}(H)}, B_{\mathfrak{N}_\mathfrak{N}}(H)\}$.
Also $\mathfrak{B} = \{d_1, d_2, d_3\}$ and $\mathfrak{B} \setminus \mathfrak{R} = \{ d_1, \{d_2, d_3\}\}$.
Define an Neut. subset $T \subseteq \mathfrak{B} \ni$
$T = \left\{\left\langle \frac{d_1}{0.4,0.5,0.8}\right\rangle, \left\langle \frac{d_2}{0.2,0.5,0.1}\right\rangle, \left\langle \frac{d_3}{0.1,0.5,0.2}\right\rangle, \right\}$
Then $\overline{\mathfrak{N}_\mathfrak{N}(T)} = \left\{\left\langle \frac{d_1}{0.4,0.5,0.8}\right\rangle, \left\langle \frac{d_2,d_3}{0.2,0.5,0.1}\right\rangle, \right\}$, $\underline{\mathfrak{N}_\mathfrak{N}(T)} = \left\{\left\langle \frac{d_1}{0.4,0.5,0.8}\right\rangle, \left\langle \frac{d_2,d_3}{0.1,0.5,0.2}\right\rangle\right\}$
and $B_{\mathfrak{N}_\mathfrak{N}}(T) = \left\{\left\langle \frac{d_1}{0.4,0.5,0.8}\right\rangle, \left\langle \frac{d_2,d_3}{0.2,0.5,0.1}\right\rangle\right\}$.
Thus $\tau_{\mathfrak{N}_\mathfrak{N}}(\mathfrak{B}) = \{0_N, 1_N, \overline{\mathfrak{N}_\mathfrak{N}(T)}, \underline{\mathfrak{N}_\mathfrak{N}(T)}, B_{\mathfrak{N}_\mathfrak{N}}(T)\}$.
Then $k : (\mathfrak{U}, \tau_{\mathfrak{N}_\mathfrak{N}}(\mathfrak{U})) \to (\mathfrak{B}, \tau_{\mathfrak{N}_\mathfrak{N}}(\mathfrak{B}))$ is a function specified by $k(c_i) = d_i$, $i = 1, 2, 3, 4$.
Consider the set $B = \left\langle \frac{d_1}{0.4,0.5,0.8}\right\rangle, \left\langle \frac{d_2,d_3}{0.2,0.5,0.1}\right\rangle$,
then $k^{-1}(B) = \left\langle \frac{c_1}{0.4,0.5,0.8}\right\rangle, \left\langle \frac{c_2,c_3}{0.2,0.5,0.1}\right\rangle$ is a $\mathfrak{N}_\mathfrak{N}$ β - OS but not $\mathfrak{N}_\mathfrak{N}$ α -OS .
Hence $k:(\mathfrak{U}, \tau_{\mathfrak{N}_\mathfrak{N}}(\mathfrak{U})) \to (\mathfrak{B}, \tau_{\mathfrak{N}_\mathfrak{N}}(\mathfrak{B}))$ is a $\mathfrak{N}_\mathfrak{N}$ β -cts function but not $\mathfrak{N}_\mathfrak{N}$ α - cts function.

Example 3.9. Consider $\mathfrak{U} = \{c_1, c_2, c_3, c_4\}$ and $\mathfrak{U} \setminus \mathfrak{R} = \{\{c_1, c_3\}, c_2, c_4\}$. Define an Neut. subset $H \subseteq \mathfrak{U} \ni$
$H = \left\{\left\langle \frac{c_1}{0.2,0.4,0.5}\right\rangle, \left\langle \frac{c_2}{0.1,0.5,0.7}\right\rangle, \left\langle \frac{c_3}{0.3,0.4,0.5}\right\rangle, \left\langle \frac{c_4}{0.1,0.5,0.8}\right\rangle\right\}$
Then $\overline{\mathfrak{N}_\mathfrak{N}(H)} = \left\{\left\langle \frac{c_1,c_3}{0.3,0.4,0.5}\right\rangle, \left\langle \frac{c_2}{0.1,0.5,0.7}\right\rangle, \left\langle \frac{c_4}{0.1,0.5,0.8}\right\rangle\right\}$, $\underline{\mathfrak{N}_\mathfrak{N}(H)} = \left\{\left\langle \frac{c_1,c_3}{0.2,0.4,0.5}\right\rangle, \left\langle \frac{c_2}{0.1,0.5,0.7}\right\rangle, \left\langle \frac{c_4}{0.1,0.5,0.8}\right\rangle\right\}$
and $B_{\mathfrak{N}_\mathfrak{N}}(H) = \left\{\left\langle \frac{c_1,c_3}{0.3,0.4,0.5}\right\rangle, \left\langle \frac{c_2}{0.1,0.5,0.7}\right\rangle, \left\langle \frac{c_4}{0.1,0.5,0.8}\right\rangle\right\}$.
Thus $\tau_{\mathfrak{N}_\mathfrak{N}}(\mathfrak{U}) = \{0_N, 1_N, \overline{\mathfrak{N}_\mathfrak{N}(H)}, \underline{\mathfrak{N}_\mathfrak{N}(H)}, B_{\mathfrak{N}_\mathfrak{N}}(H)\}$.
Also $\mathfrak{B} = \{d_1, d_2, d_3, d_4\}$ and $\mathfrak{B} \setminus \mathfrak{R} = \{ d_1, d_2, \{d_3, d_4\}\}$.
Define an Neut. subset $T \subseteq \mathfrak{B} \ni$
$T = \left\{\left\langle \frac{d_1}{0.1,0.5,0.6}\right\rangle, \left\langle \frac{d_2}{0.4,0.5,0.7}\right\rangle, \left\langle \frac{d_3}{0.3,0.5,0.9}\right\rangle, \left\langle \frac{d_4}{0.4,0.5,0.7}\right\rangle\right\}$
Then $\overline{\mathfrak{N}_\mathfrak{N}(T)} = \left\{\left\langle \frac{d_1}{0.1,0.5,0.6}\right\rangle, \left\langle \frac{d_2}{0.4,0.5,0.7}\right\rangle, \left\langle \frac{d_3,d_4}{0.4,0.5,0.7}\right\rangle\right\}$, $\underline{\mathfrak{N}_\mathfrak{N}(T)} = \left\{\left\langle \frac{d_1}{0.1,0.5,0.6}\right\rangle, \left\langle \frac{d_2}{0.4,0.5,0.7}\right\rangle, \left\langle \frac{d_3,d_4}{0.3,0.5,0.9}\right\rangle\right\}$,
and $B_{\mathfrak{N}_\mathfrak{N}}(T) = \left\{\left\langle \frac{d_1}{0.1,0.5,0.6}\right\rangle, \left\langle \frac{d_2}{0.4,0.5,0.7}\right\rangle, \left\langle \frac{d_3,d_4}{0.4,0.5,0.7}\right\rangle\right\}$.
Thus $\tau_{\mathfrak{N}_\mathfrak{N}}(\mathfrak{B}) = \{0_N, 1_N, \overline{\mathfrak{N}_\mathfrak{N}(T)}, \underline{\mathfrak{N}_\mathfrak{N}(T)}, B_{\mathfrak{N}_\mathfrak{N}}(T)\}$.
Then $k : (\mathfrak{U}, \tau_{\mathfrak{N}_\mathfrak{N}}(\mathfrak{U})) \to (\mathfrak{B}, \tau_{\mathfrak{N}_\mathfrak{N}}(\mathfrak{B}))$ is a function specified by $k(c_i) = d_i$, $i = 1, 2, 3, 4$.

Consider the set $B = \left\{\left\langle \frac{d_1}{0.1,0.5,0.6}\right\rangle, \left\langle \frac{d_2}{0.4,0.5,0.7}\right\rangle, \left\langle \frac{d_3,d_4}{0.3,0.5,0.9}\right\rangle\right\}$,

then $k^{-1}(B) = \left\{\left\{\left\langle \frac{c_1}{0.1,0.5,0.6}\right\rangle, \left\langle \frac{c_2}{0.4,0.5,0.7}\right\rangle, \left\langle \frac{c_3,c_4}{0.3,0.5,0.9}\right\rangle\right\}\right\}$ is a $\mathfrak{N}_{\mathfrak{N}}$ β - OS but not $\mathfrak{N}_{\mathfrak{N}}$ regular OS .

Hence k:($\mathfrak{U}$, $\tau_{\mathfrak{N}_{\mathfrak{N}}}(\mathfrak{U})) \to (\mathfrak{B}$, $\tau_{\mathfrak{N}_{\mathfrak{N}}}(\mathfrak{B}))$ is a $\mathfrak{N}_{\mathfrak{N}}$ β -cts function but not $\mathfrak{N}_{\mathfrak{N}}$ - regular cts function.

Example 3.10. Consider $\mathfrak{U} = \{c_1, c_2, c_3\}$ and $\mathfrak{U} \setminus \mathfrak{R} = \{\{c_1, c_2\}, c_3\}$. Define an Neut. subset $H \subseteq \mathfrak{U} \ni$

$H = \left\{\left\langle \frac{c_1}{0.2,0.5,0.8}\right\rangle, \left\langle \frac{c_2}{0.3,0.5,0.6}\right\rangle, \left\langle \frac{c_3}{0.4,0.5,0.9}\right\rangle\right\}$

Then $\overline{\mathfrak{N}_{\mathfrak{N}}(H)} = \left\{\left\langle \frac{c_1,c_2}{0.3,0.5,0.6}\right\rangle, \left\langle \frac{c_3}{0.4,0.5,0.9}\right\rangle\right\}$, $\underline{\mathfrak{N}_{\mathfrak{N}}(H)} = \left\{\left\langle \frac{c_1,c_2}{0.2,0.5,0.8}\right\rangle, \left\langle \frac{c_3}{0.4,0.5,0.9}\right\rangle\right\}$

and $B_{\mathfrak{N}_{\mathfrak{N}}}(H) = \left\{\left\langle \frac{c_1,c_2}{0.3,0.5,0.6}\right\rangle, \left\langle \frac{c_3}{0.4,0.5,0.9}\right\rangle\right\}$.

Thus $\tau_{\mathfrak{N}_{\mathfrak{N}}}(\mathfrak{U}) = \{0_N, 1_N, \overline{\mathfrak{N}_{\mathfrak{N}}(H)}, \underline{\mathfrak{N}_{\mathfrak{N}}(H)}, B_{\mathfrak{N}_{\mathfrak{N}}}(H)\}$.

Also $\mathfrak{B} = \{d_1, d_2, d_3\}$ and $\mathfrak{B}\ \mathfrak{R} = \{\ \overline{d_1,\ \{d_2}, d_3\}\}$. Define an Neut. subset $T \subseteq \mathfrak{B} \ni$

$T = \left\{\left\langle \frac{d_1}{0.3,0.2,0.8}\right\rangle, \left\langle \frac{d_2}{0.4,0.3,0.9}\right\rangle, \left\langle \frac{d_3}{0.5,0.3,0.9}\right\rangle, \right\}$

Then $\overline{\mathfrak{N}_{\mathfrak{N}}(T)} = \left\{\left\langle \frac{d_1}{0.3,0.2,0.8}\right\rangle, \left\langle \frac{d_2,d_3}{0.5,0.3,0.9}\right\rangle, \right\}$, $\underline{\mathfrak{N}_{\mathfrak{N}}(T)} = \left\{\left\langle \frac{d_1}{0.3,0.2,0.8}\right\rangle, \left\langle \frac{d_2,d_3}{0.4,0.3,0.9}\right\rangle\right\}$

and $B_{\mathfrak{N}_{\mathfrak{N}}}(T) = \left\{\left\langle \frac{d_1}{0.3,0.2,0.8}\right\rangle, \left\langle \frac{d_2,d_3}{0.5,0.3,0.9}\right\rangle\right\}$.

Thus $\tau_{\mathfrak{N}_{\mathfrak{N}}}(\mathfrak{B}) = \{0_N, 1_N, \overline{\mathfrak{N}_{\mathfrak{N}}(T)}, \underline{\mathfrak{N}_{\mathfrak{N}}(T)}, B_{\mathfrak{N}_{\mathfrak{N}}}(T)\}$.

Then $k : (\mathfrak{U},\ \tau_{\mathfrak{N}_{\mathfrak{N}}}(\mathfrak{U})) \to (\mathfrak{B}, \tau_{\mathfrak{N}_{\mathfrak{N}}}(\mathfrak{B}))$ is a function specified by $k(c_i) = d_i$, i = 1, 2, 3, 4.

Consider the set $B = \left\{\left\langle \frac{d_1}{0.3,0.2,0.8}\right\rangle, \left\langle \frac{d_2,d_3}{0.4,0.3,0.9}\right\rangle\right\}$,

then $k^{-1}(B) = \left\langle \frac{c_1}{0.3,0.2,0.8}\right\rangle, \left\langle \frac{c_2,c_3}{0.4,0.3,0.9}\right\rangle$ is a $\mathfrak{N}_{\mathfrak{N}}$ β - OS but not $\mathfrak{N}_{\mathfrak{N}}$ δ -OS .

Hence k:($\mathfrak{U}$, $\tau_{\mathfrak{N}_{\mathfrak{N}}}(\mathfrak{U})) \to (\mathfrak{B}$, $\tau_{\mathfrak{N}_{\mathfrak{N}}}(\mathfrak{B}))$ is a $\mathfrak{N}_{\mathfrak{N}}$ β -cts function but not $\mathfrak{N}_{\mathfrak{N}}$ δ - cts function.

Example 3.11. Consider $\mathfrak{U} = \{c_1, c_2, c_3\}$ and $\mathfrak{U}\mathfrak{R} = \{\{c_1, c_3\}, c_2\}$. Define an Neut. subset $H \subseteq \mathfrak{U} \ni$

$H = \left\{\left\langle \frac{c_1}{0.4,0.7,0.2}\right\rangle, \left\langle \frac{c_2}{0.4,0.7,0.4}\right\rangle, \left\langle \frac{c_3}{0.3,0.7,0.5}\right\rangle\right\}$

Then $\overline{\mathfrak{N}_{\mathfrak{N}}(H)} = \left\{\left\langle \frac{c_1,c_3}{0.4,0.7,0.2}\right\rangle, \left\langle \frac{c_2}{0.4,0.7,0.4}\right\rangle\right\}$, $\underline{\mathfrak{N}_{\mathfrak{N}}(H)} = \left\{\left\langle \frac{c_1,c_3}{0.3,0.7,0.5}\right\rangle, \left\langle \frac{c_2}{0.4,0.7,0.4}\right\rangle\right\}$

and $B_{\mathfrak{N}_{\mathfrak{N}}}(H) = \left\{\left\langle \frac{c_1,c_3}{0.4,0.7,0.2}\right\rangle, \left\langle \frac{c_2}{0.4,0.7,0.4}\right\rangle\right\}$.

Thus $\tau_{\mathfrak{N}_{\mathfrak{N}}}(\mathfrak{U}) = \{0_N, 1_N, \overline{\mathfrak{N}_{\mathfrak{N}}(H)}, \underline{\mathfrak{N}_{\mathfrak{N}}(H)}, B_{\mathfrak{N}_{\mathfrak{N}}}(H)\}$.

Also $\mathfrak{B} = \{d_1, d_2, d_3\}$ and $\mathfrak{B} \setminus \mathfrak{R} = \overline{\{\{d_1,\ d_2}\}, d_3\}$. Define an Neut. subset $T \subseteq \mathfrak{B} \ni$

$T = \left\{\left\langle \frac{d_1}{0.2,0.5,0.7}\right\rangle, \left\langle \frac{d_2}{0.1,0.4,0.8}\right\rangle, \left\langle \frac{d_3}{0.4,0.5,0.6}\right\rangle, \right\}$

Then $\overline{\mathfrak{N}_{\mathfrak{N}}(T)} = \left\{\left\langle \frac{d_1,d_2}{0.2,0.5,0.7}\right\rangle, \left\langle \frac{d_3}{0.4,0.5,0.6}\right\rangle\right\}$, $\underline{\mathfrak{N}_{\mathfrak{N}}(T)} = \left\{\left\langle \frac{d_1,d_2}{0.1,0.4,0.8}\right\rangle, \left\langle \frac{d_3}{0.4,0.5,0.6}\right\rangle\right\}$

and $B_{\mathfrak{N}_{\mathfrak{N}}}(T) = \left\{\left\langle \frac{d_1,d_2}{0.2,0.5,0.7}\right\rangle, \left\langle \frac{d_3}{0.4,0.5,0.6}\right\rangle\right\}$

Thus $\tau_{\mathfrak{N}_{\mathfrak{N}}}(\mathfrak{B}) = \{0_N, 1_N, \overline{\mathfrak{N}_{\mathfrak{N}}(T)}, \underline{\mathfrak{N}_{\mathfrak{N}}(T)}, B_{\mathfrak{N}_{\mathfrak{N}}}(T)\}$.

Then $k : (\mathfrak{U},\ \tau_{\mathfrak{N}_{\mathfrak{N}}}(\mathfrak{U})) \to (\mathfrak{B}$, $\tau_{\mathfrak{N}_{\mathfrak{N}}}(\mathfrak{B}))$ is a function specified by $k(c_i) = d_i$, i = 1, 2, 3, 4.

Consider the set $B = \left\{\left\langle \frac{d_1,d_2}{0.7,0.5,0.2}\right\rangle, \left\langle \frac{d_3}{0.6,0.5,0.4}\right\rangle\right\}$, then $k^{-1}(B) = \left\langle \frac{c_1,c_2}{0.7,0.5,0.2}\right\rangle, \left\langle \frac{c_3}{0.6,0.5,0.4}\right\rangle$ is a $\mathfrak{N}_{\mathfrak{N}}$ β - OS but not $\mathfrak{N}_{\mathfrak{N}}$ g -OS .

Hence k:($\mathfrak{U}$, $\tau_{\mathfrak{N}_{\mathfrak{N}}}(\mathfrak{U})) \to (\mathfrak{B}$, $\tau_{\mathfrak{N}_{\mathfrak{N}}}(\mathfrak{B}))$ is a $\mathfrak{N}_{\mathfrak{N}}$ β -cts function but not $\mathfrak{N}_{\mathfrak{N}}$ g - cts function.

Theorem 3.12. *A function k:($\mathfrak{U}$, $\tau_{\mathfrak{N}_{\mathfrak{N}}}(\mathfrak{U})) \to (\mathfrak{B}$, $\tau_{\mathfrak{N}_{\mathfrak{N}}}(\mathfrak{B}))$ is $\mathfrak{N}_{\mathfrak{N}}$ β - cts function ⇔ the inverse image of every $\mathfrak{N}_{\mathfrak{N}}$ closed in $\mathfrak{B}$ is $\mathfrak{N}_{\mathfrak{N}}$ β - closed in $\mathfrak{U}$.*

Proof. Let k be a $\mathfrak{N}_{\mathfrak{N}}$ β -cts and G is $\mathfrak{N}_{\mathfrak{N}}$ OS in $\mathfrak{B} \Longrightarrow \mathfrak{B}$ - G is $\mathfrak{N}_{\mathfrak{N}}$ closed in $\mathfrak{B}$. As k is $\mathfrak{N}_{\mathfrak{N}}$ β - cts , $k^{-1}(G^c)$ is $\mathfrak{N}_{\mathfrak{N}}$ β-open in $\mathfrak{U} \Longrightarrow [k^{-1}(G)]^c$ is $\mathfrak{N}_{\mathfrak{N}}$ β-open in $\mathfrak{U}$. Hence $k^{-1}(G)$ is $\mathfrak{N}_{\mathfrak{N}}$ β-closed in $\mathfrak{U}$. Conversely, let the inverse image of every $\mathfrak{N}_{\mathfrak{N}}$ closed is $\mathfrak{N}_{\mathfrak{N}}$ β - closed. Let P be $\mathfrak{N}_{\mathfrak{N}}$ open in $\mathfrak{B} \Longrightarrow \mathfrak{B}$-P is $\mathfrak{N}_{\mathfrak{N}}$- closed in $\mathfrak{B} \Longrightarrow k^{-1}(P^c)$ is $\mathfrak{N}_{\mathfrak{N}}$- closed in $\mathfrak{U}$, Then by our assumption $k^{-1}(P^c)$ is $\mathfrak{N}_{\mathfrak{N}}$ β - closed in $\mathfrak{U} \Longrightarrow [k^{-1}(P)]^c$ is $\mathfrak{N}_{\mathfrak{N}}$ β - closed in $\mathfrak{U}$. Hence $k^{-1}(P)$ is $\mathfrak{N}_{\mathfrak{N}}$ β - open in $\mathfrak{U}$ which dictates k is a $\mathfrak{N}_{\mathfrak{N}}$ β - cts function. □

Theorem 3.13. *Let k : ($\mathfrak{U}$, $\tau_{\mathfrak{N}_{\mathfrak{N}}}(\mathfrak{U})) \to (\mathfrak{B}$, $\tau_{\mathfrak{N}_{\mathfrak{N}}}(\mathfrak{B}))$ be a function. Then*

(i) *k is* $\mathfrak{N}_\mathfrak{N}$ β - cts .
(ii) *For each u in* $\mathfrak{U}$ *and each* $\mathfrak{N}_\mathfrak{N}$ *OS N containing k(u), there is a* $\mathfrak{N}_\mathfrak{N}$ β *OS M containing u such that* $k(M) \subseteq N$.
(iii) $k(\mathfrak{N}_\mathfrak{N}\ \beta\ cl(M)) \subseteq \mathfrak{N}_\mathfrak{N}\ cl(k(M))$ *for each subset M of* $\mathfrak{U}$.
(iv) $\mathfrak{N}_\mathfrak{N}$ β *cl(*k^{-1}*(N))* $\subseteq \mathfrak{N}_\mathfrak{N}$ *cl(*k^{-1}*(N)) for each subset N of* $\mathfrak{B}$.

Proof. (i)$\Longrightarrow$(ii) Let u $\in \mathfrak{U}$ and N be $\mathfrak{N}_\mathfrak{N}$ OS containing k(u). Then by (i) k^{-1}(N) is $\mathfrak{N}_\mathfrak{N}$ β OS of $\mathfrak{U}$ containing u. If M = k^{-1}(N) , then k(M) = k(k^{-1}(N)) $\subseteq$ N .

(ii)$\Longrightarrow$(iii) Let M $\subseteq \mathfrak{U}$ and k(u) $\notin \mathfrak{N}_\mathfrak{N}$ cl (k(M)). Then $\exists$ a $\mathfrak{N}_\mathfrak{N}$ OS N of $\mathfrak{B}$ containing k(u) $\ni$ N $\cap$ k(M) = $\emptyset$. Now by (ii), there is a $\mathfrak{N}_\mathfrak{N}$ β OS G containing u $\ni$ k(u) $\in$ k(G) $\subseteq$ N. Hence k(M) $\cap$ k(N) = $\emptyset$. i. e. , k(M $\cap$ N) = $\emptyset$ which dictates M $\cap$ N = $\emptyset$. Therefore, u $\notin \mathfrak{N}_\mathfrak{N}$ β cl(k(M)). Therefore k($\mathfrak{N}_\mathfrak{N}$ β cl(M)) $\subseteq \mathfrak{N}_\mathfrak{N}$ cl(k(M)).

(iii)$\Longrightarrow$(iv) Let N $\subseteq \mathfrak{B} \ni$ M = k^{-1}(N). By (iii), k($\mathfrak{N}_\mathfrak{N}$ β cl(M)) $\subseteq \mathfrak{N}_\mathfrak{N}$ cl(k(M)) for each subset M of $\mathfrak{U}$. Therefore $\mathfrak{N}_\mathfrak{N}$ β cl(k^{-1}(N)) $\subseteq \mathfrak{N}_\mathfrak{N}$ cl(k(k^{-1}(N)). i. e. , k($\mathfrak{N}_\mathfrak{N}$ β cl(k^{-1}(N)) $\subseteq \mathfrak{N}_\mathfrak{N}$ cl(N) which dictates $\mathfrak{N}_\mathfrak{N}$ β cl(k^{-1}(N)) $\subseteq k^{-1}$($\mathfrak{N}_\mathfrak{N}$ cl(N)). □

Theorem 3.14. *A function k is* $\mathfrak{N}_\mathfrak{N}$ β cts- $\Leftrightarrow$ *k(*$\mathfrak{N}_\mathfrak{N}$ β *cl (G))* $\subseteq \mathfrak{N}_\mathfrak{N}$β *cl (k (G)) for each subset G of* $\mathfrak{B}$.

Proof. Let G refers a $\mathfrak{N}_\mathfrak{N}$ OS in $\mathfrak{B}$ and k be $\mathfrak{N}_\mathfrak{N}$ β - cts function . Clearly k(G) $\subseteq$ k($\mathfrak{N}_\mathfrak{N}$ β cl (G)). Now G $\subseteq k^{-1}$(k(G)) $\subseteq k^{-1}$(k($\mathfrak{N}_\mathfrak{N}$ β cl (G)). Then $\mathfrak{N}_\mathfrak{N}$β cl (G) $\subseteq \mathfrak{N}_\mathfrak{N}$β cl ($k^{-1}$($\mathfrak{N}_\mathfrak{N}$ β cl(k(G))). As k is $\mathfrak{N}_\mathfrak{N}$ β - cts function and $\mathfrak{N}_\mathfrak{N}$ β cl(G) is $\mathfrak{N}_\mathfrak{N}$ β - CS. Thus $\mathfrak{N}_\mathfrak{N}$β cl ($k^{-1}$($\mathfrak{N}_\mathfrak{N}$ β cl (k(G)))) = k^{-1}($\mathfrak{N}_\mathfrak{N}$ β cl (k(G))). Therefore, k($\mathfrak{N}_\mathfrak{N}$ β cl (G)) $\subseteq \mathfrak{N}_\mathfrak{N}$β cl (k (G)).

Conversely , k($\mathfrak{N}_\mathfrak{N}$ β cl (G)) $\subseteq \mathfrak{N}_\mathfrak{N}$β cl (k (G)) for each subset G of $\mathfrak{B}$. Let M refers a $\mathfrak{N}_\mathfrak{N}$ β- CS in $\mathfrak{B}$. As every $\mathfrak{N}_\mathfrak{N}$ CS is $\mathfrak{N}_\mathfrak{N}$ β - CS, $\mathfrak{N}_\mathfrak{N}$β cl (k($k^{-1}$(M))) $\subseteq \mathfrak{N}_\mathfrak{N}$ β cl (M) = M. Then by our assumption, k ($\mathfrak{N}_\mathfrak{N}$ β cl (k^{-1}(k (M)) $\subseteq \mathfrak{N}_\mathfrak{N}$β cl ($k^{-1}$(k (M)) $\subseteq$ M which dictates $\mathfrak{N}_\mathfrak{N}$β cl ($k^{-1}$ (M)) $\subseteq k^{-1}$(M). But $\mathfrak{N}_\mathfrak{N}$β cl ($k^{-1}$ (M)) $\supseteq k^{-1}$(M) dictates that $\mathfrak{N}_\mathfrak{N}$β cl ($k^{-1}$ (M)) = k^{-1}(M). Hence k^{-1} (M) is $\mathfrak{N}_\mathfrak{N}$ β - CS in $\mathfrak{U}$. Thus by theorem 3. 12, k is $\mathfrak{N}_\mathfrak{N}$ β - cts function. □

Theorem 3.15. *Let k be a bijective function, then k is* $\mathfrak{N}_\mathfrak{N}$ β - cts *function* $\Leftrightarrow \mathfrak{N}_\mathfrak{N}$ β *int(k (N))* $\subseteq$ k ($\mathfrak{N}_\mathfrak{N}$ β *int(N)) for each subset N of* $\mathfrak{B}$.

Proof. Let N refers a $\mathfrak{N}_\mathfrak{N}$ set in $\mathfrak{B}$ and k be a bijective and $\mathfrak{N}_\mathfrak{N}$ β - cts function . Let mapping k(N) = M, Clearly, k^{-1}($\mathfrak{N}_\mathfrak{N}$ β int(N)) $\subseteq k^{-1}$(N). As k is an injective mapping k^{-1}(M) = N, so that k^{-1}($\mathfrak{N}_\mathfrak{N}$ β int(M)) $\subseteq$ N. Therefore $\mathfrak{N}_\mathfrak{N}$ β int(k^{-1}($\mathfrak{N}_\mathfrak{N}$ β int(M)) $\subseteq \mathfrak{N}_\mathfrak{N}$ β int(N). As k is $\mathfrak{N}_\mathfrak{N}$ β - cts , k^{-1}($\mathfrak{N}_\mathfrak{N}$ β int(M) is $\mathfrak{N}_\mathfrak{N}$ β-OS in $\mathfrak{U}$, and k^{-1}($\mathfrak{N}_\mathfrak{N}$ β int(M)) $\subseteq \mathfrak{N}_\mathfrak{N}$ β int(N) $\Longrightarrow$ k(k^{-1}($\mathfrak{N}_\mathfrak{N}$ β int(M))) $\subseteq$ k($\mathfrak{N}_\mathfrak{N}$ β int(N)). Hence $\mathfrak{N}_\mathfrak{N}$ β int(k (N)) $\subseteq$ k ($\mathfrak{N}_\mathfrak{N}$ β int(N)).

Conversely, $\mathfrak{N}_\mathfrak{N}$ β int(k (N)) $\subseteq$ k ($\mathfrak{N}_\mathfrak{N}$ β int(N)) for each $\mathfrak{N}_\mathfrak{N}$ set N in $\mathfrak{B}$. Let D be a $\mathfrak{N}_\mathfrak{N}$ OS in $\mathfrak{B}$. Then D is $\mathfrak{N}_\mathfrak{N}$ β - open in $\mathfrak{B}$. As k is surjective, and so D = $\mathfrak{N}_\mathfrak{N}$ β int(D) =$\mathfrak{N}_\mathfrak{N}$ β int(k(k^{-1} (D)) $\subseteq$ k ($\mathfrak{N}_\mathfrak{N}$ β int(k^{-1} (D)) Then k^{-1}(D) = $\mathfrak{N}_\mathfrak{N}$ β int(k^{-1}(D)) $\subseteq k^{-1}$($\mathfrak{N}_\mathfrak{N}$ β int(D)) $\Longrightarrow k^{-1}$(D) $\subseteq k^{-1}$($\mathfrak{N}_\mathfrak{N}$ β int(D)) $\Longrightarrow$ k^{-1}(D) $\subseteq \mathfrak{N}_\mathfrak{N}$ β int(k^{-1}(D)), Thus k^{-1}(D) is $\mathfrak{N}_\mathfrak{N}$ β - OS in $\mathfrak{U}$. Hence k is $\mathfrak{N}_\mathfrak{N}$ β - cts function. □

Theorem 3.16. *A function k is* $\mathfrak{N}_\mathfrak{N}$ β -cts *function* $\Leftrightarrow k^{-1}$($\mathfrak{N}_\mathfrak{N}$ β *int(N))* $\subseteq \mathfrak{N}_\mathfrak{N}$ β *int(*k^{-1}*(N)) for each subset N of* $\mathfrak{B}$.

Proof. Let N refers a $\mathfrak{N}_\mathfrak{N}$ set in $\mathfrak{B}$ and k be a $\mathfrak{N}_\mathfrak{N}$ β - cts function . Clearly, k^{-1}($\mathfrak{N}_\mathfrak{N}$ β int(N)) $\subseteq k^{-1}$(N) $\Longrightarrow \mathfrak{N}_\mathfrak{N}$ β int(k^{-1}($\mathfrak{N}_\mathfrak{N}$ β int(N)) $\subseteq \mathfrak{N}_\mathfrak{N}$ β int(k^{-1}(N)). As $\mathfrak{N}_\mathfrak{N}$ β int(N) is is $\mathfrak{N}_\mathfrak{N}$ β - OS in $\mathfrak{B}$ and k is $\mathfrak{N}_\mathfrak{N}$ β - cts function, k^{-1}($\mathfrak{N}_\mathfrak{N}$ β int(N)) is $\mathfrak{N}_\mathfrak{N}$ β - OS in $\mathfrak{U}$. Hence $\mathfrak{N}_\mathfrak{N}$ β int(k^{-1}($\mathfrak{N}_\mathfrak{N}$ β int(N)) $\subseteq k^{-1}$($\mathfrak{N}_\mathfrak{N}$ β int(N)) $\subseteq \mathfrak{N}_\mathfrak{N}$ β int(k^{-1}(N)).

Conversely, k^{-1}($\mathfrak{N}_\mathfrak{N}$ β int(N)) $\subseteq \mathfrak{N}_\mathfrak{N}$ β int(k^{-1}(N)) $\subseteq k^{-1}$($\mathfrak{N}_\mathfrak{N}$ β int(N)) for each $\mathfrak{N}_\mathfrak{N}$ set N in $\mathfrak{B}$. Let D be a $\mathfrak{N}_\mathfrak{N}$ β - OS in $\mathfrak{B}$. Then k^{-1}(D) = $\mathfrak{N}_\mathfrak{N}$ β int(k^{-1}(D)) $\subseteq k^{-1}$($\mathfrak{N}_\mathfrak{N}$ β int(D)) $\Longrightarrow k^{-1}$(D) $\subseteq k^{-1}$($\mathfrak{N}_\mathfrak{N}$ β int(D)) , but k^{-1}(D) $\supseteq k^{-1}$($\mathfrak{N}_\mathfrak{N}$ β int(D)), hence k^{-1}(D) = k^{-1}($\mathfrak{N}_\mathfrak{N}$ β int(D)). Thus k^{-1}(D) is $\mathfrak{N}_\mathfrak{N}$ β - OS in $\mathfrak{U}$. Hence k is $\mathfrak{N}_\mathfrak{N}$ β - cts function. □

Theorem 3.17. *A function k is* $\mathfrak{N}_\mathfrak{N}$ β - cts $\Leftrightarrow \mathfrak{N}_\mathfrak{N}$β *cl (*$k^{-1}$*(H))* $\subseteq k^{-1}$*(*$\mathfrak{N}_\mathfrak{N}$ β *cl (H)) for each subset of* $\mathfrak{B}$.

Proof. Let H refers a $\mathfrak{N}_\mathfrak{N}$ OS in $\mathfrak{B}$ and k be $\mathfrak{N}_\mathfrak{N}$ β - cts function . Clearly $k^{-1}(H) \subseteq k^{-1}(\mathfrak{N}_\mathfrak{N}\ \beta\ cl\ (H))$. Then $\mathfrak{N}_\mathfrak{N}\beta\ cl\ (k^{-1}(H)) \subseteq \mathfrak{N}_\mathfrak{N}\beta\ cl\ (k^{-1}(\mathfrak{N}_\mathfrak{N}\ \beta\ cl\ (H)))$ As $\mathfrak{N}_\mathfrak{N}\ \beta$ cl(H) is $\mathfrak{N}_\mathfrak{N}$ β - CS. Thus $\mathfrak{N}_\mathfrak{N}\beta\ cl\ (k^{-1}(\mathfrak{N}_\mathfrak{N}\ \beta\ cl\ (H))) = k^{-1}(\mathfrak{N}_\mathfrak{N}\ \beta\ cl\ (H))$. therefore, $\mathfrak{N}_\mathfrak{N}\beta\ cl\ (k^{-1}(H)) \subseteq k^{-1}(\mathfrak{N}_\mathfrak{N}\ \beta\ cl\ (H))$. Conversely , $\mathfrak{N}_\mathfrak{N}\beta\ cl\ (k^{-1}(H)) \subseteq k^{-1}(\mathfrak{N}_\mathfrak{N}\ \beta\ cl\ (H))$ for each subset H of $\mathfrak{B}$.

Let M refers a $\mathfrak{N}_\mathfrak{N}$ β- CS in $\mathfrak{B}$. As every $\mathfrak{N}_\mathfrak{N}$ CS is $\mathfrak{N}_\mathfrak{N}$ β - CS, $\mathfrak{N}_\mathfrak{N}\beta\ cl\ (k^{-1}(M)) \subseteq k^{-1}(\mathfrak{N}_\mathfrak{N}\ \beta\ cl\ (M)) = k^{-1}(M)$ which dictates $k^{-1}(M)$ is $\mathfrak{N}_\mathfrak{N}$ β - CS in $\mathfrak{U}$. Thus by theorem 3. 12, k is $\mathfrak{N}_\mathfrak{N}$ β - cts function. □

Theorem 3.18. *A function k is $\mathfrak{N}_\mathfrak{N}$ β - cts function ⇔ $k^{-1}(\mathfrak{N}_\mathfrak{N}\ int(R)) \subseteq \mathfrak{N}_\mathfrak{N}\ \beta\ int(k^{-1}(R))$ for each subset R of $\mathfrak{B}$.*

Proof. The proof is clear □

Theorem 3.19. *A function k is $\mathfrak{N}_\mathfrak{N}$ β - cts ⇔ $\mathfrak{N}_\mathfrak{N}\beta cl\ (k^{-1}(H)) \subseteq k^{-1}(\mathfrak{N}_\mathfrak{N}\ cl\ (H))$ for each subset H of $\mathfrak{B}$.*

Proof. The proof is clear. □

Definition 3.20. Let $x_{(j,k,l)}$ be a $\mathfrak{N}_\mathfrak{N}$ pt of a $\mathfrak{N}_\mathfrak{N}$ topological space $(\mathfrak{U},\ \tau_{\mathfrak{N}_\mathfrak{N}}(\mathfrak{U}))$. A $\mathfrak{N}_\mathfrak{N}$ set B of $\mathfrak{U}$ is called the $\mathfrak{N}_\mathfrak{N}$ neighborhood(nbhd) of $x_{(j,k,l)}$ if ∃ a $\mathfrak{N}_\mathfrak{N}$ OS C such that $x_{(j,k,l)} \in C \subseteq B$.

Theorem 3.21. *Let k : $(\mathfrak{U},\ \tau_{\mathfrak{N}_\mathfrak{N}}(\mathfrak{U})) \to (\mathfrak{B}\ ,\ \tau_{\mathfrak{N}_\mathfrak{N}}(\mathfrak{B}))$ be a function. Then the subsequent conditions are equivalent.*

(i) *k is $\mathfrak{N}_\mathfrak{N}$ β - cts .*
(ii) *For each $\mathfrak{N}_\mathfrak{N}$ pt. $x_{(j,k,l)} \in \mathfrak{U}$ and every $\mathfrak{N}_\mathfrak{N}$ nbhd B of $k(x_{(j,k,l)})$, ∃ a $\mathfrak{N}_\mathfrak{N}$ β - OS C such that $x_{(j,k,l)} \in C \subseteq k^{-1}(B)$*
(iii) *For each $\mathfrak{N}_\mathfrak{N}$ pt. $x_{(j,k,l)} \in \mathfrak{U}$ and every $\mathfrak{N}_\mathfrak{N}$ nbhd B of $k(x_{(j,k,l)})$, ∃ a $\mathfrak{N}_\mathfrak{N}$ β - OS C in $\mathfrak{U}$ such that $x_{(j,k,l)} \in$ C and k(C) = B.*

Proof. (i)⟹ (ii) : Let $x_{(j,k,l)} \in \mathfrak{U}$ be a $\mathfrak{N}_\mathfrak{N}$ pt in $\mathfrak{U}$ and let B be a $\mathfrak{N}_\mathfrak{N}$ nbhd of $k(x_{(j,k,l)})$. Then ∃ a $\mathfrak{N}_\mathfrak{N}$ OS C in $\mathfrak{B}$ such that $k(x_{(j,k,l)}) \in C \subseteq B$. As k is $\mathfrak{N}_\mathfrak{N}$ β - , therefore $k^{-1}(B)$ is $\mathfrak{N}_\mathfrak{N}$ β OS in $\mathfrak{U}$ and $x_{(j,k,l)} \in k^{-1}(k(x_{(j,k,l)})) \subseteq k^{-1}(C) \subseteq k^{-1}(B)$. Hence the (ii).

(ii) ⟹ (iii) : Let $x_{(j,k,l)}$ be $\mathfrak{N}_\mathfrak{N}$ pt in $\mathfrak{U}$ and let B be a $\mathfrak{N}_\mathfrak{N}$ nbhd of $k(x_{(j,k,l)})$. the condition (ii) dictates that ∃ a $\mathfrak{N}_\mathfrak{N}$ β - OS C such that $x_{(j,k,l)} \in C \subseteq k^{-1}(B)$. Thus $x_{(j,k,l)} \in k(C) \subseteq k(k^{-1}(B)) \subseteq B$. This dictates (ii) is true.

(iii) ⟹ (i) : Let C be $\mathfrak{N}_\mathfrak{N}$ OS in $\mathfrak{B}$ and let $x_{(j,k,l)} \in C \subseteq k^{-1}(B)$. As C is a $\mathfrak{N}_\mathfrak{N}$ OS, $k(x_{(j,k,l)}) \in C$ and so C is $\mathfrak{N}_\mathfrak{N}$ nbhd of $k(x_{(j,k,l)})$. It follows from (iii) that ∃ a $\mathfrak{N}_\mathfrak{N}$ β OS B in $\mathfrak{U}$ such that $x_{(j,k,l)} \in B$ and k(B) ⊆ C so that $x_{(j,k,l)} \in B \subseteq k^{-1}(k(B)) \subseteq k^{-1}(C)$ which dictates that $k^{-1}(C)$ is a $\mathfrak{N}_\mathfrak{N}$ β OS in $\mathfrak{U}$. Therefore, k is a $\mathfrak{N}_\mathfrak{N}$ β cts function. □

Theorem 3.22. *Let k : $(\mathfrak{U},\ \tau_{\mathfrak{N}_\mathfrak{N}}(\mathfrak{U})) \to (\mathfrak{B}\ ,\ \tau_{\mathfrak{N}_\mathfrak{N}}(\mathfrak{B}))$ be a function. Then the subsequent conditions are equivalent.*

(i) *k is $\mathfrak{N}_\mathfrak{N}$ β - cts*
(ii) *For each pt. $x_{(j,k,l)} \in \mathfrak{U}$ each $\mathfrak{N}_\mathfrak{N}$OS P containing $k(x_{(j,k,l)})$, ∃ a $\mathfrak{N}_\mathfrak{N}$ β OS H ⊂ $\mathfrak{U}$ containing $x_{(j,k,l)}$ such that k(H) ⊂ P.*

Proof. (i)⟹(ii) Let P refers a $\mathfrak{N}_\mathfrak{N}$ OS of $\mathfrak{B}$ containing $k(x_{(j,k,l)})$, then $k^{-1}(P) \in \mathfrak{N}_\mathfrak{N}$ β OS of $\mathfrak{U}$.

Then the set $H = k^{-1}(P)$ which containing $x_{(j,k,l)}$ then k(H) ⊂ P.

(ii)⟹(iii) Let P ⊂ $\mathfrak{B}$ be $\mathfrak{N}_\mathfrak{N}$ OS and let $x_{(j,k,l)} \in k^{-1}(P)$. Then $k(x_{(j,k,l)}) \in P$ and thus ∃ $H_o \in \mathfrak{N}_\mathfrak{N}$ β OS of $\mathfrak{U}$ such that $k(x_{(j,k,l)}) \in H_o$ and $k(H_o) \subset P$ which dictates $k(x_{(j,k,l)}) \in H_o \subset k^{-1}(P)$ and so $k^{-1}(P) = \bigcup_{o\in k^{-1}(P)} H_o$ but $\bigcup_{o\in k^{-1}(P)} H_o \in \mathfrak{N}_\mathfrak{N}$ β OS of $\mathfrak{U}$ which dictates $k^{-1}(P) \in \mathfrak{N}_\mathfrak{N}$ β OS of $\mathfrak{U}$. Therefore k is $\mathfrak{N}_\mathfrak{N}$ β - cts . □

Theorem 3.23. *Let k : $(\mathfrak{U},\ \tau_{\mathfrak{N}_\mathfrak{N}}(\mathfrak{U})) \to (\mathfrak{B}\ ,\ \tau_{\mathfrak{N}_\mathfrak{N}}(\mathfrak{B}))$ be a function. Then the subsequent conditions are equivalent.*

(i) *k is $\mathfrak{N}_\mathfrak{N}$ β - cts .*

(ii) *The inverse image of each* $\mathfrak{N}_{\mathfrak{N}}$ *CS in* $\mathfrak{B}$ *is* $\mathfrak{N}_{\mathfrak{N}}$ β *CS in* $\mathfrak{U}$.

(iii) $\mathfrak{N}_{\mathfrak{N}}$ *int*($\mathfrak{N}_{\mathfrak{N}}$*cl* ($k^{-1}(P)$)) $\cap$ $\mathfrak{N}_{\mathfrak{N}}$*cl*($\mathfrak{N}_{\mathfrak{N}}$ *int* ($k^{-1}(P)$)) $\subset k^{-1}$($\mathfrak{N}_{\mathfrak{N}}$*cl(P)) for each* $P \subset \mathfrak{B}$.

(iv) k($\mathfrak{N}_{\mathfrak{N}}$ *int*($\mathfrak{N}_{\mathfrak{N}}$*cl(H)))* $\cap$ k($\mathfrak{N}_{\mathfrak{N}}$*cl*($\mathfrak{N}_{\mathfrak{N}}$ *int(H)))* $\subset$ $\mathfrak{N}_{\mathfrak{N}}$*cl(k(H)) for each* $H \subset \mathfrak{U}$.

Proof. (i)$\Longrightarrow$(ii) Similar to the proof of Theorem 3. 12.

(ii)$\Longrightarrow$(iii) Let $P \subseteq \mathfrak{B}$, then $k^{-1}(\mathfrak{N}_{\mathfrak{N}}\,cl(P))$ is $\mathfrak{N}_{\mathfrak{N}}$ β closed in $\mathfrak{U}$, i. e. , $\mathfrak{N}_{\mathfrak{N}}$ int($\mathfrak{N}_{\mathfrak{N}}$cl ($k^{-1}(P)$)) $\cap$ $\mathfrak{N}_{\mathfrak{N}}$cl($\mathfrak{N}_{\mathfrak{N}}$ int ($k^{-1}(P)$)) $\subset$ $\mathfrak{N}_{\mathfrak{N}}$ int($\mathfrak{N}_{\mathfrak{N}}$cl ($k^{-1}$($\mathfrak{N}_{\mathfrak{N}}$cl (P)))) $\cap$ $\mathfrak{N}_{\mathfrak{N}}$cl($\mathfrak{N}_{\mathfrak{N}}$ int (k^{-1}($\mathfrak{N}_{\mathfrak{N}}$cl(P.)))) $\subset k^{-1}$($\mathfrak{N}_{\mathfrak{N}}$cl(P)).

(iii)$\Longrightarrow$(iv) Let $H \subseteq \mathfrak{U}$, $P = k(H)$ in (iii) then $\mathfrak{N}_{\mathfrak{N}}$ int($\mathfrak{N}_{\mathfrak{N}}$cl ($k^{-1}$ (k(H)))) $\cap$ $\mathfrak{N}_{\mathfrak{N}}$cl($\mathfrak{N}_{\mathfrak{N}}$ int (k^{-1} (k(H)))) $\subset k^{-1}$($\mathfrak{N}_{\mathfrak{N}}$cl(k(H))). This dictates $\mathfrak{N}_{\mathfrak{N}}$ int($\mathfrak{N}_{\mathfrak{N}}$cl(H)) $\cap$ $\mathfrak{N}_{\mathfrak{N}}$cl($\mathfrak{N}_{\mathfrak{N}}$ int(H)) $\subset k^{-1}$($\mathfrak{N}_{\mathfrak{N}}$cl(k(H))). Hence k($\mathfrak{N}_{\mathfrak{N}}$ int($\mathfrak{N}_{\mathfrak{N}}$cl(H))) $\cap$ k($\mathfrak{N}_{\mathfrak{N}}$cl($\mathfrak{N}_{\mathfrak{N}}$ int(H))) $\subset$ $\mathfrak{N}_{\mathfrak{N}}$cl(k(H)).

(iv)$\Longrightarrow$(v) Let $C \subseteq \mathfrak{B}$ be $\mathfrak{N}_{\mathfrak{N}}$ OS, $R = \mathfrak{B} \setminus C$ and $H = k^{-1}(R)$ in (iv), then k($\mathfrak{N}_{\mathfrak{N}}$ int($\mathfrak{N}_{\mathfrak{N}}$cl($k^{-1}$(R)))) $\cap$ k($\mathfrak{N}_{\mathfrak{N}}$cl($\mathfrak{N}_{\mathfrak{N}}$ int(k^{-1}(R)))) $\subset$ $\mathfrak{N}_{\mathfrak{N}}$cl(k($k^{-1}$(R))). $\subset$ $\mathfrak{N}_{\mathfrak{N}}$cl(R) = R which dictates $k^{-1}(R)$ is $\mathfrak{N}_{\mathfrak{N}}$ β closed in $\mathfrak{U}$, so k is $\mathfrak{N}_{\mathfrak{N}}$ β - cts . □

4. NEUTROSOPHIC NANO β-IRRESOLUTE FUNCTION

Definition 4.1. A function k:($\mathfrak{U}$, $\tau_{\mathfrak{N}_{\mathfrak{N}}}(\mathfrak{U})$) $\rightarrow$ ($\mathfrak{B}$, $\tau_{\mathfrak{N}_{\mathfrak{N}}}(\mathfrak{B})$) is called Neutrosophic Nano β-irresolute($\mathfrak{N}_{\mathfrak{N}}$ β - irres.) function if $k^{-1}(R)$ is $\mathfrak{N}_{\mathfrak{N}}$ β-open in $\mathfrak{U}$ for each $\mathfrak{N}_{\mathfrak{N}}$ β- OS R in V.

Example 4.2. Consider $\mathfrak{U} = \{c_1, c_2, c_3, c_4, c_5\}$ and $\mathfrak{U} \setminus \mathfrak{R} = \{c_1, \{c_2, c_3\}, \{c_4, c_5\}\}$. Define an Neut. subset H $\subseteq \mathfrak{U} \ni H = \left\{\left\langle\frac{c_1}{0.4,0.5,0.7}\right\rangle, \left\langle\frac{c_2}{0.5,0.5,0.8}\right\rangle, \left\langle\frac{c_3}{0.6,0.5,0.7}\right\rangle, \left\langle\frac{c_4}{0.6,0.7,0.8}\right\rangle, \left\langle\frac{c_5}{0.6,0.8,0.8}\right\rangle\right\}$

Then $\overline{\mathfrak{N}_{\mathfrak{N}}(H)} = \left\{\left\langle\frac{c_1}{0.4,0.5,0.7}\right\rangle, \left\langle\frac{c_2,c_3}{0.6,0.5,0.7}\right\rangle, \left\langle\frac{c_4,c_5}{0.6,0.8,0.8}\right\rangle\right\}$, $\underline{\mathfrak{N}_{\mathfrak{N}}(H)} = \left\{\left\langle\frac{c_1}{0.4,0.5,0.7}\right\rangle, \left\langle\frac{c_2,c_3}{0.5,0.5,0.8}\right\rangle, \left\langle\frac{c_4,c_5}{0.6,0.7,0.8}\right\rangle\right\}$ and $B_{\mathfrak{N}_{\mathfrak{N}}}(H) = \left\{\left\langle\frac{c_1}{0.4,0.5,0.7}\right\rangle, \left\langle\frac{c_2,c_3}{0.6,0.5,0.7}\right\rangle, \left\langle\frac{c_4,c_5}{0.6,0.8,0.8}\right\rangle\right\}$.

Thus $\tau_{\mathfrak{N}_{\mathfrak{N}}}(\mathfrak{U}) = \{0_N, 1_N, \overline{\mathfrak{N}_{\mathfrak{N}}(H)}, \underline{\mathfrak{N}_{\mathfrak{N}}(H)}, B_{\mathfrak{N}_{\mathfrak{N}}}(H)\}$.

Also $\mathfrak{B} = \{d_1, d_2, d_3, d_4, d_5\}$ and $\mathfrak{B} \setminus \mathfrak{R} = \{\{d_1, d_2\}, \{d_3, d_4\}, d_5\}$. Define an Neut. subset $T \subseteq \mathfrak{B} \ni$ $T = \left\{\left\langle\frac{d_1}{0.1,0.2,0.5}\right\rangle, \left\langle\frac{d_2}{0.2,0.2,0.4}\right\rangle, \left\langle\frac{d_3}{0.1,0.2,0.7}\right\rangle, \left\langle\frac{d_4}{0.2,0.2,0.6}\right\rangle, \left\langle\frac{d_5}{0.1,0.3,0.6}\right\rangle\right\}$

Then $\overline{\mathfrak{N}_{\mathfrak{N}}(T)} = \left\{\left\langle\frac{d_1,d_2}{0.2,0.2,0.4}\right\rangle, \left\langle\frac{d_3,d_4}{0.2,0.2,0.6}\right\rangle, \left\langle\frac{d_5}{0.1,0.3,0.6}\right\rangle\right\}$, $\underline{\mathfrak{N}_{\mathfrak{N}}(T)} = \left\{\left\langle\frac{d_1,d_2}{0.1,0.2,0.5}\right\rangle, \left\langle\frac{d_3,d_4}{0.1,0.2,0.7}\right\rangle, \left\langle\frac{d_5}{0.1,0.3,0.6}\right\rangle\right\}$, and $B_{\mathfrak{N}_{\mathfrak{N}}}(T) = \left\{\left\langle\frac{d_1,d_2}{0.2,0.2,0.4}\right\rangle, \left\langle\frac{d_3,d_4}{0.2,0.2,0.6}\right\rangle, \left\langle\frac{d_5}{0.1,0.3,0.6}\right\rangle\right\}$.

Thus $\tau_{\mathfrak{N}_{\mathfrak{N}}}(\mathfrak{B}) = \{0_N, 1_N, \overline{\mathfrak{N}_{\mathfrak{N}}(T)}, \underline{\mathfrak{N}_{\mathfrak{N}}(T)}, B_{\mathfrak{N}_{\mathfrak{N}}}(T)\}$.

Then k : ($\mathfrak{U}$, $\tau_{\mathfrak{N}_{\mathfrak{N}}}(\mathfrak{U})$) $\rightarrow$ ($\mathfrak{B}$, $\tau_{\mathfrak{N}_{\mathfrak{N}}}(\mathfrak{B})$) is a function specified by $k(c_i) = d_i$, i = 1, 2, 3, 4, 5.

Consider the set $B = \left\{\left\langle\frac{d_1}{0.3,0.5,0.4}\right\rangle, \left\langle\frac{d_2}{0.3,0.4,0.4}\right\rangle, \left\langle\frac{d_3}{0.3,0.5,0.4}\right\rangle, \left\langle\frac{d_4}{0.3,0.5,0.3}\right\rangle, \left\langle\frac{d_5}{0.4,0.5,0.4}\right\rangle\right\}$,

then $k^{-1}(B) = \left\{\left\langle\frac{c_1}{0.3,0.5,0.4}\right\rangle, \left\langle\frac{c_2}{0.3,0.4,0.4}\right\rangle, \left\langle\frac{c_3}{0.3,0.5,0.4}\right\rangle, \left\langle\frac{c_4}{0.3,0.5,0.3}\right\rangle, \left\langle\frac{c_5}{0.4,0.5,0.4}\right\rangle\right\}$ is a $\mathfrak{N}_{\mathfrak{N}}\beta$ - OS .

Hence k:($\mathfrak{U}$, $\tau_{\mathfrak{N}_{\mathfrak{N}}}(\mathfrak{U})$) $\rightarrow$ ($\mathfrak{B}$, $\tau_{\mathfrak{N}_{\mathfrak{N}}}(\mathfrak{B})$) is a $\mathfrak{N}_{\mathfrak{N}}$ β - irres. function.

Theorem 4.3. *Let j : (* $\mathfrak{U}$ *,* $\tau_{\mathfrak{N}_{\mathfrak{N}}}(\mathfrak{U})$*)* $\rightarrow$ *(*$\mathfrak{B}$ *,* $\tau_{\mathfrak{N}_{\mathfrak{N}}}(\mathfrak{B})$*) and k : (*$\mathfrak{B}$ *,* $\tau_{\mathfrak{N}_{\mathfrak{N}}}(\mathfrak{B})$*)* $\rightarrow$ *(*$\mathfrak{N}$ *,* $\tau_{\mathfrak{N}_{\mathfrak{N}}}(\mathfrak{M})$*) denote functions. Then*

(i) $k \circ j$*: (*$\mathfrak{U}$*,* $\tau_{\mathfrak{N}_{\mathfrak{N}}}(\mathfrak{U})$*)* $\rightarrow$ *(*$\mathfrak{N}$ *,* $\tau_{\mathfrak{N}_{\mathfrak{N}}}(\mathfrak{M})$*) is* $\mathfrak{N}_{\mathfrak{N}}$ β - cts *function if k is* $\mathfrak{N}_{\mathfrak{N}}$ cts *function and j is* $\mathfrak{N}_{\mathfrak{N}}$ β - cts *function .*

(ii) $k \circ j$*: (*$\mathfrak{U}$*,* $\tau_{\mathfrak{N}_{\mathfrak{N}}}(\mathfrak{U})$*)* $\rightarrow$ *(*$\mathfrak{N}$ *,* $\tau_{\mathfrak{N}_{\mathfrak{N}}}(\mathfrak{M})$*) is* $\mathfrak{N}_{\mathfrak{N}}$ *β - irres. function if both k and j are* $\mathfrak{N}_{\mathfrak{N}}$ *β - irres. functions.*

(iii) $k \circ j$ *: (*$\mathfrak{U}$*,* $\tau_{\mathfrak{N}_{\mathfrak{N}}}(\mathfrak{U})$*)* $\rightarrow$ *(*$\mathfrak{N}$ *,* $\tau_{\mathfrak{N}_{\mathfrak{N}}}(\mathfrak{M})$*) is* $\mathfrak{N}_{\mathfrak{N}}$ β -cts *function if k is* $\mathfrak{N}_{\mathfrak{N}}$ β - cts *function and j is* $\mathfrak{N}_{\mathfrak{N}}$ *β - irres. function.*

Proof. (i) Let T be a $\mathfrak{N}_{\mathfrak{N}}$ OS in ($\mathfrak{M}$, $\tau_{\mathfrak{N}_{\mathfrak{N}}}(\mathfrak{M})$). As k is $\mathfrak{N}_{\mathfrak{N}}$-cts function , $k^{-1}(T)$ is $\mathfrak{N}_{\mathfrak{N}}$ OS in ($\mathfrak{B}$, $\tau_{\mathfrak{N}_{\mathfrak{N}}}(\mathfrak{B})$). As j is $\mathfrak{N}_{\mathfrak{N}}$ β - cts function , $j^{-1}(k^{-1}(T)) = k \circ j^{-1}(T)$ is $\mathfrak{N}_{\mathfrak{N}}$ β - OS in ($\mathfrak{U}$, $\tau_{\mathfrak{N}_{\mathfrak{N}}}(\mathfrak{U})$). Therefore $k \circ j$ is $\mathfrak{N}_{\mathfrak{N}}$ β - cts function.

(ii) Let T be a $\mathfrak{N}_\mathfrak{N}$ β - OS in ($\mathfrak{M}$, $\tau_{\mathfrak{N}_\mathfrak{N}}(\mathfrak{M})$). As k is $\mathfrak{N}_\mathfrak{N}$ β - irres. function , $k^{-1}(T)$ is $\mathfrak{N}_\mathfrak{N}$ β - OS in ($\mathfrak{B}$, $\tau_{\mathfrak{N}_\mathfrak{N}}(\mathfrak{B})$). As j is $\mathfrak{N}_\mathfrak{N}$ β - irres. function , $j^{-1}(k^{-1}(T)) = k \circ j^{-1}(T)$ is $\mathfrak{N}_\mathfrak{N}$ β - OS in ($\mathfrak{U}$, $\tau_{\mathfrak{N}_\mathfrak{N}}(\mathfrak{U})$). Therefore $k \circ j$ is $\mathfrak{N}_\mathfrak{N}$ β - cts function.

(iii) Let T be a $\mathfrak{N}_\mathfrak{N}$ OS in ($\mathfrak{M}$, $\tau_{\mathfrak{N}_\mathfrak{N}}(\mathfrak{M})$). As k is $\mathfrak{N}_\mathfrak{N}$ β - cts function, $k^{-1}(T)$ is $\mathfrak{N}_\mathfrak{N}$ β - OS in ($\mathfrak{B}$, $\tau_{\mathfrak{N}_\mathfrak{N}}(\mathfrak{B})$). As j is $\mathfrak{N}_\mathfrak{N}$ β - irres. function , $j^{-1}(k^{-1}(T)) = k \circ j^{-1}(T)$ is $\mathfrak{N}_\mathfrak{N}$ β - OS in ($\mathfrak{U}$, $\tau_{\mathfrak{N}_\mathfrak{N}}(\mathfrak{U})$). Therefore $k \circ j$ is $\mathfrak{N}_\mathfrak{N}$ β - cts function.

□

Theorem 4.4. *Let k : ($\mathfrak{U}$, $\tau_{\mathfrak{N}_\mathfrak{N}}(\mathfrak{U})$) → ($\mathfrak{B}$, $\tau_{\mathfrak{N}_\mathfrak{N}}(\mathfrak{B})$) be a function. Then the subsequent conditions are equivalent.*

(i) *k is $\mathfrak{N}_\mathfrak{N}$ β irres.*
(ii) $k^{-1}(H)$ *is $\mathfrak{N}_\mathfrak{N}$ β CS in ($\mathfrak{U}$, $\tau_{\mathfrak{N}_\mathfrak{N}}(\mathfrak{U})$) for each $\mathfrak{N}_\mathfrak{N}$ CS in ($\mathfrak{B}$, $\tau_{\mathfrak{N}_\mathfrak{N}}(\mathfrak{B})$).*
(iii) *k($\mathfrak{N}_\mathfrak{N}$ β cl(H_1)) ⊆ $\mathfrak{N}_\mathfrak{N}$ βcl(k(H_1)) for all $H_1 \subseteq \mathfrak{U}$.*
(iv) *$\mathfrak{N}_\mathfrak{N}$ βcl($k^{-1}(H_2)$) ⊆ k^{-1}($\mathfrak{N}_\mathfrak{N}$ βcl(H_2)) for all $H_2 \subseteq \mathfrak{B}$.*
(v) *k^{-1}($\mathfrak{N}_\mathfrak{N}$ β int(H_2)) ⊆ $\mathfrak{N}_\mathfrak{N}$ β int($k^{-1}(H_2)$) for all $H_2 \subseteq \mathfrak{B}$.*
(vi) *k is $\mathfrak{N}_\mathfrak{N}$ β irres. for each $\mathfrak{w} \in$ ($\mathfrak{U}$, $\tau_{\mathfrak{N}_\mathfrak{N}}(\mathfrak{U})$).*

Proof. (i)⟹(ii) It is obvious.

(ii)⟹(iii) Let $H_1 \subseteq \mathfrak{U}$. In that case, $\mathfrak{N}_\mathfrak{N}$ β cl(k(H_1)) is a $\mathfrak{N}_\mathfrak{N}$ β CS of ($\mathfrak{B}$, $\tau_{\mathfrak{N}_\mathfrak{N}}(\mathfrak{B})$). By (ii), k^{-1}($\mathfrak{N}_\mathfrak{N}$ βcl(H_1)) is a $\mathfrak{N}_\mathfrak{N}$ β CS in ($\mathfrak{U}$, $\tau_{\mathfrak{N}_\mathfrak{N}}(\mathfrak{U})$) and $\mathfrak{N}_\mathfrak{N}$ β cl(k(H_1)) ⊆ $\mathfrak{N}_\mathfrak{N}$ β clk^{-1}k((H_1)) ⊆ $\mathfrak{N}_\mathfrak{N}$ β cl(k^{-1}($\mathfrak{N}_\mathfrak{N}$ β cl(k(H_1))) = k^{-1}($\mathfrak{N}_\mathfrak{N}$ β cl(k(H_1)) . So k($\mathfrak{N}_\mathfrak{N}$ β cl(k(H_1)) ⊆ $\mathfrak{N}_\mathfrak{N}$ β cl (k((H_1))).

(iii)⟹(iv)Let $H_2 \subseteq \mathfrak{B}$. By (iii) k($\mathfrak{N}_\mathfrak{N}$βcl($k^{-1}(H_2)$)) ⊆ $\mathfrak{N}_\mathfrak{N}$βcl($kk^{-1}(H_2)$) ⊆ $\mathfrak{N}_\mathfrak{N}$βcl($H_2$)) . So $\mathfrak{N}_\mathfrak{N}$β cl$k^{-1}(H_2)$ ⊆ k^{-1}($\mathfrak{N}_\mathfrak{N}$ β cl(H_2)).

(iv)⟹(v) Let $H_2 \subseteq \mathfrak{B}$. By (iv), k^{-1}($\mathfrak{N}_\mathfrak{N}$ β cl($\mathfrak{B} - H_2$)) ⊇ $\mathfrak{N}_\mathfrak{N}$β cl($k^{-1}(\mathfrak{B} - H_2)$) = $\mathfrak{N}_\mathfrak{N}$β cl($\mathfrak{U}$-$k^{-1}(H_2)$)). As $\mathfrak{U}$-$\mathfrak{N}_\mathfrak{N}$β cl($\mathfrak{U}$-$H_2$) = $\mathfrak{N}_\mathfrak{N}$ β int(H_2)) subsecuently, k^{-1}($\mathfrak{N}_\mathfrak{N}$ β int(H_2)) = k^{-1}($\mathfrak{B}$ -$\mathfrak{N}_\mathfrak{N}$β cl($\mathfrak{B}$-$H_2$)) = $\mathfrak{U}$-k^{-1}($\mathfrak{N}_\mathfrak{N}$β cl($\mathfrak{B}$-$H_2$)) ⊆ $\mathfrak{U}$-$\mathfrak{N}_\mathfrak{N}$β cl($\mathfrak{U}$-$k^{-1}(H_2)$) = $\mathfrak{N}_\mathfrak{N}$ β int($k^{-1}(H_2)$).

(v)⟹(vi) Let H refers a $\mathfrak{N}_\mathfrak{N}$β OS of ($\mathfrak{B}$, $\tau_{\mathfrak{N}_\mathfrak{N}}(\mathfrak{B})$), subsecuently H = $\mathfrak{N}_\mathfrak{N}$ β int(H)). By (v), $k^{-1}(H)$ = k^{-1}($\mathfrak{N}_\mathfrak{N}$ β int(H)) ⊆ $\mathfrak{N}_\mathfrak{N}$ β int($k^{-1}(H)$) ⊆ $k^{-1}(H)$. So, $k^{-1}(H)$ = $\mathfrak{N}_\mathfrak{N}$ β int($k^{-1}(H)$) . Thus $k^{-1}(H)$ is a $\mathfrak{N}_\mathfrak{N}$β OS of ($\mathfrak{U}$, $\tau_{\mathfrak{N}_\mathfrak{N}}(\mathfrak{U})$). Therefore k is $\mathfrak{N}_\mathfrak{N}$β irres.

(i)⟹(vi) Let k be a $\mathfrak{N}_\mathfrak{N}$β irres. , $\mathfrak{w} \in$ ($\mathfrak{U}$, $\tau_{\mathfrak{N}_\mathfrak{N}}(\mathfrak{U})$) and any $\mathfrak{N}_\mathfrak{N}$β OS H of ($\mathfrak{B}$, $\tau_{\mathfrak{N}_\mathfrak{N}}(\mathfrak{B})$) ∋
k($\mathfrak{w}$) ⊆ H . Then $\mathfrak{w} \in k^{-1}(H)$ = $\mathfrak{N}_\mathfrak{N}$ β int($k^{-1}(H)$). Let G = $k^{-1}(H)$ where G is a $\mathfrak{N}_\mathfrak{N}$β OS of (, $\tau_{\mathfrak{N}_\mathfrak{N}}(\mathfrak{U})$) and so k(G) = k($k^{-1}(H)$) ⊆ H. Thus k is $\mathfrak{N}_\mathfrak{N}$β irres. for each $\mathfrak{w} \in$ ($\mathfrak{U}$, $\tau_{\mathfrak{N}_\mathfrak{N}}(\mathfrak{U})$).

(vi)⟹(i) Let O be a $\mathfrak{N}_\mathfrak{N}$β OS of ($\mathfrak{B}$, $\tau_{\mathfrak{N}_\mathfrak{N}}(\mathfrak{B})$), $\mathfrak{w} \in k^{-1}(O)$. Then k($\mathfrak{N}$) ∈ O. By hypothesis, ∃ a $\mathfrak{N}_\mathfrak{N}$β OS P of ($\mathfrak{U}$, $\tau_{\mathfrak{N}_\mathfrak{N}}(\mathfrak{U})$) ∋ $\mathfrak{w}$ ∈ P and k (P) ⊆ O which dictates $\mathfrak{w}$ ∈ P ⊆ k^{-1}(k(P)) ⊆ k^{-1} (O) and $\mathfrak{w}$ ∈ P = $\mathfrak{N}_\mathfrak{N}$β int (P) ⊆ $\mathfrak{N}_\mathfrak{N}$β int($k^{-1}$ (O))⟹ k^{-1}(O) ⊆ $\mathfrak{N}_\mathfrak{N}$β int($k^{-1}$ (O)). Hence k^{-1} (O) = $\mathfrak{N}_\mathfrak{N}$β int($k^{-1}$ (O)). Hence k is $\mathfrak{N}_\mathfrak{N}$β irres. □

Theorem 4.5. *Let k be a bijective function, then k is $\mathfrak{N}_\mathfrak{N}$ β - irres. function ⇔ $\mathfrak{N}_\mathfrak{N}$ β int(k (N)) ⊆ k ($\mathfrak{N}_\mathfrak{N}$ β int(N)) for each subset N of $\mathfrak{U}$.*

Proof. Let N ⊆ ($\mathfrak{U}$, $\tau_{\mathfrak{N}_\mathfrak{N}}(\mathfrak{U})$). By theorem 4. 4 and As k is bijective, k^{-1}($\mathfrak{N}_\mathfrak{N}$ β int(N)) ⊆
$\mathfrak{N}_\mathfrak{N}$ β int(k(k^{-1}(N))) . Hence k(k^{-1}($\mathfrak{N}_\mathfrak{N}$ β int(M))) ⊆ k($\mathfrak{N}_\mathfrak{N}$ β int(N)). Therefore $\mathfrak{N}_\mathfrak{N}$ β int(k (N)) ⊆ k ($\mathfrak{N}_\mathfrak{N}$ β int(N)).

Conversely, let M be a $\mathfrak{N}_\mathfrak{N}$β OS of ($\mathfrak{U}$, $\tau_{\mathfrak{N}_\mathfrak{N}}(\mathfrak{U})$) . Then M = $\mathfrak{N}_\mathfrak{N}$ β int(M). By assumption k ($\mathfrak{N}_\mathfrak{N}$ β int(k^{-1}(M))) ⊇ $\mathfrak{N}_\mathfrak{N}$ β int(k(k^{-1}((M))) = $\mathfrak{N}_\mathfrak{N}$ β int(M) = M which dictates k^{-1}(k ($\mathfrak{N}_\mathfrak{N}$ β int(k^{-1}(M)))) ⊇ k^{-1}(M). Hence k^{-1}(M) = $\mathfrak{N}_\mathfrak{N}$ β int(k^{-1}(M)). Thus k^{-1}(M) is a $\mathfrak{N}_\mathfrak{N}$ β OS of ($\mathfrak{U}$, $\tau_{\mathfrak{N}_\mathfrak{N}}(\mathfrak{U})$). Thus k is $\mathfrak{N}_\mathfrak{N}$β irres. □

5. Conclusion

This work introduces and studies neutrosophic nano β - continuous functions, extending continuity concepts within neutrosophic nano topological spaces to better handle uncertainty and indeterminacy. The findings enhance the theoretical foundation of neutrosophic nano topology and provide scope for broader applications. As a continuation, further research will focus on the development and investigation of neutrosophic nano β-open maps and β-closed maps, which are expected to enrich the structure of neutrosophic mappings and offer deeper insights into topological properties under uncertainity.

References

[1] Anbarasi Rodrigo, P. & Charlet Jenisha, J. Neutrosophic Nano β - Open Sets:Definitions and Chararcterizations in Neutrosophic Nano Topological Spaces, AIP Conference proceeding(communicated). 2.7, 2.8

[2] Dhavaseelan, R., & Jafari, S. (2019). Generalized neutrosophic closed sets, New Trends in Neutrosophic Theory and Applications, Vol. II,(2017), 261-273. R. Dhavaseelan and Md. Hanif PAGE, Neutrosophic Almost α-contra-continuous function, Neutrosophic Sets and Systems, 29, 71-77. 2.9

[3] Smarandache, F. (1999). A unifying field in Logics: Neutrosophic Logic. In Philosophy. American Research Press, 1-141. 2.1, 2.6

[4] Thivagar, M. L., Jafari, S., Devi, V. S., & Antonysamy, V. (2018). A novel approach to nano topology via neutrosophic sets. Infinite Study. 2.3, 2.4, 2.5

[5] Thivagar, M. L., & Richard, C. (2013). On nano forms of weakly open sets. International journal of mathematics and statistics invention, 1(1), 31-37. 2.2