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An interpretation of separation axioms via Neutrosophic Λ_P-open sets

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Abstract

The main purpose of this paper is to present a novel concept of separation axioms in neutrosophic topological spaces by means of neutrosophic $N_{tr}\Lambda_P-$open sets. The concepts of neutrosophic spaces $N_{tr}\Lambda_P - T_i$ spaces $(i = 0, 1, 2)$ are introduced and their properties are studied.

Keywords: Neutrosophic $N_{tr}\Lambda_P-$open, Neutrosophic $N_{tr}\Lambda_P - T_0$ space, Neutrosophic $N_{tr}\Lambda_P - T_1$ space, Neutrosophic $N_{tr}\Lambda_P - T_2$ space.

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1. Introduction

Neutrophic sets were created as a result of Smarandache's 1998 introduction of the idea of neutrosophy, which aims to investigate the nature and applications of neutrality. Numerous scholars have investigated different topological notions, including the introduction of neutrosophic topological spaces by Salama and Albowi. Ahu Acikgoz and F. Esenbel introduced the idea of separation axioms in neutrosophic topological spaces by using the concept of quasi-coincidence. Later, several fundamental findings on separation axioms in neutrosophic topological spaces were also specified and established by Suman Das et al. The idea of neutrosophic Λ_P separation axioms in neutrosophic topological spaces is presented in this study. Neutrosophic $\Lambda_P - T_i$ spaces $(i = 0, 1, 3)$ have been defined, and their characteristics and relationships have been noted. Furthermore, in order to describe the spaces, the concept of neutrosophic Λ_P-kernel has also been defined.

2. Preliminaries

Throughout this paper, X refers to an initial universe, E be a set of parameters, $P(X)$ denote the power set of X.

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Definition 2.1. [6] Let U be a non-empty fixed set.A **Neutrosophic set** K is an object having the form $K = \{\langle u, \mu_K(u), \sigma_K(u), \gamma_K(u)\rangle : u \in U\}$ where $\mu_K(u), \sigma_K(u)$ and $\gamma_K(u)$ represents the degree of membership, the degree of indeterminacy and the degree of non-membership respectively of each element $u \in U$ to the set U. A neutrosophic set $K = \{\langle u, \mu_K(u), \sigma_K(u), \gamma_K(u)\rangle : u \in U\}$ can be identified to an ordered triple $\langle u, \mu_K(u), \sigma_K(u), \gamma_K(u)\rangle$ in $]^-0, 1^+[$ on U.
A neutrosophic set has membership, indeterminacy, and non-membership functions that are defined on the non-standard unit interval. But doing so makes it difficult to demonstrate specific outcomes. From here on, we will focus on the neutrosophic sets whose membership, non-membership, and indeterminacy functions are specified on $[0, 1]$.

Definition 2.2. [6] Let U be a non-empty set and $K = \{\langle u, \mu_K(u), \sigma_K(u), \gamma_K(u)\rangle : u \in U\}$ and
$M = \{\langle u, \mu_M(u), \sigma_M(u), \gamma_M(u)\rangle : u \in U\}$ are neutrosophic sets, then i. $K \subseteq M \Leftrightarrow \mu_K(u) \leqslant \mu_M(u), \sigma_K(u) \leqslant \sigma_M(u)$ and $\gamma_K(u) \geqslant \gamma_M(u) \forall u \subseteq U$
ii. $K \cup M = \{\langle u, max(\mu_K(u), \mu_M(u)), max(\sigma_K(u), \sigma_M(u)), min(\gamma_K(u), \gamma_M(u))\rangle : u \in U\}$
iii. $K \cap M = \{\langle u, min(\mu_K(u), \mu_M(u)), min(\sigma_K(u), \sigma_M(u)), max(\gamma_K(u), \gamma_M(u))\rangle : u \in U\}$
iv. $K^C = \{\langle u, (\gamma_K(u), 1 - \sigma_K(u), \mu_K(u))\rangle : u \in U\}$
v. $0_{N_{tr}} = \{\langle u, 0, 0, 1\rangle : u \in U\}$ and $1_{N_{tr}} = \{\langle u, 1, 1, 0\rangle : u \in U\}$

Definition 2.3. [6] A **Neutrosophic topology** on a non-empty set U is a family $T_{N_{tr}}$ of neutrosophic sets in U satisfying the following axioms:
i. $0_{N_{tr}}, 1_{N_{tr}} \in \tau_{N_{tr}}$
ii. $K_1 \cap K_2 \in \tau_{N_{tr}}$ for any $K_1, K_2 \in \tau_{N_{tr}}$
iii. $\cup K_i \in \tau_{N_{tr}}$ for every $K_i : i \in \subseteq \tau_{N_{tr}}$
In this case the ordered pair $(U, \tau_{N_{tr}})$ is called a neutrosophic topological space. The members of $\tau_{N_{tr}}$ are neutrosophic open set and its complements are neutrosophic closed.

Definition 2.4. [1] A neutrosophic set $K = \{\langle u, \mu_K(u), \sigma_K(u), \gamma_K(u)\rangle : u \in U\}$ is called a **neutrosophic point** if for any element $v \in U, \mu_K(v) = a, \sigma_K(v) = b, \gamma_K(v) = c$ and for $u = v$ and $\mu_K(v) = 0, \sigma_K = 0, \gamma_K(v) = 1$ for $u \neq v$, where a, b, c are real standard or non-standard sunsets of $]^-0, 1^+[$. A neutrosophic point is denoted by $u_{a,b,c}$. For the neutrosophic point $u_{a,b,c}$, u will be called its support.

Definition 2.5. [1] A neutrosophic point $u_{a,b,c}$ is said to be **neutrosophic quasi-coincident** with a neutrosophic set K, denoted by $u_{a,b,c}qK$ if $u_{a,b,c} \notin K^c$. If $u_{a,b,c}$ is not neutrosophic quasi-coincident with K, we denote it by $u_{a,b,c}\hat{q}K$.

Definition 2.6. [3] A neutrosophic set K of a neutrosophic topological space $(U, T_{N_{tr}})$ is said to be **neutrosophic Λ_P-open** if there exist a N_{tr}pre-open set $E \neq 0_{N_{tr}}, 1_{N_{tr}}$ such that $K \subseteq N_{tr}cl(K \cap E)$. The complement of neutrosophic Λ_p- open set is neutrosophic Λ_P-closed. The class of neutrosophic Λ_P- open sets is denoted by $N_{tr}\Lambda_P O(U, \tau_{N_{tr}})$.

Theorem 2.7. *[3] Every $N_{tr}-$open set is $N_{tr}\Lambda_P-$open.*

Definition 2.8. [4] Let $(U, \tau_{N_{tr}})$ and $(V, \rho_{N_{tr}})$ be neutrosophic topological spaces. Then the function $f_{N_{tr}} : (U, \tau_{N_{tr}}) \to (V, \rho_{N_{tr}})$ is said to be **neutrosophic Λ_P-continuous** if $f^{-1}_{N_{tr}}(M)$ is $N_{tr}\Lambda_P-$open in $(U, \tau_{N_{tr}})$ for every N_{tr} open set M in $(V, \rho_{N_{tr}})$.

Definition 2.9. [4] Let $(U, \tau_{N_{tr}})$ and $(V, \rho_{N_{tr}})$ be neutrosophic topological spaces. Then the function $f_{N_{tr}} : (U, \tau_{N_{tr}}) \to (V, \rho_{N_{tr}})$ is said to be **neutrosophic Λ_P-irresolute** if $f^{-1}_{N_{tr}}(M)$ is $N_{tr}\Lambda_P-$open in $(U, \tau_{N_{tr}})$ for every $N_{tr}\Lambda_P-$ open set M in $(V, \rho_{N_{tr}})$.

Definition 2.10. [5] Let $(U, \tau_{N_{tr}})$ and $(V, \rho_{N_{tr}})$ be neutrosophic topological spaces. Then the mapping $f_{N_{tr}} : (U, \tau_{N_{tr}}) \to (V, \rho_{N_{tr}})$ is said to be **neutrosophic Λ_P-open** if $f_{N_{tr}}(K)$ is $N_{tr}\Lambda_P-$open in $(V, \rho_{N_{tr}})$ for every N_{tr} open set K in $(U, \tau_{N_{tr}})$.

Definition 2.11. [2] A neutrosophic topological space $(U, \tau_{N_{tr}})$ is said to be a $N_{tr}T_0-$**space** if for every pair of neutrosophic points $u_{a,b,c}$ and $v_{a',b',c'}, u \neq v$ in U, there exists N_{tr} open set K in $(U, \tau_{N_{tr}})$ such that $u_{a,b,c} \in K, v_{a',b',c'} \notin K$ or $u_{a,b,c} \notin K, v_{a',b',c'} \in K$.

Definition 2.12. [2] A neutrosophic topological space $(U, \tau_{N_{tr}})$ is said to be a $N_{tr}T_1-$**space** if for every pair of neutrosophic points $u_{a,b,c}$ and $v_{a',b',c'}, u \neq v$, there exists N_{tr} open sets K and M in $(U, \tau_{N_{tr}})$ such that $u_{a,b,c} \in K, v_{a',b',c'} \notin K$ and $u_{a,b,c} \notin M, v_{a',b',c'} \in M$.

Definition 2.13. [2] A neutrosophic topological space $(U, \tau_{N_{tr}})$ is said to be a $N_{tr}T_2-$**space** or neutrosophic hausdorff space if for every pair of neutrosophic points $u_{a,b,c}$ and $v_{a',b',c'}, u \neq v$, there exists N_{tr} open sets K and M in $(U, \tau_{N_{tr}})$ such that $u_{a,b,c} \in K, v_{a',b',c'} \in M$ and $K \cap M = 0_{N_{tr}}$.

3. Main Results

3.1. Neutrosophic $\Lambda_P - T_0$ Spaces

Definition 3.1. A neutrosophic topological space $(U, \tau_{N_{tr}})$ is said to be a $N_{tr}\Lambda_P - T_0$ space if for every pair of neutrosophic points $u_{a,b,c}$ and $v_{a',b',c'}, u \neq v$ in U, there exists $N_{tr}\Lambda_P-$open set K in $(U, \tau_{N_{tr}})$ such that $u_{a,b,c} \in K, v_{a',b',c'} \notin K$ or $u_{a,b,c} \notin K, v_{a',b',c'} \in K$.

Example 3.2. Let $U = \{u, v\}, \tau_{N_{tr}} = \{0_{N_{tr}}, \mathcal{K}, 1_{N_{tr}}\}$ where $\mathcal{K} = \{\{< u, a, b, c >< v, 0, 0, 1 >\} : 0 < a \leqslant 1; 0 < b \leqslant 1; 0 \leqslant c < 1\}$ is the collection of neutrosophic sets in U. Clearly $(U, \tau_{N_{tr}})$ is space.

Theorem 3.3. *Every $N_{tr}T_0-$ space is $N_{tr}\Lambda_P - T_0$.*

Proof. Let $(U, \tau_{N_{tr}})$ be a $N_{tr}T_0-$space. Then, for every pair of neutrosophic points $u_{a,b,c}$ and $v_{a',b',c'}, u \neq v$, there exists N_{tr}open set K in $(U, \tau_{N_{tr}})$ such that $u_{a,b,c} \in K, v_{a',b',c'} \notin K$ or $u_{a,b,c} \notin K, v_{a',b',c'} \in K$. By theorem 2.7, K is $N_{tr}\Lambda_P-$open. Hence $(U, \tau_{N_{tr}})$is $N_{tr}\Lambda_P - T_0$. □

Remark 3.4. The above theorem's converse need not hold.

Example 3.5. Let $U = \{u, v\}, \tau_{N_{tr}} = \{0_{N_{tr}}, \mathcal{K}, 1_{N_{tr}}\}$ where $\mathcal{K} = \{\{< u, a, b, 1-a >< v, 0, 0, 1 >\} : a, b \in (0, 0.6]\}$ is the collection of neutrosophic sets in U. Clearly $(U, \tau_{N_{tr}})$ is $N_{tr}\Lambda_P - T_0$ space. Now for any two neutrosophic points $u_{0.6,0.3,0.2}$ and $v_{0.1,0.2,0.7}, u \neq v$ in U, there exist a $N_{tr}\Lambda_P-$open set $\{< u, 0.6, 0.6, 0.6 >< v, 0, 0, 1 >\}$ containing $v_{0.1,0.2,0.7}$ but not $u_{0.6,0.3,0.2}$. However, there exist no $N_{tr}\Lambda_P-$open set K such that $u_{0.6,0.3,0.2} \in K$, $v_{0.1,0.2,0.7} \notin K$ or $u_{0.6,0.3,0.2} \notin K$, $v_{0.1,0.2,0.7} \in K$. Hence $(U, \tau_{N_{tr}})$ is not $N_{tr}T_0-$space.

Definition 3.6. The **neutrosophic** Λ_P-**kernel** of a neutrosophic set K in $(U, \tau_{N_{tr}})$ is the intersection of all $N_{tr}\Lambda_P-$open sets containing K. It is denoted by $N_{tr}\Lambda_P ker(K)$.

Theorem 3.7. *A neutrosophic topological space $(U, \tau_{N_{tr}})$ is $N_{tr}\Lambda_P - T_0$ if and only if for every pair of neutrosophic points $u_{a,b,c}$ and $v_{a',b',c'}, u \neq v$ in U,either $u_{a,b,c} \notin N_{tr}\Lambda_P ker(v_{a',b',c'})$ or $v_{a',b',c'} \notin N_{tr}\Lambda_P ker(u_{a,b,c})$*

Proof. Let U be a $N_{tr}\Lambda_P - T_0$ space. Then, for every pair of neutrosophic points $u_{a,b,c}$ and $v_{a',b',c'}, u \neq v$ in U, there exists $N_{tr}\Lambda_P-$open set K in U such that $u_{a,b,c} \in K, v_{a',b',c'} \notin K$ or $u_{a,b,c} \notin K, v_{a',b',c'} \in K$. Now, $u_{a,b,c} \in K, v_{a',b',c'} \notin K$ implies $v_{a',b',c'} \notin N_{tr}\Lambda_P ker(u_{a,b,c})$ for suppose $v_{a',b',c'} \in N_{tr}\Lambda_P ker(u_{a,b,c})$, there exists no $N_{tr}\Lambda_P-$open set Kin U such that $u_{a,b,c} \in K, v_{a',b',c'} \notin K$ which is a contradiction. Similiarly, $u_{a,b,c} \notin K, v_{a',b',c'} \in K$ implies $u_{a,b,c} \notin N_{tr}\Lambda_P ker(v_{a',b',c'})$. Conversely, suppose $u_{a,b,c} \notin N_{tr}\Lambda_P ker(v_{a',b',c'})$ or $v_{a',b',c'} \notin N_{tr}\Lambda_P ker(u_{a,b,c})$ for every pair of neutrosophic points $u_{a,b,c}$ and $v_{a',b',c'}, u \neq v$ in U. Then, there exists a $N_{tr}\Lambda_P-$open set K in U such that $u_{a,b,c} \notin K, v_{a',b',c'} \in K$ or $u_{a,b,c} \in K, v_{a',b',c'} \notin K$. Hence Uis $N_{tr}\Lambda_P - T_0$. □

Theorem 3.8. *Let $f_{N_{tr}} : (U, \tau_{N_{tr}}) \to (V, \rho_{N_{tr}})$ be a one-one $N_{tr}\Lambda_P-$continuous function. If $(V, \rho_{N_{tr}})$ is $N_{tr}T_0$,then $(U, \tau_{N_{tr}})$is $N_{tr}\Lambda_P - T_0$.*

Proof. Let $u_{1_{a,b,c}}$ and $u_{2_{a,b,c}}, u_1 \neq u_2$ be any two neutrosophic points in U. Since $f_{N_{tr}}$ is one-one, there exists neutrosophic points $v_{1_{a,b,c}}$ and $v_{2_{a,b,c}}$ in V such that $f_{N_{tr}}(u_{1_{a,b,c}}) = v_{1_{a,b,c}}$, $f_{N_{tr}}(u_{2_{a,b,c}}) = v_{2_{a,b,c}}$ and $v_1 \neq v_2$. Hence $u_{1_{a,b,c}} = f^{-1}_{N_{tr}}(v_{1_{a,b,c}})$ and $u_{2_{a,b,c}} = f^{-1}_{N_{tr}}(v_{2_{a,b,c}})$. Now, since V is $N_{tr}T_0$, there exists a $N_{tr}-$open set K in V such that $v_{1_{a,b,c}} \in K, v_{2_{a,b,c}} \notin K$ or $v_{1_{a,b,c}} \notin K, v_{2_{a,b,c}} \in K$. Again, since $f_{N_{tr}}$ is $N_{tr}\Lambda_P-$continuous,$f^{-1}_{N_{tr}}$ is $N_{tr}\Lambda_P-$open in U. Also, $v_{1_{a,b,c}} \in K$ implies $f^{-1}_{N_{tr}}(v_{1_{a,b,c}}) \in f^{-1}_{N_{tr}}(K)$ implies $u_{1_{a,b,c}} \in f^{-1}_{N_{tr}}(K)$ and $v_{2_{a,b,c}} \notin K$ implies $u_{2_{a,b,c}} \notin f^{-1}_{N_{tr}}(K)$. Hence, for any two neutrosophic points $u_{1_{a,b,c}}$ and $u_{2_{a,b,c}}, u_1 \neq u_2$ in U, there exists a $N_{tr}\Lambda_P-$open set $f^{-1}_{N_{tr}}(K)$ in U such that $u_{1_{a,b,c}} \in f^{-1}_{N_{tr}}(K), u_{2_{a,b,c}} \notin f^{-1}_{N_{tr}}(K)$ or $u_{1_{a,b,c}} \notin f^{-1}_{N_{tr}}(K), u_{2_{a,b,c}} \in f^{-1}_{N_{tr}}(K)$. Therefore, U is $N_{tr}\Lambda_P - T_0$. □

Theorem 3.9. *Let* $f_{N_{tr}} : (U, \tau_{N_{tr}}) \to (V, \rho_{N_{tr}})$ *be a one-one* $N_{tr}\Lambda_P-$*irresolute function. If* $(V, \rho_{N_{tr}})$ *is* $N_{tr}\Lambda_P - T_0$,*then* $(U, \tau_{N_{tr}})$*is* $N_{tr}\Lambda_P - T_0$.

Proof. Let $u_{1_{a,b,c}}$ and $u_{2_{a,b,c}}, u_1 \neq u_2$ be any two neutrosophic points in U. Since $f_{N_{tr}}$ is one-one, there exists neutrosophic points $v_{1_{a,b,c}}$ and $v_{2_{a,b,c}}$ in V such that $f_{N_{tr}}(u_{1_{a,b,c}}) = v_{1_{a,b,c}}$, $f_{N_{tr}}(u_{2_{a,b,c}}) = v_{2_{a,b,c}}$ and $v_1 \neq v_2$. Hence $u_{1_{a,b,c}} = f^{-1}_{N_{tr}}(v_{1_{a,b,c}})$ and $u_{2_{a,b,c}} = f^{-1}_{N_{tr}}(v_{2_{a,b,c}})$. Now, since V is $N_{tr}\Lambda_P - T_0$, there exists a $N_{tr}\Lambda_p-$open set K in V such that $v_{1_{a,b,c}} \in K, v_{2_{a,b,c}} \notin K$ or $v_{1_{a,b,c}} \notin K, v_{2_{a,b,c}} \in K$. Again, since $f_{N_{tr}}$ is $N_{tr}\Lambda_P-$irresolute,$f^{-1}_{N_{tr}}$ is $N_{tr}\Lambda_P-$open in U. Also, $v_{1_{a,b,c}} \in K$ implies $f^{-1}_{N_{tr}}(v_{1_{a,b,c}}) \in f^{-1}_{N_{tr}}(K)$ implies $u_{1_{a,b,c}} \in f^{-1}_{N_{tr}}(K)$ and $v_{2_{a,b,c}} \notin K$ implies $u_{2_{a,b,c}} \notin f^{-1}_{N_{tr}}(K)$. Hence, for any two neutrosophic points $u_{1_{a,b,c}}$ and $u_{2_{a,b,c}}, u_1 \neq u_2$ in U, there exists a $N_{tr}\Lambda_P-$open set $f^{-1}_{N_{tr}}(K)$ in U such that $u_{1_{a,b,c}} \in f^{-1}_{N_{tr}}(K), u_{2_{a,b,c}} \notin f^{-1}_{N_{tr}}(K)$ or $u_{1_{a,b,c}} \notin f^{-1}_{N_{tr}}(K), u_{2_{a,b,c}} \in f^{-1}_{N_{tr}}(K)$. Therefore, U is $N_{tr}\Lambda_P - T_0$. □

Theorem 3.10. *Let* $f_{N_{tr}} : (U, \tau_{N_{tr}}) \to (V, \rho_{N_{tr}})$ *be a bijective* $N_{tr}\Lambda_P-$*open function. If* $(U, \tau_{N_{tr}})$ *is* $N_{tr}T_0$,*then* $(V, \rho_{N_{tr}})$*is* $N_{tr}\Lambda_P - T_0$.

Proof. Let $v_{1_{a,b,c}}$ and $v_{2_{a,b,c}}, v_1 \neq v_2$ be any two neutrosophic points in V. Since $f_{N_{tr}}$ is bijective, there exists neutrosophic points $u_{1_{a,b,c}}$ and $u_{2_{a,b,c}}$, $u_1 \neq u_2$ in U such that $f_{N_{tr}}(u_{1_{a,b,c}}) = v_{1_{a,b,c}}$ and $f_{N_{tr}}(u_{2_{a,b,c}}) = v_{2_{a,b,c}}$. Now, since U is $N_{tr}T_0$, there exists a $N_{tr}-$open set K in U such that $u_{1_{a,b,c}} \in L, u_{2_{a,b,c}} \notin K$ or $u_{1_{a,b,c}} \notin K, v_{2_{a,b,c}} \in K$. Since, $f_{N_{tr}}$ is $N_{tr}\Lambda_P-$open,$f_{N_{tr}}(K)$ is $N_{tr}\Lambda_P-$open in V. Also, $u_{1_{a,b,c}} \in K$ implies $f_{N_{tr}}(u_{1_{a,b,c}}) \in f_{N_{tr}}(K)$ implies $v_{1_{a,b,c}} \in f_{N_{tr}}(K)$ and $u_{2_{a,b,c}} \notin K$ implies $v_{1_{a,b,c}} \notin f_{N_{tr}}(K)$. Hence, for any two neutrosophic points $v_{1_{a,b,c}}$ and $v_{2_{a,b,c}}, v_1 \neq v_2$ in V, there exists a $N_{tr}\Lambda_P-$open set $f_{N_{tr}}(K)$ in V such that $v_{1_{a,b,c}} \in f_{N_{tr}}(K), v_{2_{a,b,c}} \notin f_{N_{tr}}(K)$ or $v_{1_{a,b,c}} \notin f_{N_{tr}}(K), v_{2_{a,b,c}} \in f_{N_{tr}}(K)$. Therefore, V is $N_{tr}\Lambda_P - T_0$. □

4. Neutrosophic $\Lambda_P - T_1$ Spaces

Definition 4.1. A neutrosophic topological space $(U, \tau_{N_{tr}})$ is said to be a $N_{tr}\Lambda_P - T_1$ space if for every pair of neutrosophic points $u_{a,b,c}$ and $v_{a',b',c'}, u \neq v$, there exists $N_{tr}\Lambda_P-$open set K and M in $(U, \tau_{N_{tr}})$ such that $u_{a,b,c} \in K, v_{a',b',c'} \notin K$ and $u_{a,b,c} \notin M, v_{a',b',c'} \in M$.

Example 4.2. Let $U = \{u, v\}, \tau_{N_{tr}} = \{0_{N_{tr}}, \mathcal{K}, \mathcal{M}, \mathcal{N}, 1_{N_{tr}}\}$ where
$\mathcal{K} = \{K_i = \{< u, 0, 0, 1 >< v, a_i^{'}, b_i^{'}, c_i^{'} >\} : 0 < a_i \leqslant 1, 0 < b_i \leqslant 1, 0 \leqslant c_i < 1\}$, $\mathcal{M} = \{M_i = \{< u, a_i^{'}, b_i^{'}, c_i^{'} >< v, 0, 0, 1 >\} : 0 < a_i \leqslant 1, 0 < b_i \leqslant 1, 0 \leqslant c_i < 1\}$ and $\mathcal{N} = \{K_i \cup M_i : K_i \in \mathcal{K}, M_i \in \mathcal{M}\}$ are collections of neutrosophic sets in U. Clearly $(U, \tau_{N_{tr}})$ is a $N_{tr}\Lambda_P - T_1$ space.

Theorem 4.3. *Every* $N_{tr}T_1-$ *space is* $N_{tr}\Lambda_P - T_1$.

Proof. Let $(U, \tau_{N_{tr}})$ be a $N_{tr}T_1-$space. Then, for every pair of neutrosophic points $u_{a,b,c}$ and $v_{a',b',c'}, u \neq v$, there exists N_{tr}open sets K and M in $(U, \tau_{N_{tr}})$ such that $u_{a,b,c} \in K, v_{a',b',c'} \notin K$ and $u_{a,b,c} \notin M, v_{a',b',c'} \in M$. By theorem 2.7, K is $N_{tr}\Lambda_P-$open. Hence $(U, \tau_{N_{tr}})$is $N_{tr}\Lambda_P - T_1$. □

Remark 4.4. The above theorem's converse need not hold.

Example 4.5. Let $U = \{u, v\}, \tau_{N_{tr}} = \{0_{N_{tr}}, K_1, K_2, K_3, 1_{N_{tr}}\}$ where $K_1 = \{< u, 0.3, 0.2, 0.1 >< v, 0.4, 0.2, 0.2 > \}$,$K_2 = \{< u, 0.4, 0.2, 0.1 >< v, 0.3, 0.2, 0.1 >\}$ and $K_3 = \{< u, 0.4, 0.2, 0.1 >< v, 0.4, 0.2, 0.1 >\}$. Clearly $(U, \tau_{N_{tr}})$ is $N_{tr}\Lambda_P - T_1$ space. Now for any two neutrosophic points $u_{0.4,0.1,0.6}$ and $v_{0.1,0.7,0.3}$ in U, there exists $N_{tr}\Lambda_P-$open sets K_2 and $M = \{< u, 0.3, 0.6, 0.1 >< v, 0.4, 0.8, 0.1 >\}$ such that $u_{0.4,0.1,0.6} \in K_2$ $v_{0.1,0.7,0.3} \notin K_2$ and $u_{0.4,0.1,0.6} \notin M, v_{0.1,0.7,0.3} \in M$. However, there exists no $N_{tr}-$open set K in U such that $u_{0.4,0.1,0.6} \notin K$, $v_{0.1,0.7,0.3} \in K$. Hence $(U, \tau_{N_{tr}})$ is not $N_{tr}T_1-$space.

Theorem 4.6. *Every* $N_{tr}\Lambda_P - T_1$ *space* $N_{tr}\Lambda_P - T_0$.

Proof. Let $(U, \tau_{N_{tr}})$ be a $N_{tr}\Lambda_P - T_1$ space. Then, for every pair of neutrosophic points $u_{a,b,c}$ and $v_{a',b',c'}$, $u \neq v$, there exists $N_{tr}\Lambda_{P_1}-$open sets K and M in $(U, \tau_{N_{tr}})$ such that $u_{a,b,c} \in K, v_{a',b',c'} \notin K$ or $u_{a,b,c} \notin M, v_{a',b',c'} \in M$. Hence $(U, \tau_{N_{tr}})$is $N_{tr}\Lambda_P - T_0$. □

Remark 4.7. The converse of the above theorem need not be true.

Example 4.8. Consider the neutrosophic topological space $(U, \tau_{N_{tr}})$ which is $N_{tr}\Lambda_P - T_0$. Now, for any two neutrosopjic points $u_{0.6,0.3,0.2}$ and $v_{0.1,0.2,0.6}, u \neq v$ in U, there exists a $N_{tr}\Lambda_P-$open set $\{< u, 0.5, 0.5, 0.5 >$

$< v, 1, 1, 0 >\}$ containing $v_{0.1,0.2,0.6}$ but not $u_{0.6,0.3,0.2}$. However, there exists no $N_{tr}\Lambda_P-$open set K such that $u_{0.6,0.3,0.2} \in K$ and $v_{0.1,0.2,0.6} \notin K$. Hence $(U, \tau_{N_{tr}})$is $N_{tr}\Lambda_P - T_1$.

Theorem 4.9. *A neutrosophic topological space* $(U, \tau_{N_{tr}})$ *is* $N_{tr}\Lambda_P - T_1$ *if and only if for every pair of neutrosophic point* $u_{a,b,c}$ *in* U, $N_{tr}\Lambda_P ker(u_{a,b,c}) = u_{p,q,r}, a \leqslant p; b \leqslant q; c \geqslant r$.

Proof. Let U be a $N_{tr}\Lambda_P - T_1$ space and $N_{tr}\Lambda_P ker(u_{a,b,c}) \neq u_{p,q,r}, a \leqslant p; b \leqslant q; c \geqslant r$. Then, there exists a neutrosophic point $v_{a',b',c'}$ distinct from $u_{a,b,c}$ such that in $v_{a',b',c'} \in N_{tr}\Lambda_P ker(u_{a,b,c})$. This implies that there exists a $N_{tr}\Lambda_P-$open set K in U such that $u_{a,b,c} \in K$ and $v_{a',b',c'} \in K$ which is a contradiction. Conversely, suppose $N_{tr}\Lambda_P ker(u_{a,b,c}) = u_{p,q,r}, a \leqslant p; b \leqslant q; c \geqslant r$. and U is not $N_{tr}\Lambda_P - T_1$. Then, for every pair of neutrosophic points $u_{a,b,c}$ and $v_{a',b',c'}, u \neq v$, there exists a $N_{tr}\Lambda_P-$open set K in U containing $v_{a',b',c'}$ whenever $u_{a,b,c} \in K$. Hence $v_{a',b',c'} \in N_{tr}\Lambda_P ker(u_{a,b,c})$ which is a contradiction. Therefore $U$is a $N_{tr}\Lambda_P - T_1$ space.

□

Theorem 4.10. *A neutrosophic topological space* $(U, \tau_{N_{tr}})$ *is* $N_{tr}\Lambda_P - T_1$ *if and only if for every pair of neutrosophic points* $u_{a,b,c}$ *and* $v_{a',b',c'}, u \neq v$ *in* $U, u_{a,b,c} \notin N_{tr}\Lambda_P ker(v_{a',b',c'})$ *and* $v_{a',b',c'}) \notin N_{tr}\Lambda_P ker(u_{a,b,c})$.

Proof. Proof is similar to theorem 3.7.

□

Theorem 4.11. *A neutrosophic topological space* $(U, \tau_{N_{tr}})$ *is* $N_{tr}\Lambda_P - T_1$ *iff for every pair of neutrosophic points* $u_{a,b,c}$, $v_{a',b',c'}, u \neq v$ *in* $U, N_{tr}\Lambda_P ker(u_{a,b,c}) \cap N_{tr}\Lambda_P ker(v_{a',b',c'}) = 0_{N_{tr}}$

Proof. Let U be a $N_{tr}\Lambda_P - T_1$ space. Then, by theorem 4.10, for every pair of neutrosophic points $u_{a,b,c}$ and $v_{a',b',c'}, u \neq v$ in $U, u_{a,b,c} \notin N_{tr}\Lambda_P ker(v_{a',b',c'})$ and $v_{a',b',c'}) \notin N_{tr}\Lambda_P ker(u_{a,b,c})$. Hence $N_{tr}\Lambda_P ker(u_{a,b,c}) \cap N_{tr}\Lambda_P ker(v_{a',b',c'}) = 0_{N_{tr}}$. Conversely, suppose U is not $N_{tr}\Lambda_P - T_1$. Then, for every pair of neutrosophic points $u_{a,b,c}$ and $v_{a',b',c'}, u \neq v$ in U,either $u_{a,b,c} \in N_{tr}\Lambda_P ker(v_{a',b',c'})$ or $v_{a',b',c'}) \in N_{tr}\Lambda_P ker(u_{a,b,c})$. This implies that $N_{tr}\Lambda_P ker(u_{a,b,c}) \cap$
$N_{tr}\Lambda_P ker(v_{a',b',c'}) \neq 0_{N_{tr}}$. which is a contradiction. Hence U be a $N_{tr}\Lambda_P - T_1$ space.

□

Theorem 4.12. *If* $(U, \tau_{N_{tr}})$ *is a neutrosophic topological space in which every neutrosophic point is* $N_{tr}\Lambda_P-$*closed, then* $(U, \tau_{N_{tr}})$ *is a* $N_{tr}\Lambda_P - T_1$ *space.*

Proof. Let $u_{a,b,c}$ and $v_{a',b',c'}, u \neq v$ be any two neutrosophic points in U. Since $u \neq v, u_{a,b,c} \in (v_{1,1,0})^C$. By assumption, $v_{1,1,0}$ is $N_{tr}\Lambda_p-$closed. Therefore $(v_{1,1,0})^C$ is $N_{tr}\Lambda_p-$open in U and obviously $v_{a',b',c'} \notin (v_{1,1,0})^C$. Hence there exixts a $N_{tr}\Lambda_p-$open set such that $u_{a,b,c} \in (v_{1,1,0})^C$, $v_{a',b',c'} \notin (v_{1,1,0})^C$. Similarly, there exists a $N_{tr}\Lambda_p-$open set $(u_{1,1,0})^C$ such that $u_{a,b,c} \notin (u_{1,1,0})^C$, $v_{a',b',c'} \notin (v_{1,1,0})^C$. Hence $(U, \tau_{N_{tr}})$is $N_{tr}\Lambda_P - T_1$.

□

Theorem 4.13. *Let* $f_{N_{tr}} : (U, \tau_{N_{tr}}) \to (V, \rho_{N_{tr}})$ *be a function between two neutrosophic topological spaces. If* $f_{N_{tr}}$ *is* $N_{tr}\Lambda_P-$*open bijective and* $(U, \tau_{N_{tr}})$ *is* $N_{tr}T_1$,*then* $(V, \rho_{N_{tr}})$*is* $N_{tr}\Lambda_P - T_1$.

Proof. Let $v_{1_{a,b,c}}$and $v_{2_{a,b,c}}, v_1 \neq v_2$ be any two neutrosophic points in V. Since $f_{N_{tr}}$ is bijective, there exists neutrosophic points $u_{1_{a,b,c}}$and $u_{2_{a,b,c}}, u_1 \neq u_2$ in U such that $f_{N_{tr}}(u_{1_{a,b,c}}) = v_{1_{a,b,c}}$and $f_{N_{tr}}(u_{2_{a,b,c}}) = v_{2_{a,b,c}}$. Now, since U is $N_{tr}T_1$, there exist a $N_{tr}-$open sets $K$and M in U such that $u_{1_{a,b,c}} \in K, u_{2_{a,b,c}} \notin K$ and $u_{1_{a,b,c}} \notin M, u_{2_{a,b,c}} \in M$. Since $f_{N_{tr}}$ is $N_{tr}\Lambda_P-$open,$f_{N_{tr}}(K)$ and $f_{N_{tr}}(M)$ are $N_{tr}\Lambda_P-$open in V. Also, $u_{1_{a,b,c}} \in K$ implies $f_{N_{tr}}(u_{1_{a,b,c}}) \in f_{N_{tr}}(K)$ implies $v_{1_{a,b,c}} \in f_{N_{tr}}(K)$ and $u_{2_{a,b,c}} \notin K$ implies $v_{2_{a,b,c}} \notin f_{N_{tr}}(K)$. Similarly,$v_{1_{a,b,c}} \notin f_{N_{tr}}(M)$ and $v_{2_{a,b,c}} \in f_{N_{tr}}(M)$. Hence, for any two neutrosophic points $v_{1_{a,b,c}}$ and $v_{2_{a,b,c}}, v_1 \neq v_2$ in V, there exists a $N_{tr}\Lambda_P-$open set $f_{N_{tr}}(K)$ in V such that $v_{1_{a,b,c}} \in f_{N_{tr}}(K), v_{2_{a,b,c}} \notin f_{N_{tr}}(K)$ and $v_{1_{a,b,c}} \notin f_{N_{tr}}(M), v_{2_{a,b,c}} \in f_{N_{tr}}(M)$. Therefore, V is $N_{tr}\Lambda_P - T_1$.

□

Theorem 4.14. *Let* $f_{N_{tr}} : (U, \tau_{N_{tr}}) \to (V, \rho_{N_{tr}})$ *be a function between two neutrosophic topological spaces. If* $f_{N_{tr}}$ *is one-one,* $N_{tr}\Lambda_P-$*continuous and* $(V, \rho_{N_{tr}})$ *is* $N_{tr}T_1$,*then* $(U, \tau_{N_{tr}})$*is* $N_{tr}\Lambda_P - T_1$.

Proof. Let $u_{1_{a,b,c}}$and $u_{2_{a,b,c}}, u_1 \neq u_2$ be any two neutrosophic points in U. Since $f_{N_{tr}}$ is one-one, there exists neutrosophic points $v_{1_{a,b,c}}$and $v_{2_{a,b,c}}$, in V such that $f_{N_{tr}}(u_{1_{a,b,c}}) = v_{1_{a,b,c}}$, $f_{N_{tr}}(u_{2_{a,b,c}}) = v_{2_{a,b,c}}$ and $v_1 \neq v_2$. Hence $u_{1_{a,b,c}} = f^{-1}_{N_{tr}}(v_{1_{a,b,c}})$ and $u_{2_{a,b,c}} = f^{-1}_{N_{tr}}(v_{2_{a,b,c}})$. Now, since V is $N_{tr}T_1$, there exists a $N_{tr}-$open sets K and M in V such that $v_{1_{a,b,c}} \in K, v_{2_{a,b,c}} \notin K$ and $v_{1_{a,b,c}} \notin M, v_{2_{a,b,c}} \in M$. Again, since $f_{N_{tr}}$ is $N_{tr}\Lambda_P-$continuous,$f^{-1}_{N_{tr}}(K)$ and $f^{-1}_{N_{tr}}(M)$ is $N_{tr}\Lambda_P-$open in U. Also, $v_{1_{a,b,c}} \in K$ implies $f^{-1}_{N_{tr}}(v_{1_{a,b,c}}) \in f^{-1}_{N_{tr}}(K)$ implies $u_{1_{a,b,c}} \in f^{-1}_{N_{tr}}(K)$ and $v_{2_{a,b,c}} \notin K$ implies $u_{2_{a,b,c}} \notin f^{-1}_{N_{tr}}(K)$.Similarly $v_{1_{a,b,c}} \notin M$ implies $u_{1_{a,b,c}} \notin f^{-1}_{N_{tr}}(M)$ and $v_{2_{a,b,c}} \in M$ implies $u_{2_{a,b,c}} \notin f^{-1}_{N_{tr}}(M)$. Hence, for any two neutrosophic points $u_{1_{a,b,c}}$ and $u_{2_{a,b,c}}, u_1 \neq u_2$ in U, there exists a $N_{tr}\Lambda_P-$open sets $f^{-1}_{N_{tr}}(K)$ and $f^{-1}_{N_{tr}}(M)$ in U such that $u_{1_{a,b,c}} \in f^{-1}_{N_{tr}}(K), u_{2_{a,b,c}} \notin f^{-1}_{N_{tr}}(K)$ and $u_{1_{a,b,c}} \notin f^{-1}_{N_{tr}}(M), u_{2_{a,b,c}} \in f^{-1}_{N_{tr}}(M)$. Therefore, U is $N_{tr}\Lambda_P - T_1$.

□

Theorem 4.15. *Let* $f_{N_{tr}} : (U, \tau_{N_{tr}}) \to (V, \rho_{N_{tr}})$ *be a function between two neutrosophic topological spaces. If* $f_{N_{tr}}$ *is one-one,* $N_{tr}\Lambda_P-$*irresolute and* $(V, \rho_{N_{tr}})$ *is* $N_{tr}\Lambda_P - T_1$,*then* $(U, \tau_{N_{tr}})$*is* $N_{tr}\Lambda_P - T_1$.

Proof. Let $u_{1_{a,b,c}}$and $u_{2_{a,b,c}}, u_1 \neq u_2$ be any two neutrosophic points in U. Since $f_{N_{tr}}$ is one-one, there exists neutrosophic points $v_{1_{a,b,c}}$and $v_{2_{a,b,c}}$ in V such that $f_{N_{tr}}(u_{1_{a,b,c}}) = v_{1_{a,b,c}}$, $f_{N_{tr}}(u_{2_{a,b,c}}) = v_{2_{a,b,c}}$ and $v_1 \neq v_2$. Hence $u_{1_{a,b,c}} = f^{-1}_{N_{tr}}(v_{1_{a,b,c}})$ and $u_{2_{a,b,c}} = f^{-1}_{N_{tr}}(v_{2_{a,b,c}})$. Now, since V is $N_{tr}\Lambda_P - T_1$, there exists a $N_{tr}\Lambda_p-$open sets K and M in V such that $v_{1_{a,b,c}} \in K, v_{2_{a,b,c}} \notin K$ and $v_{1_{a,b,c}} \notin M, v_{2_{a,b,c}} \in M$. Again, since $f_{N_{tr}}$ is $N_{tr}\Lambda_P-$irresolute,$f^{-1}_{N_{tr}}(K)$ and $f^{-1}_{N_{tr}}(M)$ are $N_{tr}\Lambda_P-$open in U. Also, $v_{1_{a,b,c}} \in K$ implies $f^{-1}_{N_{tr}}(v_{1_{a,b,c}}) \in f^{-1}_{N_{tr}}(K)$ implies $u_{1_{a,b,c}} \in f^{-1}_{N_{tr}}(K)$ and $v_{2_{a,b,c}} \notin K$ implies $u_{2_{a,b,c}} \notin f^{-1}_{N_{tr}}(K)$. Similarly, $v_{1_{a,b,c}} \notin M$ implies $u_{1_{a,b,c}} \notin f^{-1}_{N_{tr}}(M)$ and $v_{2_{a,b,c}} \in M$ implies $u_{2_{a,b,c}} \in f^{-1}_{N_{tr}}(M)$. Hence, for any two neutrosophic points $u_{1_{a,b,c}}$ and $u_{2_{a,b,c}}, u_1 \neq u_2$ in U, there exists a $N_{tr}\Lambda_P-$open sets $f^{-1}_{N_{tr}}(K)$ and $f^{-1}_{N_{tr}}(K)$ in U such that $u_{1_{a,b,c}} \in f^{-1}_{N_{tr}}(K), u_{2_{a,b,c}} \notin f^{-1}_{N_{tr}}(K)$ and $u_{1_{a,b,c}} \notin f^{-1}_{N_{tr}}(M), u_{2_{a,b,c}} \in f^{-1}_{N_{tr}}(M)$. Therefore, U is $N_{tr}\Lambda_P - T_1$.

□

5. Neutrosophic $\Lambda_P - T_2$ Spaces

Definition 5.1. A neutrosophic topological space $(U, \tau_{N_{tr}})$ is said to be a $N_{tr}\Lambda_P - T_2$ space or $N_{tr}\Lambda_P-$ hausdorff space if for every pair of neutrosophic points $u_{a,b,c}$ and $v_{a',b',c'}, u \neq v$, there exist $N_{tr}\Lambda_P-$ open sets K and M in $(U, \tau_{N_{tr}})$ such that $u_{a,b,c} \in K, v_{a',b',c'} \in M$ and $K \cap M = 0_{N_{tr}}$.

Example 5.2. Let $U = \{u, v\}, \tau_{N_{tr}} = \{0_{N_{tr}}, K_1, K_2, 1_{N_{tr}}\}$ where $K_1 = \{< u, 0, 0, 1 >< v, 1, 1, 0 >\}, K_2 = \{< u, 1, 1, 0 >< v, 0, 0, 1 >\}$ Clearly $(U, \tau_{N_{tr}})$ is $N_{tr}\Lambda_P - T_2$ space.

Theorem 5.3. *Every* $N_{tr}\Lambda_P - T_2$ *space* $N_{tr}\Lambda_P - T_1$.

Proof. Let $u_{a,b,c}, v_{a',b',c'}, u \neq v$ be any two neutrosophic points in U. Since U is $N_{tr}\Lambda_P - T_2$, there exist $N_{tr}\Lambda_P-$open sets K and M in $(U, \tau_{N_{tr}})$ such that $u_{a,b,c} \in K, v_{a',b',c'}, \in M$ and $K \cap M = 0_{N_{tr}}$. Since $u_{a,b,c} \in K$and $K \cap M = 0_{N_{tr}}, u_{a,b,c} \notin M$. Similarly,$v_{a',b',c'}, \notin K$. Hence there exist $N_{tr}\Lambda_P-$open sets K and M in $(U, \tau_{N_{tr}})$ such that $u_{a,b,c} \in K, v_{a',b',c'} \notin K$ and $u_{a,b,c} \notin M, v_{a',b',c'} \in M$. Hence $(U, \tau_{N_{tr}})$is $N_{tr}\Lambda_P - T_1$. □

Remark 5.4. The converse of the above theorem need not be true.

Example 5.5. Consider the neutrosophic topological space $(U, \tau_{N_{tr}})$ which is $N_{tr}\Lambda_P - T_1$. Now, for any two neutrosophic points $u_{0.4,0.1,0.6}$ and $v_{0.1,0.7,0.5}$ in U, there exists $N_{tr}\Lambda_P-$open sets $K = \{< u, 0.4, 0.2, 0.1 >< v, 0.3, 0.2, 0.1 >\}$ and $M = \{< u, 0.3, 0.6, 0.1 >< v, 0.4, 0.8, 0.1 >\}$ such that $u_{0.4,0.1,0.6} \in K$, $v_{0.1,0.7,0.3} \notin K$ and $u_{0.4,0.1,0.6} \notin M, v_{0.1,0.7,0.3} \in M$. However,$K \cap M \neq 0_{N_{tr}}$. Therefore U is not $N_{tr}\Lambda_P - T_2$.

Theorem 5.6. *Let* $f_{N_{tr}} : (U, \tau_{N_{tr}}) \to (V, \rho_{N_{tr}})$ *be a function between two neutrosophic topological spaces. If* $f_{N_{tr}}$ *is* $N_{tr}\Lambda_P-$*open bijective and* $(U, \tau_{N_{tr}})$ *is* $N_{tr}T_2$,*then* $(V, \rho_{N_{tr}})$*is* $N_{tr}\Lambda_P - T_2$.

Proof. Let $v_{1_{a,b,c}}$and $v_{2_{a,b,c}}, v_1 \neq v_2$ be any two neutrosophic points in V. Since $f_{N_{tr}}$ is bijective, there exists neutrosophic points $u_{1_{a,b,c}}$and $u_{2_{a,b,c}}, u_1 \neq u_2$ in U such that $f_{N_{tr}}(u_{1_{a,b,c}}) = v_{1_{a,b,c}}$and $f_{N_{tr}}(u_{2_{a,b,c}}) = v_{2_{a,b,c}}$. Now, since U is $N_{tr}T_2$, there exist a $N_{tr}-$open set Kand M in U such that $u_{1_{a,b,c}} \in K, u_{2_{a,b,c}} \in M$ and $K \cap M \neq 0_{N_{tr}}$. Since $f_{N_{tr}}$ is $N_{tr}\Lambda_P-$open,$f_{N_{tr}}(K)$ and $f_{N_{tr}}(M)$ are $N_{tr}\Lambda_P-$open in V. Also, $u_{1_{a,b,c}} \in K$ implies $f_{N_{tr}}(u_{1_{a,b,c}}) \in f_{N_{tr}}(K)$ implies $v_{1_{a,b,c}} \in f_{N_{tr}}(K)$ and $u_{2_{a,b,c}} \in M$ implies $v_{2_{a,b,c}} \in f_{N_{tr}}(M)$. Hence, for any two neutrosophic points $v_{1_{a,b,c}}$ and $v_{2_{a,b,c}}, v_1 \neq v_2$ in V, there exists a $N_{tr}\Lambda_P-$open sets $f_{N_{tr}}(K)$ and $f_{N_{tr}}(M)$ in V such that $v_{1_{a,b,c}} \in f_{N_{tr}}(K), v_{2_{a,b,c}} \in f_{N_{tr}}(M)$ and $f_{N_{tr}}(K) \cap f_{N_{tr}}(M) = 0_{N_{tr}}$. Therefore, V is $N_{tr}\Lambda_P - T_2$. □

Theorem 5.7. *Let* $f_{N_{tr}} : (U, \tau_{N_{tr}}) \to (V, \rho_{N_{tr}})$ *be a function between two neutrosophic topological spaces. If* $f_{N_{tr}}$ *is one-one,* $N_{tr}\Lambda_P-$*continuous and* $(V, \rho_{N_{tr}})$ *is* $N_{tr}T_2$,*then* $(U, \tau_{N_{tr}})$*is* $N_{tr}\Lambda_P - T_2$.

Proof. Let $u_{1_{a,b,c}}$and $u_{2_{a,b,c}}, u_1 \neq u_2$ be any two neutrosophic points in U. Since $f_{N_{tr}}$ is one-one, there exists neutrosophic points $v_{1_{a,b,c}}$and $v_{2_{a,b,c}}$ in V such that $f_{N_{tr}}(u_{1_{a,b,c}}) = v_{1_{a,b,c}}$, $f_{N_{tr}}(u_{2_{a,b,c}}) = v_{2_{a,b,c}}$ and $v_1 \neq v_2$. Then $u_{1_{a,b,c}} = f^{-1}_{N_{tr}}(v_{1_{a,b,c}})$ and $u_{2_{a,b,c}} = f^{-1}_{N_{tr}}(v_{2_{a,b,c}})$. Now, since V is $N_{tr}T_2$, there exists a $N_{tr}-$open sets K and M in V such that $v_{1_{a,b,c}} \in K, v_{2_{a,b,c}} \in M$ and $K \cap M \neq 0_{N_{tr}}$. Again, since $f_{N_{tr}}$ is $N_{tr}\Lambda_P-$continuous,$f^{-1}_{N_{tr}}(K)$ and $f^{-1}_{N_{tr}}(M)$ is $N_{tr}\Lambda_P-$open in U. Also, $v_{1_{a,b,c}} \in K$ implies $f^{-1}_{N_{tr}}(v_{1_{a,b,c}}) \in f^{-1}_{N_{tr}}(K)$ implies $u_{1_{a,b,c}} \in f^{-1}_{N_{tr}}(K)$. Similarly, $v_{2_{a,b,c}} \in M$ implies $u_{2_{a,b,c}} \in f^{-1}_{N_{tr}}(M)$. Hence, for any two neutrosophic points $u_{1_{a,b,c}}$ and $u_{2_{a,b,c}}, u_1 \neq u_2$ in U, there exists a $N_{tr}\Lambda_P-$open sets $f^{-1}_{N_{tr}}(K)$ and $f^{-1}_{N_{tr}}(M)$ in U such that $u_{1_{a,b,c}} \in f^{-1}_{N_{tr}}(K), u_{2_{a,b,c}} \in f^{-1}_{N_{tr}}(M)$ and $f_{N_{tr}}(K) \cap f_{N_{tr}}(M) = 0_{N_{tr}}$. Therefore, U is $N_{tr}\Lambda_P - T_2$. □

Theorem 5.8. *Let* $f_{N_{tr}} : (U, \tau_{N_{tr}}) \to (V, \rho_{N_{tr}})$ *be a function between two neutrosophic topological spaces. If* $f_{N_{tr}}$ *is one-one,* $N_{tr}\Lambda_P-$*irresolute and* $(V, \rho_{N_{tr}})$ *is* $N_{tr}\Lambda_P - T_2$,*then* $(U, \tau_{N_{tr}})$*is* $N_{tr}\Lambda_P - T_2$.

Proof. Proof is similar to theorem 5.7 □

Definition 5.9. A neutrosophic topological space $(U, \tau_{N_{tr}})$ is said to be a $N_{tr}\Lambda_P - qT_2$ space or $N_{tr}\Lambda_P-$quasi hausdorff space if for every pair of neutrosophic points $u_{a,b,c}$ and $v_{a',b',c'}, u \neq v$, there exist $N_{tr}\Lambda_P-$open sets K and M in $(U, \tau_{N_{tr}})$ such that $u_{a,b,c} \in K, v_{a',b',c'} \in M$ and $K\hat{q}M$.

Example 5.10. Let $U = \{u, v\}, \tau_{N_{tr}} = \{0_{N_{tr}}, K_1, K_2, K_3, K_4, 1_{N_{tr}}\}$ where $K_1 = \{< u, 0.2, 0.7, 0.3 >< v, 0, 0, 1 >\}, K_2 = \{< u, 0, 0.8, 1 >< v, 0.3, 0.1, 0.1 >\}, K_3 = \{< u, 0, 0.7, 1 >< v, 0, 0, 1 >\}$ and $K_4 = \{< u, 0.2, 0.8, 0.3 >< v, 0.3, 0.2, 0.1 >\}$ Clearly $(U, \tau_{N_{tr}})$ is a $N_{tr}\Lambda_P - qT_2$ space.

Theorem 5.11. *A neutrosophic topological space $(U, \tau_{N_{tr}})$ is $N_{tr}\Lambda_P - qT_2$ if and only if for every neutrosophic point $v_{a',b',c'}$ distinct from $u_{a,b,c}$, there exists a $N_{tr}\Lambda_P-$open set K containing $u_{a,b,c}$ such that $v_{a',b',c'}\hat{q}N_{tr}\Lambda_P Cl(K)$.*

Proof. Let U be a $N_{tr}\Lambda_P - qT_2$ space and $u_{a,b,c}, v_{a',b',c'}$ be neutrosophic points in U. Since U is $N_{tr}\Lambda_P - qT_2$ and $u \neq v$, there exists $N_{tr}\Lambda_P-$open sets K and M in U such that $u_{a,b,c} \in K, v_{a',b',c'} \in M$ and $K\hat{q}M$. Now $K\hat{q}M$ implies $K \subseteq M^C$ implies $N_{tr}\Lambda_P Cl(K) \subseteq M^C$ implies $M \subseteq (N_{tr}\Lambda_P Cl(K))^C$ implies $v_{a',b',c'} \in (N_{tr}\Lambda_P Cl(K))^C$. Hence $v_{a',b',c'}\hat{q}N_{tr}\Lambda_P Cl(K)$. Conversely,suppose for every neutrosophic point $v_{a',b',c'}$ distinct from $u_{a,b,c}$, there exists a $N_{tr}\Lambda_P-$open set K such that $u_{a,b,c} \in K$ and $v_{a',b',c'}\hat{q}N_{tr}\Lambda_P Cl(K)$. Then, for every pair of neutrosophic points $u_{a,b,c}$ and $v_{a',b',c'}, u \neq v$, there exist $N_{tr}\Lambda_P-$open sets K and $M = (N_{tr}\Lambda_P Cl(K))^C$ such that $u_{a,b,c} \in K, v_{a',b',c'} \in M$ and $K \subseteq M^C$. Hence, U is $N_{tr}\Lambda_P - qT_2$ space. □

Theorem 5.12. *Let $(U, \tau_{N_{tr}})$ be a $N_{tr}\Lambda_P - qT_2$ space if and only if for every neutrosophic point $u_{a,b,c}$ in $U, \cap\{N_{tr}\Lambda_P Cl(K) : u_{a,b,c} \in K$ and $K \in N_{tr}\Lambda_P O(U, \tau_{N_{tr}})\} = u_{p,q,r}, a \leqslant p; b \leqslant q; c \geqslant r$.*

Proof. Let U be a $N_{tr}\Lambda_P - qT_2$ space and $u_{a,b,c}, v_{1,1,0}, u \neq v$ be neutrosophic points in U. Then, by theorem 5.11, there exists a $N_{tr}\Lambda_P-$open set K containg $u_{a,b,c}$ such that $v_{1,1,0}\hat{q}N_{tr}\Lambda_P Cl(K)$. This implies $v_{1,1,0} \in (N_{tr}\Lambda_P Cl(K))^C$ implies $v_{1,1,0} \cap N_{tr}\Lambda_P Cl(K) = 0_{N_{tr}}$ implies $v_{a',b',c'} \cap N_{tr}\Lambda_P Cl(K) = 0_{N_{tr}}$ implies $v_{a',b',c'} \notin N_{tr}\Lambda_P Cl(K)$. Hence, for every neutrosophic point $v_{a',b',c'}$ distinct from $u_{a,b,c}, v_{a',b',c'} \notin \cap\{N_{tr}\Lambda_P Cl(K) : u_{a,b,c} \in K$ and $K \in N_{tr}\Lambda_P O(U, \tau_{N_{tr}})\}$. Obviously, $u_{a,b,c}, \in \cap\{N_{tr}\Lambda_P Cl(K) : u_{a,b,c} \in K$ and $K \in N_{tr}\Lambda_P O(U, \tau_{N_{tr}})\}$ since $u_{a,b,c} \in K$. Therefore $\cap\{N_{tr}\Lambda_P Cl(K) : u_{a,b,c} \in K$ and $K \in N_{tr}\Lambda_P O(U, \tau_{N_{tr}})\} = u_{p,q,r}, a \leqslant p; b \leqslant q; c \geqslant r$. The converse part can be proved by retracing the above steps. □

Theorem 5.13. *Let $(U, \tau_{N_{tr}})$ be a $N_{tr}\Lambda_P - T_i (i = 0, 1, 2)$ space in which the class of all $N_{tr}\Lambda_P-$open sets is closed under finite intersection. Then every neutrosophic open subspace of $(U, \tau_{N_{tr}})$ is also a $N_{tr}\Lambda_P - T_i (i = 0, 1, 2)$ space.*

Proof. The following proof is given for $i = 0$. The other cases $(i = 1, 2)$ can be proved similarly. Let $(S, \tau^S_{N_{tr}})$ be a neutrosophic open subspace of $(U, \tau_{N_{tr}}), u_{a,b,c}$ and $v_{a',b',c'}, u \neq v$ be any two neutrosophic points in S. Then $u_{a,b,c}$ and $v_{a',b',c'}, u \neq v$ are neutrosophic points in U. Since U is $N_{tr}\Lambda_P - T_0$, there exists a $N_{tr}\Lambda_P-$open set K in U such that $u_{a,b,c} \in K, v_{a',b',c'} \notin K$ or $u_{a,b,c} \notin K, v_{a',b',c'} \in K$. Now, without loss of generality, let us assume that $u_{a,b,c} \in K, v_{a',b',c'} \notin K$. Then $u_{a,b,c} \in K \cap 1^S_{N_{tr}}$ and $v_{a',b',c'} \notin K \cap 1^S_{N_{tr}}$. By theorem 2.7,$1^S_{N_{tr}}$ is $N_{tr}\Lambda_P-$open in U and by hypothesis,$K \cap 1^S_{N_{tr}}$ is $N_{tr}\Lambda_P-$open in U. Thus, $K \cap 1^S_{N_{tr}}$ is $N_{tr}\Lambda_P-$open in S. Hence there exists a $N_{tr}\Lambda_P-$open set$K \cap 1^S_{N_{tr}}$ in S such that $u_{a,b,c} \in K \cap 1^S_{N_{tr}}$ and $v_{a',b',c'} \notin K \cap 1^S_{N_{tr}}$. Therefore, $(S, \tau^S_{N_{tr}})$ is $N_{tr}\Lambda_P - T_0$. □

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