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Intuitionistic complex Fuzzy sets in decision support systems: A choquet operated data mining-ANN approach

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Abstract

This work introduces a novel set, the Intuitionistic Complex Fuzzy Set (ICFS), that expands traditional intuitionistic fuzzy sets into a complex-valued domain and captures interacting attributes more effectively in the decision support framework based on ICFS. A new aggregation operator called the Intuitionistic Complex Fuzzy Einstein Correlated Geometric (ICFECG) operator and a new score and accuracy function for the ICFS are proposed and to ensure theoretical robustness, rigorous proofs are provided for multiple theorems associated with the newly developed ICFECG operator, the score and the accuracy functions. This operator effectively combines expert opinions while preserving both the amplitude and phase components of complex uncertainty, thereby ensuring that the aggregated information accurately reflects the full structure of the intuitionistic complex fuzzy evaluations. To improve efficiency in solving MAGDM problems, a data miningbased dimensionality reduction strategy that helps identify and remove redundant or weakly influential attributes is introduced. Artificial Neural Network (ANN) techniques are also incorporated to enhance the learning ability and optimization of the decision-support process. A new defuzzification function is proposed to integrate all the ICFS components, yielding a crisp value for enhancing the data mining and ANN computations. The final hybrid model combines ICFS theory, the ICFECG operator, data mining, and ANN optimization which effectively handles high-dimensional, correlated, and uncertain information arising in the decision making environment. A numerical case study shows that our methodology reduces the computational load, removes insignificant alternatives, and significantly improves decision accuracy, stability, and reliability.

Keywords: Intuitionistic complex Fuzzy set, MAGDM, Choquet integral operator, Data mining, ANN.

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1. Introduction

In this paper, we introduce a new theoretical framework called the Intuitionistic Complex Fuzzy Set (ICFS). Traditional Intuitionistic Fuzzy Sets (IFS) measure how much an element belongs to a fuzzy concept based on a single attribute. However, many real-world decision-making problems involve several related characteristics that a one-dimensional structure cannot capture effectively. The ICFS addresses this issue by extending the traditional IFS into a two-dimensional complex domain. Each dimension represents an interacting attribute while keeping a real-valued uncertainty measure in the form of an

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IFS, which allows for a better description of multidimensional and connected uncertainty. Driven by this need, the study evaluates the ability of the ICFS framework to provide a clearer and more accurate representation of alternatives in MAGDM settings compared to traditional IFS formulations. It also assesses how well the proposed ICFS-based aggregation mechanism, particularly the Intuitionistic Complex Fuzzy Einstein Correlated Geometric (ICFECG) operator, improves the accuracy and distinction of alternative rankings. Additionally, the study looks at how integrating complementary techniques like data mining and perceptron learning helps strengthen the robustness, consistency, and understanding of the overall ICFS-driven decision-making model. These factors guide the methodological development and form the analytical foundation of this work.

Literature Survey

Atanassov [1] presented the Intuitionistic Fuzzy Sets (IFSs) and demonstrated a number of features related to the operations. Mitchell [11] assumed the problem of defining a correlation coefficient between two intuitionistic fuzzy sets and, by looking at a number of test scenarios, demonstrated that the results were reasonable. Popescu et al. [12] presented the possibility of controlling the induction driving using neural systems. Lin et al. [10] constructed a new way to find relationships between fuzzy itemsets using fuzzy statistics and the rules they produced are known as fuzzy correlation rules. Du & Swamy [4] introduced neural networks and machine learning in a statistical framework. Robinson & Amirtharaj [13] proposed a new framework called the MAGDM-Miner for mining correlation rules from trapezoidal intuitionistic fuzzy data efficiently. Using this MAGDM-Miner, a decision-maker can overcome the drawbacks in the conventional methods of Decision Support Systems (DSS), especially when dealing with large datasets. Robinson & Jeeva [14] proposed a new framework for mining correlation rules from trapezoidal intuitionistic fuzzy information. Garg [7] proposed a novel correlation coefficient and weighted correlation coefficient formulation to measure the relationship between two Pythagorean Fuzzy Sets (PFSs). Giveki et al [8] proposed a new approach for training Radial Basis Function Neural Networks (RBFNN) using the fuzzy C-means clustering algorithm based on Atanassovs intuitionistic fuzzy set (A-IFS) theory. Chaira [3] defined various types of fuzzy operations that are performed on fuzzy sets, such as fuzzy union, fuzzy intersection, fuzzy complement, fuzzy algebraic sum, and algebraic product. Wan & Dong [15] developed a new method for solving the cloud service selection issues found in Multi-attribute group decision making (MAGDM) problems. In this method, the ratings of the alternatives on attributes in individual decision matrices are Interval-Valued Intuitionistic Fuzzy Sets (IVIFSs), which can flexibly describe the preferences of experts on qualitative attributes. Enginolu and Arslan [6] proposed the concept of intuitionistic fuzzy parameterized intuitionistic fuzzy soft matrices (ifpifs-matrices) and several of their basic properties. Atanassov et al. [2] developed an ANN with deep learning techniques that will work on IFS environments. Joshi et al. [9] discussed the basic working of neurons, compared and contrasted Biological Neural Networks (BNN) with Artificial Neural Networks (ANN), and discussed different types of learning systems used in ANN. Ejegwa et al. [5] demonstrated a new method based on the Fermatean Fuzzy Set (FFS), an advanced variant of fuzzy sets applicable for handling uncertainty and vagueness in complex decision-making scenarios. Zeeshan et al. [16] introduced some set-theoretic operations and laws of the Interval-Valued Complex Fuzzy Soft Sets (IV-CFSSs) and developed a new decision-making method using the interval-valued complex fuzzy distance measures under the environments of IV-CFSSs.

Research Gap and Objectives

Although fuzzy set theory and Multiple Attribute Group Decision-Making (MAGDM) methods have advanced significantly, existing models often face challenges in managing multidimensional, uncertain, and correlated information. Traditional intuitionistic fuzzy sets and complex fuzzy sets may fail to fully capture the interdependencies among attributes, and most aggregation operators do not account for such correlations, potentially reducing decision accuracy. Moreover, conventional approaches rarely incorporate data-driven preprocessing, such as dimensionality reduction, which can eliminate redundant or less relevant attributes and enhance computational efficiency. While Artificial Neural Networks (ANNs) have shown promise in adaptive decision-support systems, their integration with intuitionistic complex fuzzy environments remains limited, particularly in frameworks that combine uncertainty modelling, correlation-aware aggregation, and intelligent learning. To address these gaps, this paper proposes a hybrid decision-making framework that introduces a novel Intuitionistic Complex Fuzzy Set (ICFS) and the Intuitionistic Complex Fuzzy Einstein Correlated Geometric (ICFECG) operator for aggregating expert judgments. The framework integrates data mining-based dimensionality reduction and ANN techniques to enhance learning, adaptability, and computational efficiency. The primary objectives of this paper are to propose a hybrid ICFS-based decision-making framework that unifies the ICFECG aggregation operator, data mining-driven dimensionality reduction, and ANN-assisted optimization. The proposed model aims to establish a computationally intelligent and uncertainty-resilient approach for solving complex MAGDM problems with higher precision and interpretability.

2. A Novel Set Constructed from Complex IFS Dynamics

In this section, we introduce a new set arising naturally from the structure of complex iterated function systems. The following definition formalizes this construction.

Definition 2.1. An Intuitionistic Complex Fuzzy Set (ICFS) A in $\mathbb{C}$ is given by:

$$A = \{\langle x\,(\mu_A(x), \nu_A(x))\rangle\, | x = (a + ib) \in \mathbb{C},\ a, b \in (0, 1]\},$$

where $\mu_A\colon \mathbb{C} \to [0, 1]$ and $\nu_A\colon \mathbb{C} \to [0, 1]$ satisfy the condition

$$0 \leqslant \mu_A(x) + \nu_A(x) \leqslant 1, \forall\ x \in \mathbb{C}.$$

The numbers $\mu_A(x)$ and $\nu_A(x)$ represent respectively, the membership degree and non-membership degree of the element x to the set A.

It is assumed in the suggested Intuitionistic Complex Fuzzy (ICF) framework that the real and imaginary components of the complex element cause a monotonic variation in the membership degree $\mu_A(a, b)$. In particular, when the real part a and the imaginary part b reach greater values, $\mu_A(a, b)$ show larger magnitudes; on the other hand, a lower a and b indicate that $\mu_A(a, b)$ have smaller magnitudes. The ICFS facilitates the handling of two characteristics simultaneously using a single complex representation, thereby enriching the analysis beyond traditional IFS.

3. Basic Arithmetic Operators of ICFS

Having established the structure of the new set, we now examine how the basic arithmetic operations behave within this framework. The following results describe addition, multiplication, scalar multiplication, λ -exponentiation, subtraction and division operations on ICFS.

Definition 3.1. Let $\delta_1 = \langle (a_1 + ib_1)\,(\mu_1, \nu_1)\rangle$, $\delta_2 = \langle (a_2 + ib_2)\,(\mu_2, \nu_2)\rangle$ be two ICFNs and $0 \leqslant \lambda \leqslant 1$.

- $\delta_1 + \delta_2 = \langle ((a_1 + a_2) + i\,(b_1 + b_2))\,((\mu_1 + \mu_2 - \mu_1\mu_2), (\nu_1\nu_2))\rangle$
- $\delta_1 \times \delta_2 = \langle ((a_1 \times a_2) + i\,(b_1 \times b_2))\,((\mu_1\mu_2), (\nu_1 + \nu_2 - \nu_1\nu_2))\rangle$
- $\lambda\delta_1 = \left\langle (\lambda a_1 + i\lambda b_1)\left(\left(1 - (1 - \mu_1)^{\lambda}\right), \left({\nu_1}^{\lambda}\right)\right)\right\rangle$
- ${\delta_1}^{\lambda} = \left\langle \left({a_1}^{\lambda} + i{b_1}^{\lambda}\right)\left(\left({\mu_1}^{\lambda}\right), \left(1 - (1 - \nu_1)^{\lambda}\right)\right)\right\rangle$
- $\delta_1 - \delta_2 = \begin{cases} \left\langle (|a_1 - a_2| + i\,|b_1 - b_2|)\left(\left(\frac{\mu_1 - \mu_2}{1 - \mu_2}\right), \left(\frac{\nu_1}{\nu_2}\right)\right)\right\rangle, & \text{if } 0 \leqslant \frac{\nu_1}{\nu_2} \leqslant \frac{1 - \mu_1}{1 - \mu_2} \leqslant 1 \\ \langle (|a_1 - a_2| + i\,|b_1 - b_2|)\,(0, 1)\rangle, & \text{otherwise} \end{cases}$

- $\frac{\delta_1}{\delta_2} = \begin{cases} \left\langle \left(\frac{min(a_1,a_2)}{max(a_1,a_2)} + i\frac{min(b_1,b_2)}{max(b_1,b_2)} \right) \left(\left(\frac{\mu_1}{\mu_2} \right), \left(\frac{\nu_1 - \nu_2}{1-\nu_2} \right) \right) \right\rangle, & \text{if } 0 \leqslant \frac{\mu_1}{\mu_2} \leqslant \frac{1-\nu_1}{1-\nu_2} \leqslant 1 \\ \left\langle \left(\frac{min(a_1,a_2)}{max(a_1,a_2)} + i\frac{min(b_1,b_2)}{max(b_1,b_2)} \right) (0,1) \right\rangle, & otherwise \end{cases}$

Having defined addition and multiplication on the ICFS, we now investigate how these operations extend and behave under the broader structure of the set. The following theorems formalize these extensions.

Theorem 3.2. ***(Extension of the Addition of ICFNs up to n terms)*** *For any collection of ICFNs* $\delta_j = \left\langle (a_j + ib_j), (\mu_j, \nu_j) \right\rangle$ *(j = 1,2,. . . ,n), the combined value obtained by using the addition operator is given as*
$\delta_1 + \delta_2 + \ldots + \delta_n = \left\langle \sum_{j=1}^{n} (a_j + ib_j) \ , \left[\sum_{j=1}^{n} \mu_j - \sum_{1\leqslant i<j\leqslant n} (\mu_i\mu_j) + \ldots + \right.\right.$
$\left.\left. +(-1)^{n-1} \prod_{j=1}^{n} (\mu_j) \right], \left[\prod_{j=1}^{n} \nu_j \right] \right\rangle.$

Proof. We will demonstrate that the result holds using the mathematical induction method. Consider two ICFNs δ_1 and δ_2. Then the Addition operator for n=2 is given as:
$\delta_1 + \delta_2 = \langle (a_1 + a_2 + i(b_1 + b_2)), [\mu_1 + \mu_2 - \mu_1\mu_2], [\nu_1\nu_2] \rangle.$
Hence,

$$\delta_1 + \delta_2 = \left\langle \sum_{j=1}^{2} (a_j + ib_j), \left[\sum_{j=1}^{2} \mu_j - \prod_{j=1}^{2} (\mu_j) \right], \left[\prod_{j=1}^{2} \nu_j \right] \right\rangle.$$

Assume that the result holds for $n = m$, where m is any natural number. Then,

$$\delta_1 + \delta_2 + \ldots + \delta_m$$
$$= \left\langle \sum_{j=1}^{m} (a_j + ib_j), \left[\sum_{j=1}^{m} \mu_j - \sum_{1\leqslant i<j\leqslant m} (\mu_i\mu_j) + \ldots + (-1)^{m-1} \prod_{j=1}^{m} (\mu_j) \right], \left[\prod_{j=1}^{m} \nu_j \right] \right\rangle.$$

Now we can prove the result for $n = m + 1$;

$$\delta_1 + \delta_2 + \ldots + \delta_m + \delta_{m+1}$$
$$= \left\langle \left(\sum_{j=1}^{m} (a_j + ib_j) + a_{m+1} + ib_{m+1} \right), \left[\left(\sum_{j=1}^{m} \mu_j - \sum_{1\leqslant i<j\leqslant m} (\mu_i\mu_j) + \ldots + (-1)^{m-1} \prod_{j=1}^{m} (\mu_j) \right) \right.\right.$$
$$\left.\left. +(\mu_{m+1}) - \left(\sum_{j=1}^{m} \mu_j - \sum_{1\leqslant i<j\leqslant m} (\mu_i\mu_j) + \ldots + (-1)^{m-1} \prod_{j=1}^{m} (\mu_j) \right) (\mu_{m+1}) \right], \left[\left(\prod_{j=1}^{m} \nu_j \right) (\nu_{m+1}) \right] \right\rangle$$
$$= \left\langle \sum_{j=1}^{m+1} (a_j + ib_j) \ , \left[\sum_{j=1}^{m+1} \mu_j - \sum_{1\leqslant i<j\leqslant m+1} (\mu_i\mu_j) + \ldots + \quad (-1)^{m} \prod_{j=1}^{m+1} (\mu_j) \right], \left[\prod_{j=1}^{m+1} \nu_j \right] \right\rangle.$$

Thus, the result is true for $n = m + 1$, and hence the result holds for all natural numbers n. Therefore,

$$\delta_1 + \delta_2 + \ldots + \delta_n$$
$$= \left\langle \sum_{j=1}^{n} (a_j + ib_j), \left[\sum_{j=1}^{n} \mu_j - \sum_{1\leqslant i<j\leqslant n} (\mu_i\mu_j) + \ldots + (-1)^{n-1} \prod_{j=1}^{n} (\mu_j) \right], \left[\prod_{j=1}^{n} \nu_j \right] \right\rangle.$$

□

Theorem 3.3. *(Extension of the Multiplication of ICFNs up to n terms) For any collection of ICFNs* $\delta_j = \left\langle (a_j + ib_j), (\mu_j, \nu_j) \right\rangle$ *(j = 1,2, ... ,n), the combined value obtained by using the multiplication operator is given as*

$$\delta_1 \times \delta_2 \times \ldots \times \delta_n$$

$$= \left\langle \prod_{j=1}^{n}(a_j + ib_j), \left(\left[\prod_{j=1}^{n} \mu_j \right], \left[\sum_{j=1}^{n} \nu_j - \sum_{1 \leqslant i < j \leqslant n} (\nu_i \nu_j) + \ldots + + (-1)^{n-1} \prod_{j=1}^{n} (\nu_j) \right) \right\rangle.$$

Proof. We will demonstrate that the result holds using the mathematical induction method. Consider two ICFNs δ_1 and δ_2. Then the Addition operator for n=2 is given as:

$\delta_1 \times \delta_2$

$= \langle (a_1 \times a_2 + i(b_1 \times b_2)), [\mu_1 \mu_2], [\nu_1 + \nu_2 - \nu_1 \nu_2] \rangle.$

Hence,

$$\delta_1 \times \delta_2 = \left\langle \prod_{j=1}^{2} (a_j + ib_j), \left(\left[\prod_{j=1}^{2} \mu_j \right], \left[\sum_{j=1}^{2} \nu_j - \prod_{j=1}^{2} (\nu_j) \right] \right) \right\rangle.$$

Assume that the result holds for n=m, where m is any natural number. Then,

$\delta_1 \times \delta_2 \times \ldots \times \delta_m$

$= \left\langle \prod_{j=1}^{m} (a_j + ib_j) , \left(\left[\prod_{j=1}^{m} \mu_j \right], \left[\sum_{j=1}^{m} \nu_j - \sum_{1 \leqslant i < j \leqslant m} (\nu_i \nu_j) + \ldots + \quad +(-1)^{m-1} \prod_{j=1}^{m} (\nu_j) \right] \right\rangle.$

Now we can prove the result for n = m+1;

$\delta_1 \times \delta_2 \times \ldots \times \delta_m \times \delta_{m+1}$

$= \left\langle \left(\left(\prod_{j=1}^{m} (a_j + ib_j) \times (a_{m+1} + ib_{m+1}) \right) , \left[\left(\prod_{j=1}^{m} \mu_j \right) (\mu_{m+1}) \right], \left[\left(\sum_{j=1}^{m} \nu_j - \sum_{1 \leqslant i < j \leqslant m+1} (\nu_i \nu_j) + \quad \ldots \right.\right.\right.$

$+ \quad (-1)^{m-1} \prod_{j=1}^{m} (\nu_j) \Big) + (\nu_{m+1}) - \left(\sum_{j=1}^{m} \nu_j \; - \sum_{1 \leqslant i < j \leqslant m} (\nu_i \nu_j) + \ldots + (-1)^{m-1} \prod_{j=1}^{m} (\nu_j))(\; \nu_{m+1})]$

$= \left\langle \prod_{j=1}^{m+1} (a_j + ib_j) , \left(\left[\prod_{j=1}^{m+1} \mu_j \right], \left[\sum_{j=1}^{m+1} \nu_j - \sum_{1 \leqslant i < j \leqslant m+1} (\nu_i \nu_j) + \ldots \quad +(-1)^{m} \prod_{j=1}^{m+1} (\nu_j) \right) \right\rangle.$

Thus, the result is true for n = m+1, and hence the result holds for all natural numbers n. Therefore,

$\delta_1 \times \delta_2 \times \ldots \times \delta_n$

$= \left\langle \prod_{j=1}^{n} (a_j + ib_j) , \left(\left[\prod_{j=1}^{n} \mu_j \right], \left[\sum_{j=1}^{n} \nu_j - \sum_{1 \leqslant i < j \leqslant n} (\nu_i \nu_j) + \ldots + \quad +(-1)^{n-1} \prod_{j=1}^{n} (\nu_j) \right] \right) \right\rangle.$ □

4. Score and Accuracy Functions for ICFS

In this section, the new Score function and Accuracy function for ICFS are introduced, which will play a crucial role in ranking the alternatives.

Definition 4.1. Let $\delta = \langle a + ib, \ (\mu, \nu) \rangle$ be an ICFN. Then, a Score function S of an intuitionistic complex fuzzy value can be represented as:

$$S(\delta) = |a + ib| \cdot (\mu - \nu) \text{ where } S(\delta) \in [-\sqrt{2}, \sqrt{2}] \tag{4.1}$$

Definition 4.2. Let $\delta = \langle a + ib, \ (\mu, \nu) \rangle$ be an ICFN. Then, an Accuracy function H of an intuitionistic complex fuzzy value can be represented as:

$$H(\delta) = |a + ib| \cdot (\mu + \nu) \text{ where } H(\delta) \in [0, \sqrt{2}] \tag{4.2}$$

4.1. *Properties of Score and Accuracy Functions*

Having established the definition of the new Score and Accuracy function for ICFS, we now proceed to prove the Boundedness and Monotonicity properties for the same.

Theorem 4.3. ***(Boundedness of the Score function)*** *The Score function of the ICFS is a bounded function.*

Proof. From definition 4.1, $S(\delta) = |a + ib|.(\mu - \nu)$. We know that, $\mu, \nu \in [0, 1]$ and $0 \leqslant \mu_\delta + \nu_\delta \leqslant 1$, and this implies that, $-1 \leqslant (\mu_\delta - \nu_\delta) \leqslant 1$. We also know that, $|a + ib| = \sqrt{a^2 + b^2}$. Since, $a, b \in [0, 1]$ we have, $0 \leqslant \sqrt{a^2 + b^2} \leqslant \sqrt{2}$. This leads to the fact that, $-\sqrt{2} \leqslant |a + ib|.(\mu - \nu) \leqslant \sqrt{2}$. Hence, $S(\delta) \in \left[-\sqrt{2}, \sqrt{2}\right]$. □

Theorem 4.4. ***(Monotonicity of the Score function)*** *The Score function of the ICFS is a bounded function.*

Proof. Let $\delta_1 = \langle (a_1 + ib_1)(\mu_1, \nu_1) \rangle, \delta_2 = \langle (a_2 + ib_2)(\mu_2, \nu_2) \rangle$ be two ICFNs.
If $\delta_1 > \delta_2$, this implies that, $\mu_1 > \mu_2$ and hence $a_1 > a_2$, $b_1 > b_2$ and $\nu_1 < \nu_2$. This leads to the fact that, $\sqrt{{a_1}^2 + {b_1}^2}.(\mu_1 - \nu_1) > \sqrt{{a_2}^2 + {b_2}^2}.(\mu_2 - \nu_2)$. Hence, $S(\delta_1) > S(\delta_2)$.

□

Theorem 4.5. ***(Boundedness of the Accuracy function)*** *The Accuracy function of the ICFS is a bounded function.*

Proof. From definition 4.2, $H(\delta) = |a + ib|.(\mu + \nu)$. We know that, $\mu, \nu \in [0, 1]$, $0 \leqslant \mu_\delta + \nu_\delta \leqslant 1$ and $|a + ib| = \sqrt{a^2 + b^2}$. Since, $a, b \in [0, 1]$ we have, $0 \leqslant \sqrt{a^2 + b^2} \leqslant \sqrt{2}$. This leads to the fact that, $0 \leqslant |a + ib|.(\mu - \nu) \leqslant \sqrt{2}$. Hence, $H(\delta) \in \left[0, \sqrt{2}\right]$. □

Theorem 4.6. ***(Monotonicity of the Accuracy function)*** *The Accuracy function of the ICFS is a Monotonic function.*

Proof. Let $\delta_1 = \langle (a_1 + ib_1)(\mu_1, \nu_1) \rangle, \delta_2 = \langle (a_2 + ib_2)(\mu_2, \nu_2) \rangle$ be two ICFNs. If $\delta_1 > \delta_2$, this implies that, $\mu_1 > \mu_2$ and hence $a_1 > a_2$, $b_1 > b_2$ and $\nu_1 < \nu_2$. This leads to the fact that, $\sqrt{{a_1}^2 + {b_1}^2}.(\mu_1 + \nu_1) > \sqrt{{a_2}^2 + {b_2}^2}.(\mu_2 + \nu_2)$. Hence, $H(\delta_1) > H(\delta_2)$. □

5. Intuitionistic Complex Fuzzy Einstein Correlated Geometric (ICFECG) Operator

To enhance the analytical capabilities of the ICFS framework, we introduce a novel aggregation operator specifically designed to capture both the magnitude and phase information inherent in complex membership functions. Unlike classical real-valued aggregation methods, this ICFECG operator accounts for the interdependence between amplitude and argument components, ensuring a more accurate representation of uncertainty in complex domains.

Definition 5.1. Let $\delta_j = \left\langle \left(a_j + ib_j\right)\left(\mu_j, \nu_j\right) \right\rangle$ (*j* = 1,2,...,n) be a collection of intuitionistic Complex fuzzy values and ρ be a fuzzy measure. Then, the Intuitionistic Complex Fuzzy Einstein Correlated Geometric (ICFECG) operator of $\widetilde{\delta}_j$ with respect to ρ is defined by:

$$
\begin{aligned}
&\mathrm{ICFECG}_\rho(\delta_1, \delta_2, \ldots, \delta_n) \\
&= (\delta_{\sigma(1)})^{(\rho(B_1)-\rho(B_2))} \bigotimes \left(\delta_{\sigma(2)}\right)^{(\rho(B_2)-\rho(B_3))} \bigotimes \ldots \bigotimes \left(\delta_{\sigma(n)}\right)^{(\rho(B_n)-\rho(B_{n+1}))} \\
&= \bigotimes_{j=1}^{n} \left(\delta_{\sigma(j)}\right)^{\left(\rho\left(B_{(j)}\right)-\rho\left(B_{(j+1)}\right)\right)} \\
&= \left\langle \left(\prod_{j=1}^{n} a_{ij}^{\left(\rho\left(B_{(j)}\right)-\rho\left(B_{(j+1)}\right)\right)} + i \prod_{i=1}^{n} b_{ij}^{\left(\rho\left(B_{(j)}\right)-\rho\left(B_{(j+1)}\right)\right)} \right) \right. \\
&\left(\frac{2\prod_{j=1}^{n} {\mu_{\sigma(ij)}}^{\left(\rho\left(B_{(j)}\right)-\rho\left(B_{(j+1)}\right)\right)}}{\prod_{j=1}^{n}\left(2-\mu_{\sigma(ij)}\right)^{\left(\rho\left(B_{(j)}\right)-\rho\left(B_{(j+1)}\right)\right)} + \prod_{j=1}^{n}\left(\mu_{\sigma(ij)}\right)^{\left(\rho\left(B_{(j)}\right)-\rho\left(B_{(j+1)}\right)\right)}}, \right. \\
&\left.\left. \frac{\prod_{j=1}^{n}\left(1+\nu_{\sigma(ij)}\right)^{\left(\rho\left(B_{(j)}\right)-\rho\left(B_{(j+1)}\right)\right)} - \prod_{j=1}^{n}\left(1-\nu_{\sigma(ij)}\right)^{\left(\rho\left(B_{(j)}\right)-\rho\left(B_{(j+1)}\right)\right)}}{\prod_{j=1}^{n}\left(1+\nu_{\sigma(ij)}\right)^{\left(\rho\left(B_{(j)}\right)-\rho\left(B_{(j+1)}\right)\right)} + \prod_{j=1}^{n}\left(1-\nu_{\sigma(ij)}\right)^{\left(\rho\left(B_{(j)}\right)-\rho\left(B_{(j+1)}\right)\right)}} \right) \right\rangle
\end{aligned}
\tag{5.1}
$$

where $(\sigma(1), \sigma(2), \ldots, \sigma(n))$ is a permutation of (1,2,...,n), such that $\delta_{\sigma(j-1)} \leqslant \delta_{\sigma(j)}$ for all *j*=2,...,*n*, $B_{(i)} = ((i), \ldots, (n))$, $B_{(n+1)} = \phi$.

5.1. *Properties of the ICFECG Operator*

The structural properties and functional behavior of the new ICFECG operator make it a robust tool for decision-making, pattern-recognition and other applications where the ICFS information must be integrated coherently. Following are the basic theorems and proofs of the properties of the new ICFECG operator.

Theorem 5.2. ***(Idempotency)*** *Let* $\delta_j = \langle (a_j + ib_j)\,(\mu_j, \nu_j) \rangle$ (j = 1,2, ... ,n) *be a collection of intuitionistic complex fuzzy values and* ρ *be a fuzzy measure. If all* $\delta_j (j = 1, 2, ..., n)$ *are equal, i.e.* $\delta_j = \delta\ \forall\ j$, *then* $\mathrm{ICFECG}_\rho\,(\delta_1, \delta_2, \ldots, \delta_n) = \delta$.

Proof. We have,

$$\mathrm{ICFECG}_\rho\,(\delta_1, \delta_2, \ldots, \delta_n) = \bigotimes_{j=1}^{n} \left(\delta_{\sigma(j)}\right)^{\left(\rho\left(B_{(j)}\right) - \rho\left(B_{(j+1)}\right)\right)}$$

$$\begin{aligned} &= \bigotimes_{j=1}^{n} (\delta)^{\left(\rho\left(B_{(j)}\right) - \rho\left(B_{(j+1)}\right)\right)} \qquad \text{(if } \delta_j = \delta\text{)} \\ &= (\delta)^{\sum_{j=1}^{n}\left(\rho\left(B_{(j)}\right) - \rho\left(B_{(j+1)}\right)\right)} \\ &= (\delta)^{\left(\rho\left(B_{(1)}\right) - \rho\left(B_{(n+1)}\right)\right)} \\ &= (\delta)^{(1-0)} \\ &= \delta \end{aligned}$$

$\mathrm{ICFECG}_\mu\,(\delta_1, \delta_2, \ldots, \delta_n) = \delta.$ □

Theorem 5.3. ***(Boundedness)*** *Let* $\delta_j (j = 1, 2, ..., n)$ *be a collection of ICFNs, and let* $\delta^- = \min \delta_j$, $\delta^+ = \max \delta_j$. *Then*
$\delta^- \leqslant \mathrm{ICFECG}_\mu\,(\delta_1, \delta_2, \ldots, \delta_n) \leqslant \delta^+$.

Proof. Let $\delta^- = \min \delta_j = \langle x, (\mu^-, \nu^-) \rangle$, $\delta^+ = \max \delta_j = \langle x, (\mu^+, \nu^+) \rangle$.
First, let us establish the inequality for the membership term. For each *j* we have, $\mu^- \leqslant \mu_{\sigma(j)} \leqslant \mu^+$.
Raising to the non-negative power and using monotonicity, we have:

$$\left(\mu^-\right)^{\left(\rho\left(B_{(j)}\right) - \rho\left(B_{(j+1)}\right)\right)} \leqslant \mu_{\sigma(j)} \leqslant \left(\mu^+\right)^{\left(\rho\left(B_{(j)}\right) - \rho\left(B_{(j+1)}\right)\right)}.$$

Applying non-decreasing $\bigotimes$ operator, we have:

$$\bigotimes_{j=1}^{n} \left(\mu^-\right)^{\left(\rho\left(B_{(j)}\right) - \rho\left(B_{(j+1)}\right)\right)} \leqslant \bigotimes_{j=1}^{n} \left(\mu_{\sigma(j)}\right)^{\left(\rho\left(B_{(j)}\right) - \rho\left(B_{(j+1)}\right)\right)} \leqslant \bigotimes_{j=1}^{n} \left(\mu^+\right)^{\left(\rho\left(B_{(j)}\right) - \rho\left(B_{(j+1)}\right)\right)}.$$

Since,

$$\bigotimes_{j=1}^{n} \left(\mu^-\right)^{\left(\rho\left(B_{(j)}\right) - \rho\left(B_{(j+1)}\right)\right)} = \mu^- \text{ and } \bigotimes_{j=1}^{n} \left(\mu^+\right)^{\left(\rho\left(B_{(j)}\right) - \rho\left(B_{(j+1)}\right)\right)} = \mu^+,$$

we have

$$\mu^- \leqslant \bigotimes_{j=1}^{n} \left(\mu_{\sigma(j)}\right)^{\left(\rho\left(B_{(j)}\right) - \rho\left(B_{(j+1)}\right)\right)} \leqslant \mu^+.$$

Hence, $\mu^- \leqslant \mu_{ICFN} \leqslant \mu^+$.
Let us now establish the inequality for the non-membership term. For each *j*, we have

$$\nu^- \leqslant \nu_{\sigma(j)} \leqslant \nu^+.$$

Raising to the non-negative power and using monotonicity, we have:

$$\left(\nu^-\right)^{\left(\rho\left(B_{(j)}\right) - \rho\left(B_{(j+1)}\right)\right)} \leqslant \nu_{\sigma(j)} \leqslant \left(\nu^+\right)^{\left(\rho\left(B_{(j)}\right) - \rho\left(B_{(j+1)}\right)\right)}.$$

Applying non-decreasing $\bigotimes$ operator, we have:

$$\bigotimes_{j=1}^{n}\left(\nu^{-}\right)^{\left(\rho\left(B_{(j)}\right)-\rho\left(B_{(j+1)}\right)\right)} \leqslant \bigotimes_{j=1}^{n}\left(\nu_{\sigma(j)}\right)^{\left(\rho\left(B_{(j)}\right)-\rho\left(B_{(j+1)}\right)\right)} \leqslant \bigotimes_{j=1}^{n}\left(\nu^{+}\right)^{\left(\rho\left(B_{(j)}\right)-\rho\left(B_{(j+1)}\right)\right)}.$$

Since,

$$\bigotimes_{j=1}^{n}\left(\nu^{-}\right)^{\left(\rho\left(B_{(j)}\right)-\rho\left(B_{(j+1)}\right)\right)}=\nu^{-} \text{ and } \bigotimes_{j=1}^{n}\left(\nu^{+}\right)^{\left(\rho\left(B_{(j)}\right)-\rho\left(B_{(j+1)}\right)\right)}=\nu^{+},$$

we have:

$$\nu^{-} \leqslant \bigotimes_{j=1}^{n}\left(\nu_{\sigma(j)}\right)^{\left(\rho\left(B_{(j)}\right)-\rho\left(B_{(j+1)}\right)\right)} \leqslant \nu^{+}.$$

Hence, $\nu^{-} \leqslant \nu_{ICFN} \leqslant \nu^{+}$. By combining the membership and non-membership inequalities, we have:

$$\left\langle x,(\mu^{-},\nu^{-})\right\rangle \leqslant ICFECG_{\rho}\left(\delta_1,\delta_2,\ldots,\delta_n\right) \leqslant \left\langle x,(\mu^{+},\nu^{+})\right\rangle.$$

Hence,

$$\delta^{-} \leqslant ICFECG_{\rho}\left(\delta_1,\delta_2,\ldots,\delta_n\right) \leqslant \delta^{+}.$$

□

Theorem 5.4. *(Monotonicity) Let* $\delta_j (j=1,2,...,n)$ *and* $\delta_j' (j=1,2,...,n)$ *be two sets of ICFNs. If* $\delta_j \leqslant \delta_j'$, $\forall$ j, *then* $ICFECG_{\rho}\left(\delta_1,\delta_2,\ldots,\delta_n\right) \leqslant ICFECG_{\rho}\left(\delta_1',\delta_2',\ldots,\delta_n'\right)$.

Proof. If $\delta_j \leqslant \delta_j'$, then raising to the non-negative power and using monotonicity, we have: $(\delta_j)^{w_j} \leqslant \left(\delta_j'\right)^{w_j}$ followed by $\bigotimes_{j=1}^{n}(\delta_j)^{w_j} \leqslant \bigotimes_{j=1}^{n}\left(\delta_j'\right)^{w_j}$.
Hence, $ICFECG_{\rho}\left(\delta_1,\delta_2,\ldots,\delta_n\right) \leqslant ICFECG_{\rho}\left(\delta_1',\delta_2',\ldots,\delta_n'\right)$. □

Theorem 5.5. *(Commutativity) Let* $\delta_j (j=1,2,...,n)$ *and* $\delta_j' (j=1,2,...,n)$ *be two sets of ICFNs. Then* $ICFECG_{\rho}\left(\delta_1,\delta_2,\ldots,\delta_n\right) = ICFECG_{\rho}(\delta_{\pi(1)}, \delta_{\pi(2)},\ldots, \delta_{\pi(n)})$, *where* δ_j' $(j=1,2,...,n)$ *is any permutation of* $\delta_j (j=1,2,...,n)$.

Proof. For a given list $(\delta_1,\delta_2,\ldots,\delta_n)$, choose a permutation σ such that $\delta_{\sigma(1)} \leqslant \delta_{\sigma(2)} \leqslant \ldots \leqslant \delta_{\sigma(n)}$, and define the tail sets as:

$$S_j := \left\{\delta_{\sigma(j)} \leqslant \delta_{\sigma(j+1)} \leqslant \ldots \leqslant \delta_{\sigma(n)}\right\}, \qquad j=1,\ldots,n,$$

with $S_{n+1}=\varnothing$. By the definition of the ICFECG operator (with respect to the fuzzy measure ρ) we may write the aggregation in the sorted order as:

$$ICFECG_{\rho}\left(\delta_1,\delta_2,\ldots,\delta_n\right)=\bigotimes_{j=1}^{n}(\delta_{\sigma(j)})^{w_j}, w_j := \rho\left(S_j\right)-\rho\left(S_{j+1}\right),$$

where $w_j \geqslant 0$ and $\sum_{j=1}^{n} w_j = 1$. Now consider the permuted input list $\left(\delta_{\pi(1)}, \delta_{\pi(2)},\ldots, \delta_{\pi(n)}\right)$. Let σ' be a permutation that sorts this permuted list in non-decreasing order: $\delta_{\sigma'(1)} \leqslant \delta_{\sigma'(2)} \leqslant \ldots \leqslant \delta_{\sigma'(n)}$. Because the multiset of input ICFS is unchanged by reordering, the sequence of values that appears after sorting the permuted list is identical to the sequence obtained by sorting the original list. Consequently, for every *j*, the j^{th} tail of the sorted permuted list is:

$$S_j' := \left\{\delta_{\sigma'(j)} \leqslant \delta_{\sigma'(j+1)} \leqslant \ldots \leqslant \delta_{\sigma'(n)}\right\}.$$

This contains the same ICF elements as S_j (the membership of S'_j and S_j as sets of elements coincide). Therefore, $\rho\left(S'_j\right) = \rho\left(S_j\right), \quad \forall\, j,$
and hence the corresponding weights computed for the permuted input satisfy:
$w'_j \coloneqq \rho\left(S'_j\right) - \rho\left(S'_{j+1}\right) = \rho\left(S_j\right) - \rho\left(S_{j+1}\right) = w_j, \forall\, j.$
Because the sorted sequences of the terms (the δ- values in non-decreasing order) are identical for the two cases and the corresponding weights coincide term wise, the two ICFECG products are equal and given as:
$\bigotimes_{j=1}^{n}(\delta_{\sigma(j)})^{w_j} = \bigotimes_{j=1}^{n}\left(\delta_{\sigma'(j)}\right)^{w'_j}.$
Hence, $ICFECG_\rho\,(\delta_1, \delta_2, \ldots, \delta_n) = ICFECG_\rho(\delta_{\pi(1)}, \ \delta_{\pi(2)}, \ldots, \ \delta_{\pi(n)}).$ □

6. Defuzzification Function

In order to translate the ICFS information into a crisp and interpretable form, we propose a new defuzzification function tailored to the structure of the ICFS representation. The new defuzzification function integrates all the ICFS components, yielding a single representative value that reflects all the characterisation of the underlying uncertainty. This new formulation offers an effective bridge between the ICFS assessments and practical, crisp outputs.

Definition 6.1. Let $\delta = \langle (a + ib)(\mu, \nu)\rangle$ be an ICFN. Then, the defuzzification function D of an ICFS value can be represented as follows:

$$D(\delta) = \frac{(a-b)}{4} + \frac{(\mu - \nu + 1)}{4}, \quad D(\ \delta) \in [0, 1]. \tag{6.1}$$

6.1. *Theoretical Foundations of the New Defuzzification Function: Continuity, Differentiability, Stability and Validity*

To ensure the robustness and analytical soundness of the proposed defuzzification function, we establish a comprehensive set of theoretical results addressing its fundamental properties. Specifically, we examine the continuity and differentiability of the function to guarantee smooth behavior under small perturbation of ICFS inputs. Furthermore, we investigate its stability and validity which offers strong mathematical assurance of the function's reliability and suitability for practical decision-making application.

Theorem 6.2. ***(Continuity)*** *The function* $D(\delta) = \frac{(a+b)}{4} + \frac{(\mu-\nu+1)}{4}, \quad D(\delta) \in [0, 1]$ *is continuous.*

Proof. Let $f(a,\ b, \mu, \nu) = \frac{(a+b)}{4} + \frac{(\mu-\nu+1)}{4}$, and let $x = (a,\ b, \mu, \nu),\ x_0 = (a_0,\ b_0, \mu_0, \nu_0)$. Given $\varepsilon > 0$, we have to find $\tau > 0$ so that:

$$\begin{aligned}
&|f(a,\ b, \mu, \nu) - f(a_0,\ b_0, \mu_0, \nu_0)| \\
&= \left|\left[\tfrac{(a+b)}{4} + \tfrac{(\mu-\nu+1)}{4}\right] - \left[\tfrac{(a_0+b_0)}{4} + \tfrac{(\mu_0-\nu_0+1)}{4}\right]\right| \\
&= \tfrac{1}{4}\left|(a - a_0) + (b - b_0) + (\mu - \mu_0) + (\nu_0 - \nu)\right| \\
&\leqslant \tfrac{1}{4}\left(|a - a_0| + |b - b_0| + |\mu - \mu_0| + |\nu_0 - \nu|\right).
\end{aligned}$$

Consider $sup\ norm\,(\|x - x_0\|_\infty) = max\{|a - a_0|, |b - b_0|, |\mu - \mu_0|, |\nu_0 - \nu|\}$. Then for each x, $\|x - x_0\| \leqslant \|x - x_0\|_\infty$. Therefore,

$$|f(x) - f(x_0)| \leqslant \frac{1}{4} \cdot 4\, \|x - x_0\|_\infty = \|x - x_0\|_\infty\,.$$

So, if $\tau = \varepsilon$, then whenever $\|x - x_0\|_\infty < \tau$ we have: $|f(x) - f(x_0)| \leqslant \|x - x_0\|_\infty < \tau = \varepsilon$. Hence, $|f(x) - f(x_0)| < \varepsilon$. Since x_0 was arbitrary, f is continuous. □

Theorem 6.3. ***(Differentiability)*** *The function* $D(\delta) = \frac{(a+b)}{4} + \frac{(\mu-\nu+1)}{4}, \quad D(\delta) \in [0,1]$ *is differentiable.*

Proof. Let $f(a,\ b,\mu,\nu) = \frac{(a+b)}{4} + \frac{(\mu-\nu+1)}{4}$. The partial derivatives of each variable are given as:

$$\frac{\partial f}{\partial a} = \frac{1}{4}; \frac{\partial f}{\partial b} = \frac{1}{4}; \frac{\partial f}{\partial \mu} = \frac{1}{4}; \frac{\partial f}{\partial \nu} = -\frac{1}{4}\ .$$

Since the partial derivatives are constant, they are continuous. Hence, all higher derivatives exist and are zero. So, f is differentiable everywhere. □

Theorem 6.4. ***(Stability)*** *The function* $D(\delta) = \frac{(a+b)}{4} + \frac{(\mu-\nu+1)}{4}, \quad D(\delta) \in [0,1]$ *is stable.*

Proof. Let $f(x) = \frac{(a+b)}{4} + \frac{(\mu-\nu+1)}{4}$ has a coefficient vector $\mathscr{C} = \left(\frac{1}{4}, \frac{1}{4}, \frac{1}{4}, -\frac{1}{4}\right)$ and $x,\ x'$ be two inputs. Then,
$L^1 - \mathrm{stability}:\ \|\mathscr{C}\|_\infty = (\max_i |c_i|) = \frac{1}{4}$.
Hence, $|f(x) - f(x')| \leqslant \frac{1}{4} \cdot \|x - x'\|_1$.

$L^2 - \mathrm{stability}:\ \|\mathscr{C}\|_2 = \sqrt{\sum_{i=1}^{4} {}_i^2} = \sqrt{\left(\frac{1}{4}\right)^2 + \left(\frac{1}{4}\right)^2 + \left(\frac{1}{4}\right)^2 + \left(-\frac{1}{4}\right)^2} = \frac{1}{2}$.
Hence, $|f(x) - f(x')| \leqslant \frac{1}{2} \cdot \|x - x'\|_2$.

$L^\infty - \mathrm{stability}:\ \|\mathscr{C}\|_1 = \sum_{i=1}^{4} |c_i| = \left|\frac{1}{4}\right| + \left|\frac{1}{4}\right| + \left|\frac{1}{4}\right| + \left|-\frac{1}{4}\right| = 1$.
Hence, $|f(x) - f(x')| \leqslant 1 \cdot \|x - x'\|_\infty$.
So, the presented condition for such a function f is stable. □

Theorem 6.5. ***(Validity)****The function* $D(\delta) = \frac{(a+b)}{4} + \frac{(\mu-\nu+1)}{4}, \quad D(\delta) \in [0,1]$ *is valid.*

Proof. Under the intuitionistic complex fuzzy extrema $(a,b,\mu,\nu) = (a,b,1,0)$ and $(a,b,\mu,\nu) = (a,b,0,1)$, the function attains the expected shifts from the baseline $\frac{(a+b)}{4}$ as follows:
$f(a,b,1,0) = \frac{(a+b)}{4} + \frac{1}{2}$, $f(a,b,0,1) = \frac{(a+b)}{4}$.
Output rises when evidence supports the set, falls when evidence opposes it, proving that the defuzzification function $D(\delta)$ is semantically valid. □

7. Algorithm for solving MAGDM Problems with ICFECG operator and ScoreAccuracy Functions

The following assumptions or notations are used to represent the MAGDM problems for evaluating the best cloud provider based on the ICFECG operator with intuitionistic complex fuzzy information. Let $A = \{\delta_1, \delta_2, \ldots, \delta_m\}$ be a discrete set of alternatives. $B = \{B_1, B_2, \ldots, B_n\}$ be a set of attributes. Let $\Omega = \{\omega_1, \omega_2, \ldots, \omega_n\}$ be the weight vector of attributes, where $\omega_j \geqslant 0$, $j = 1, 2, \ldots, n$. Suppose that $\widetilde{R} = \left(\widetilde{r}_{ij}\right)_{m\times n} = \left\langle \left(a_{ij} + ib_{ij}\right), \left(\mu_{ij}, \nu_{ij}\right)\right\rangle_{m\times n}$ is the intuitionistic complex fuzzy decision matrix. In the following, we apply the ICFECG operator to MAGDM problems for evaluating the best cloud provider based on the ICFECG operator with intuitionistic complex fuzzy information.

Step 1: Determine the fuzzy measure of the attribute of $B_j (j = 1, 2, ..., n)$ and attribute sets of B

Step 2: Utilize the ICFECG operator

$$\widetilde{r}_i = \mathrm{ICFECG}\left(\widetilde{r}_{i1}, \widetilde{r}_{i2}, ..., \widetilde{r}_{in}\right) = \left\langle \left(\prod_{j=1}^{n} a_{ij}^{\left(\rho\left(B_{(j)}\right) - \rho\left(B_{(j+1)}\right)\right)} + i \prod_{i=1}^{n} b_{ij}^{\left(\rho\left(B_{(j)}\right) - \rho\left(B_{(j+1)}\right)\right)}\right) \right.$$
$$\left. \left(\frac{2\prod_{j=1}^{n} \mu_{\sigma(ij)}^{\left(\rho\left(B_{(j)}\right)-\rho\left(B_{(j+1)}\right)\right)}}{\prod_{j=1}^{n} \left(2-\mu_{\sigma(ij)}\right)^{\left(\rho\left(B_{(j)}\right)-\rho\left(B_{(j+1)}\right)\right)} + \prod_{j=1}^{n} \left(\mu_{\sigma(ij)}\right)^{\left(\rho\left(B_{(j)}\right)-\rho\left(B_{(j+1)}\right)\right)}},\ \frac{\prod_{j=1}^{n} \left(1+\nu_{\sigma(ij)}\right)^{\left(\rho\left(B_{(j)}\right)-\rho\left(B_{(j+1)}\right)\right)} - \prod_{j=1}^{n} \left(1-\nu_{\sigma(ij)}\right)^{\left(\rho\left(B_{(j)}\right)-\rho\left(B_{(j+1)}\right)\right)}}{\prod_{j=1}^{n} \left(1+\nu_{\sigma(ij)}\right)^{\left(\rho\left(B_{(j)}\right)-\rho\left(B_{(j+1)}\right)\right)} + \prod_{j=1}^{n} \left(1-\nu_{\sigma(ij)}\right)^{\left(\rho\left(B_{(j)}\right)-\rho\left(B_{(j+1)}\right)\right)}}\right)\right\rangle.$$

to derive the overall preference values $\widetilde{r}_i$, i being natural numbers up to m, of the alternative δ_i.

Step 3: Calculate the scores $S\left(\widetilde{r}_i\right)$ of the collective overall intuitionistic fuzzy preference values $\widetilde{r}_i$.

Step 4: Rank all the enterprises δ_i and select the best one(s) in accordance with $S\left(\widetilde{r}_i\right)$ and $H\left(\widetilde{r}_i\right)$.

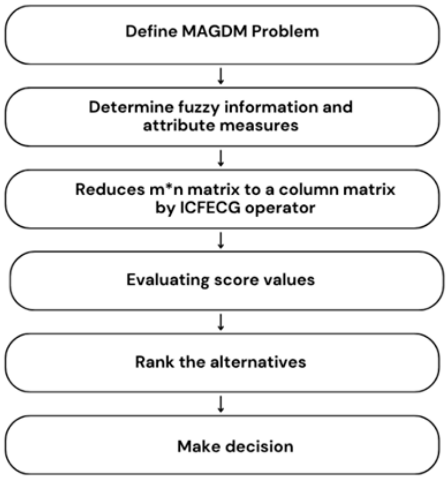

Figure 1: Workflow of the ICFECG Aggregation based MAGDM Procedure

7.1. *Calculation of Fuzzy measure attributes for Weight vectors*

The weight vector information from the fuzzy measures is given as: $\rho(B_1) = 0.20,\ \rho(B_2) = 30,\ \rho(B_3) = 0.25,\ \rho(B_4) = 0.35\ and\ \rho(B_1, B_2, B_3, B_4) = 1.00$. If the pairwise fuzzy measures and the triple fuzzy measures are involved in the computation of weights, then the following method can be employed.

1) **Solve for –:** λ satisfies $\prod_{i=1}^{4}(1 + \lambda\rho_i) = 1 + \lambda$. The nontrivial root is $\lambda \approx -0.232989393603364$. ($\lambda = 0$ is the trivial root, but would imply additivity; here, the nonzero root is required because $\sum \rho_i = 1.10 \neq 1$)
2) **Compute Pairwise fuzzy measures:** For disjoint singletons *i, j*: $\rho(\{i,j\}) = \rho_i + \rho_j + \lambda\rho_i\rho_j$. Hence, $\rho(B_1, B_2) = 0.486021$; $\rho(B_1, B_3) = 0.438351$; $\rho(B_1, B_4) = 0.533691$; $\rho(B_2, B_3) = 0.532526$; $\rho(B_2, B_4) = 0.625536$; $\rho(B_3, B_4) = 0.579613$.
3) **Triple fuzzy measures** Using associativity: $\rho(\{i,j,k\}) = \rho(\{i,j\}) + \rho_k + \lambda\rho(\{i,j\})\rho_k$. Hence, $\rho(B_1, B_2, B_3) = 0.707711$; $\rho(B_1, B_2, B_4) = 0.796387$; $\rho(B_1, B_3, B_4) = 0.752605$; $\rho(B_2, B_3, B_4) = 0.839100$.

7.2. *Numerical Illustration: Optimal Alternative Selection using ICFECG operator and Score-Accuracy Functions*

Suppose a company wants to choose the best cloud provider to host their large-scale application. Five cloud providers are under consideration and they are: δ_1-AWS, δ_2-Azure, δ_3-Google Cloud, δ_4-Oracle, δ_5-IBM. They need to evaluate based on multiple criteria given by: $B_1 : Cost\ efficiency,\ B_2 : Performance,\ B_3 : Security\ \&\ Compliance\ and\ B_4 : Technical\ Support.$

Normalized Fuzzy Measures:

For $\rho(B_1) = 0.20;\ \rho(B_2) = 0.30;\ \rho(B_3) = 0.25;\ \rho(B_4) = 0.35$, we get the normalized measures as: $\rho(B_1) = 0.18;\ \rho(B_2) = 0.27;\ \rho(B_3) = 0.23;\ \ \rho(B_4) = 0.32$.

Using the above normalized fuzzy measures and the following decision matrix, the decision maker evaluates the five cloud providers under these four criteria, with the evaluations expressed in intuitionistic complex fuzzy information.

$$\widetilde{R} = \begin{bmatrix} \langle(0.240 + 0.126i)(0.366, 0.234)\rangle & \langle(0.065 + 0.024i)(0.089, 0.111)\rangle \\ \langle(0.403 + 0.079i)(0.482, 0.318)\rangle & \langle(0.141 + 0.023i)(0.164, 0.236)\rangle \\ \langle(0.123 + 0.023i)(0.146, 0.054)\rangle & \langle(0.326 + 0.129i)(0.455, 0.445)\rangle \\ \langle(0.523 + 0.106i)(0.629, 0.342)\rangle & \langle(0.239 + 0.094i)(0.333, 0.567)\rangle \\ \langle(0.145 + 0.123i)(0.268, 0.232)\rangle & \langle(0.197 + 0.122i)(0.319, 0.222)\rangle \end{bmatrix}$$

$$\begin{matrix} \langle(0.010+0.118i)(0.128,0.772)\rangle & \langle(0.140+0.255i)(0.395,0.195)\rangle \\ \langle(0.286+0.366i)(0.652,0.118)\rangle & \langle(0.006+0.068i)(0.074,0.226)\rangle \\ \langle(0.264+0.369i)(0.633,0.167)\rangle & \langle(0.024+0.198i)(0.222,0.578)\rangle \\ \langle(0.321+0.366i)(0.687,0.113)\rangle & \langle(0.211+0.114i)(0.325,0.375)\rangle \\ \langle(0.171+0.398i)(0.569,0.338)\rangle & \langle(0.028+0.211i)(0.239,0.498)\rangle \end{matrix}\Bigg].$$

Utilize the decision information given in the matrix $\widetilde{R}$ and the ICFECG operator to obtain the overall preference values $\widetilde{r}_i$ of the cloud providers.

$$\widetilde{r}_i=\Bigg\langle\Bigg(\prod_{j=1}^{n} a_{ij}^{(\rho(B_{(j)})-\rho(B_{(j+1)}))}+i\prod_{i=1}^{n} b_{ij}^{(\rho(B_{(j)})-\rho(B_{(j+1)}))}\Bigg)$$
$$\Bigg(\frac{2\prod_{j=1}^{n}\mu_{\sigma(ij)}{}^{(\rho(B_{(j)})-\rho(B_{(j+1)}))}}{\prod_{j=1}^{n}(2-\mu_{\sigma(ij)})^{(\rho(B_{(j)})-\rho(B_{(j+1)}))}+\prod_{j=1}^{n}(\mu_{\sigma(ij)})^{(\rho(B_{(j)})-\rho(B_{(j+1)}))}},$$
$$\frac{\prod_{j=1}^{n}(1+\nu_{\sigma(ij)})^{(\rho(B_{(j)})-\rho(B_{(j+1)}))}-\prod_{j=1}^{n}(1-\nu_{\sigma(ij)})^{(\rho(B_{(j)})-\rho(B_{(j+1)}))}}{\prod_{j=1}^{n}(1+\nu_{\sigma(ij)})^{(\rho(B_{(j)})-\rho(B_{(j+1)}))}+\prod_{j=1}^{n}(1-\nu_{\sigma(ij)})^{(\rho(B_{(j)})-\rho(B_{(j+1)}))}}\Bigg)\Bigg\rangle.$$

For i = 1,

$$\widetilde{r}_1=\big\langle\big(0.240^{0.18}0.065^{0.27}0.010^{0.23}0.140^{0.32}+i0.126^{0.18}0.024^{0.27}0.118^{0.23}0.255^{0.32}\big)$$
$$\left(\frac{\left(2\left((0.366)^{0.18}\times(0.089)^{0.27}\times(0.128)^{0.23}\times(0.395)^{0.32}\right)\right)}{\left(\begin{array}{r}(2-0.366)^{0.18}\times(2-0.089)^{0.27}\times(2-0.128)^{0.23}\times(2-0.395)^{0.32}+\\(0.366)^{0.18}\times(0.089)^{0.27}\times(0.128)^{0.23}\times(0.395)^{0.32}\end{array}\right)},\right.$$
$$\left.\frac{\left(\begin{array}{r}(1+0.234)^{0.18}\times(1+0.111)^{0.27}\times(1+0.772)^{0.23}\times(1+0.195)^{0.32}-\\(1-0.234)^{0.18}\times(1-0.111)^{0.27}\times(1-0.772)^{0.23}\times(1-0.195)^{0.32}\end{array}\right)}{\left(\begin{array}{r}(1+0.234)^{0.18}\times(1+0.111)^{0.27}\times(1+0.772)^{0.23}\times(1+0.195)^{0.32}+\\(1-0.234)^{0.18}\times(1-0.111)^{0.27}\times(1-0.772)^{0.23}\times(1-0.195)^{0.32}\end{array}\right)}\right)\Bigg\rangle.$$

$$\widetilde{r}_1=\langle(0.0683+i0.0994)(0.2062,0.3558)\rangle.$$

For i = 2,

$$\widetilde{r}_2=\big\langle\big(0.403^{0.18}0.141^{0.27}0.286^{0.23}0.006^{0.32}+i0.079^{0.18}0.023^{0.27}0.366^{0.23}0.068^{0.32}\big)$$
$$\left(\frac{\left(2\left((0.482)^{0.18}\times(0.164)^{0.27}\times(0.652)^{0.23}\times(0.074)^{0.32}\right)\right)}{\left(\begin{array}{r}(2-0.482)^{0.18}\times(2-0.164)^{0.27}\times(2-0.652)^{0.23}\times(2-0.074)^{0.32}+\\(0.482)^{0.18}\times(0.164)^{0.27}\times(0.652)^{0.23}\times(0.074)^{0.32}\end{array}\right)},\right.$$
$$\left.\frac{\left(\begin{array}{r}(1+0.318)^{0.18}\times(1+0.236)^{0.27}\times(1+0.118)^{0.23}\times(1+0.226)^{0.32}-\\(1-0.318)^{0.18}\times(1-0.236)^{0.27}\times(1-0.118)^{0.23}\times(1-0.226)^{0.32}\end{array}\right)}{\left(\begin{array}{r}(1+0.318)^{0.18}\times(1+0.236)^{0.27}\times(1+0.118)^{0.23}\times(1+0.226)^{0.32}+\\(1-0.318)^{0.18}\times(1-0.236)^{0.27}\times(1-0.118)^{0.23}\times(1-0.226)^{0.32}\end{array}\right)}\right)\Bigg\rangle.$$

$$\widetilde{r}_2=\langle(0.0730+i0.0768)(0.2244,0.2214)\rangle.$$

For i = 3,

$$\widetilde{r}_3=\big\langle\big(0.123^{0.18}0.326^{0.27}0.264^{0.23}0.024^{0.32}+i0.023^{0.18}0.129^{0.27}0.369^{0.23}0.198^{0.32}\big)$$

$$\left(\frac{\left(2\left((0.146)^{0.18}\times(0.455)^{0.27}\times(0.633)^{0.23}\times(0.222)^{0.32}\right)\right)}{\left(\begin{array}{r}(2-0.146)^{0.18}\times(2-0.455)^{0.27}\times(2-0.633)^{0.23}\times(2-0.222)^{0.32}+\\(0.146)^{0.18}\times(0.455)^{0.27}\times(0.633)^{0.23}\times(0.222)^{0.32}\end{array}\right)},\right.$$

$$\left.\frac{\left(\begin{array}{r}(1+0.054)^{0.18}\times(1+0.445)^{0.27}\times(1+0.167)^{0.23}\times(1+0.578)^{0.32}-\\(1-0.054)^{0.18}\times(1-0.445)^{0.27}\times(1-0.167)^{0.23}\times(1-0.578)^{0.32}\end{array}\right)}{\left(\begin{array}{r}(1+0.054)^{0.18}\times(1+0.445)^{0.27}\times(1+0.167)^{0.23}\times(1+0.578)^{0.32}+\\(1-0.054)^{0.18}\times(1-0.445)^{0.27}\times(1-0.167)^{0.23}\times(1-0.578)^{0.32}\end{array}\right)}\right)\Big\rangle.$$

$$\widetilde{r}_3=\langle(0.1131+i0.1381)(0.3276,0.3702)\rangle.$$

For i = 4,

$$\widetilde{r}_4=\Big\langle\big(0.523^{0.18}0.239^{0.27}0.321^{0.23}0.211^{0.32}+i0.106^{0.18}0.094^{0.27}0.366^{0.23}0.114^{0.32}\big)$$

$$\left(\frac{\left(2\left((0.629)^{0.18}\times(0.333)^{0.27}\times(0.687)^{0.23}\times(0.325)^{0.32}\right)\right)}{\left(\begin{array}{r}(2-0.629)^{0.18}\times(2-0.333)^{0.27}\times(2-0.687)^{0.23}\times(2-0.325)^{0.32}+\\(0.629)^{0.18}\times(0.333)^{0.27}\times(0.687)^{0.23}\times(0.325)^{0.32}\end{array}\right)},\right.$$

$$\left.\frac{\left(\begin{array}{r}(1+0.342)^{0.18}\times(1+0.567)^{0.27}\times(1+0.113)^{0.23}\times(1+0.375)^{0.32}-\\(1-0.342)^{0.18}\times(1-0.567)^{0.27}\times(1-0.113)^{0.23}\times(1-0.375)^{0.32}\end{array}\right)}{\left(\begin{array}{r}(1+0.342)^{0.18}\times(1+0.567)^{0.27}\times(1+0.113)^{0.23}\times(1+0.375)^{0.32}+\\(1-0.342)^{0.18}\times(1-0.567)^{0.27}\times(1-0.113)^{0.23}\times(1-0.375)^{0.32}\end{array}\right)}\right)\Big\rangle.$$

$$\widetilde{r}_4=\langle(0.2830+i0.1397)(0.4458,0.3714)\rangle.$$

For i = 5

$$\widetilde{r}_4=\Big\langle\big(0.145^{0.18}0.197^{0.27}0.171^{0.23}0.028^{0.32}+i0.123^{0.18}0.122^{0.27}0.398^{0.23}0.211^{0.32}\big)$$

$$\left(\frac{\left(2\left((0.268)^{0.18}\times(0.319)^{0.27}\times(0.569)^{0.23}\times(0.239)^{0.32}\right)\right)}{\left(\begin{array}{r}(2-0.268)^{0.18}\times(2-0.319)^{0.27}\times(2-0.569)^{0.23}\times(2-0.239)^{0.32}+\\(0.268)^{0.18}\times(0.319)^{0.27}\times(0.569)^{0.23}\times(0.239)^{0.32}\end{array}\right)},\right.$$

$$\left.\frac{\left(\begin{array}{r}(1+0.232)^{0.18}\times(1+0.222)^{0.27}\times(1+0.338)^{0.23}\times(1+0.498)^{0.32}-\\(1-0.232)^{0.18}\times(1-0.222)^{0.27}\times(1-0.338)^{0.23}\times(1-0.498)^{0.32}\end{array}\right)}{\left(\begin{array}{r}(1+0.232)^{0.18}\times(1+0.222)^{0.27}\times(1+0.338)^{0.23}\times(1+0.498)^{0.32}+\\(1-0.232)^{0.18}\times(1-0.222)^{0.27}\times(1-0.338)^{0.23}\times(1-0.498)^{0.32}\end{array}\right)}\right)\Big\rangle.$$

$$\widetilde{r}_4=\langle(0.0967+i0.1911)(0.3261,0.3446)\rangle.$$

Calculate the scores $S(\widetilde{r}_i)$ of the overall intuitionistic fuzzy preference values $\widetilde{r}_i$.

$S(\widetilde{r}_1)=-0.018042$; $S(\widetilde{r}_2)=0.000318$; $S(\widetilde{r}_3)=-0.007604$; $S(\widetilde{r}_4)=0.023481$; $S(\widetilde{r}_5)=-0.003962$.

Ranking all the cloud providers in accordance with the scores $S(\widetilde{r}_i)$ of the overall intuitionistic complex fuzzy preference values $\widetilde{r}_i$: $\delta_4>\delta_2>\delta_5>\delta_3>\delta_1$, we see that the most desirable cloud provider is δ_4.

8. Algorithm for Solving MAGDM problem using Data Mining Techniques

Data mining refers to the systematic process of uncovering significant patterns, trends, relationships, and valuable insights from large datasets through the application of statistical, mathematical, and computational methods. Assume that $A = \{\delta_1, \delta_2, \ldots, \delta_m\}$ be a set of fuzzy items; $B = \{B_1, B_2, \ldots, B_n\}$ be a random sample with n fuzzy data records (attributes in the MAGDM problem), and each sample record B_i is represented as a vector with m values, $(\delta_1(B_i), \delta_2(B_i), \ldots, \delta_m(B_i))$, where $\delta_j(B_i)$ is the degree of the fuzzy item δ_j occurs in the record B_i, $\delta_j(B_i) \in [0,1]$, and propose three predefined thresholds for the process. Here, s_A is the minimal fuzzy support; c_A is the minimal fuzzy confidence; r_A is the minimal fuzzy correlation coefficient. The procedure of mining fuzzy correlation rules is described as follows:

Step 1. The fuzzy support of each fuzzy item $\delta_i \in A$, $fsupp(\delta_i)$ is computed: $fsupp(\delta_i) = \frac{\sum \delta_i}{n}$.

Step 2. Let $L_1 = \{\delta_p | \delta_p \in A, fuspp(\delta_p) \geqslant s_A\}$ be the set of frequent fuzzy itemsets whose size is equal to 1.

Step 3. Let $C_2 = \{(\delta_i, \delta_j)\}$ be the set of all combinations of two elements that belong to L_1, where $\delta_i, \delta_j \in L_1$, $\delta_i \neq \delta_j$. That is, C_2 is generated by L_1 joint with L_1. Because δ_i and δ_j are the elements of L_1, the size of each element of C_2 is 2.

Step 4. For each element of C_2, (δ_i, δ_j) the fuzzy support, $fsupp(\{\delta_i, \delta_j\})$ is computed using

$$fsupp\left(\{\delta_i, \delta_j\}\right) = \frac{\sum_{k=1}^{n} min(\delta_j(B_k) | \delta_j \in A)}{n}$$

and then the fuzzy correlation between δ_i and δ_j, r_{δ_i,δ_j}, is computed using: $r_{\delta_i,\delta_j} = \frac{S_{\delta_i,\delta_j}}{S_{\delta_i} \cdot S_{\delta_j}}$, where $S_{\delta_i,\delta_j} = \frac{\sum_{k=1}^{n}\left(\delta_i(C_k) - \overline{\delta_i}\right) \cdot \left(\delta_j(C_k) - \overline{\delta_j}\right)}{n-1}$, $\overline{\delta_j} = \frac{\sum_{k=1}^{n}(\delta_i(C_k))}{n}$, $\overline{\delta_j} = \frac{\sum_{k=1}^{n}\left(\delta_j(C_k)\right)}{n}$, $S^2_{\delta_i} = \frac{\sum_{k=1}^{n}\left(\delta_i(C_k) - \overline{\delta_i}\right)^2}{n-1}$, $S^2_{\delta_j} = \frac{\sum_{k=1}^{n}\left(\delta_j(C_k) - \overline{\delta_i}\right)^2}{n-1}$, $S_{\delta_i} = \sqrt{S^2_{\delta_i}}$, $S_{\delta_j} = \sqrt{S^2_{\delta_j}}$.
Since r_{δ_i,δ_j} is computed from the random sample T, r_{δ_i,δ_j} needs to be tested to determine if it is really greater than the minimal fuzzy correlation r_A. The formula for testing is as follows: $t = \frac{r_{\delta_i,\delta_j} - r_A}{\sqrt{\frac{1 - r^2{}_{\delta_i,\delta_j}}{n-2}}}$.
Compare the computed t-value to $t_{1-\alpha(n-2)}$, where $t_{1-\alpha(n-2)}$ is the $(1-\alpha)^{th}$ percentile in the t distribution with degrees of freedom n-1. If we obtain the t value, which is greater than $t_{1-\alpha(n-2)}$, then we can conclude that r_{δ_i,δ_j} is greater than the predefined minimal fuzzy correlation coefficient.

Step 5. For each element, whose fuzzy support is greater than or equal to s_A and the fuzzy correlation coefficient passes the test of C_2, then it is an element of L_2. Hence, L_2 is the set of the frequent combinations of two fuzzy itemsets, and still, the size of each element of L_2 is 2.

Step 6. Next, each C_k, $k \geqslant 3$, is generated by L_{k-1} joint with L_{k-1}. Assume that (δ_i, δ_j) and (δ_l, δ_m) are two elements of L_{k-1}, where $\delta_j = \delta_l$. If the size of the combinations $\left(\delta_j \{\delta_i, \delta_m\}\right)$ is k, and (δ_i, δ_m) is also a frequent combination of two fuzzy itemsets, then the combination $\left(\delta_j \{\delta_i, \delta_m\}\right)$ is an element with size k of C_k. For each element of C_k, its fuzzy support and fuzzy correlation coefficient are still used to find the elements of L_k.

Step 7. When each L_k, $k \geqslant 2$, is obtained for each element of L_k, (δ_x, δ_y), two candidate fuzzy correlation rules, $\delta_x \to \delta_y$ and $\delta_y \to \delta_x$ can be generated. If the fuzzy confidence of a rule is greater than or equal to c_A, then it is considered an interesting fuzzy correlation rule. The fuzzy confidence of the candidate fuzzy correlation rules can be generated using: $fconf(\delta_x \to \delta_y) = \frac{fsupp(\{\delta_x, \delta_y\})}{fsupp(\{\delta_x\})}$.

The algorithm won't stop until no next C_{k+1} can be generated. The above algorithm efficiently solves the same problem by filtering out unimportant alternatives.

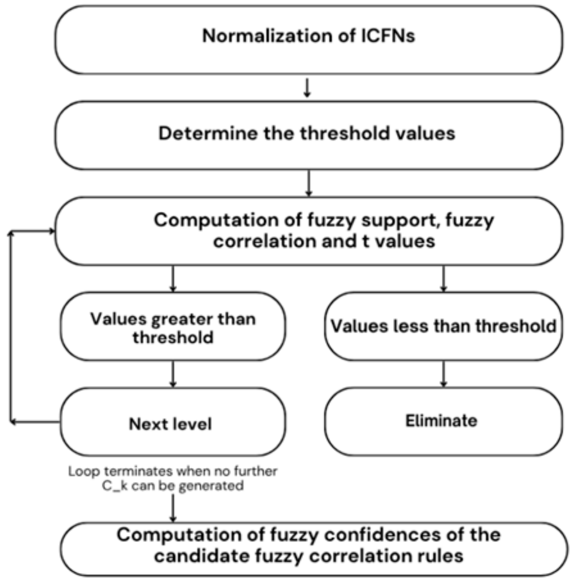

Figure 2: Workflow of the Data Mining Procedure

8.1. *Numerical Illustration: Optimal Alternative Selection Using Data Mining (Correlation Rule Mining)*

The normalized form of the intuitionistic complex fuzzy information from $\widetilde{R}$ using the proposed defuzzification function $D(\delta) = \frac{(a+b)}{4} + \frac{(\mu-\nu+1)}{4}$ is $\widetilde{R}_D$ and is presented in the following table.

Table 1: $\widetilde{R}_D$ – The Normalized values of ICFS from the decision matrix $\widetilde{R}$

	δ_1	δ_2	δ_3	δ_4	δ_5
Criteria-1 (B_1)	0.3745	0.4115	0.3095	0.4790	0.3260
Criteria-2 (B_2)	0.2668	0.2730	0.3663	0.2748	0.3540
Criteria-3 (B_3)	0.1210	0.5465	0.5248	0.5653	0.3600
Criteria-4 (B_4)	0.3988	0.2305	0.2165	0.3188	0.2450

Assume that $A = \{\delta_1, \delta_2, \delta_3, \delta_4, \delta_5\}$ is the set of alternatives with 4 different criteria. Here $s_A = 0.01$, $C_A = 0.9$, $r_A = 0.50$, $\alpha = 0.20$ and thus $t_{0.80,2} = 1.061$.

Step 1. The fuzzy support of each fuzzy item of A is given below:

Table 2: Fuzzy support of each δ_i's

Alternative	fsupp
δ_1	0.2903
δ_2	0.3654
δ_3	0.3543
δ_4	0.4095
δ_5	0.3213

Step 2. Since all $fsupp\,(\delta_i)$ are greater than $s_A(0.01)$, the set of the frequent fuzzy itemsets whose size is equal to 1 is $L_1 = \{\delta_1, \delta_2, \delta_3, \delta_4, \delta_5\}$.

Step 3. Next C_2, the set of all combinations of two elements of L_1, is generated by L_1 joint with L_1.

$$C_2 = \{(\delta_1, \delta_2), (\delta_1, \delta_3), (\delta_1, \delta_4), (\delta_1, \delta_5), (\delta_2, \delta_3), (\delta_2, \delta_4), (\delta_2, \delta_5), (\delta_3, \delta_4), (\delta_3, \delta_5), (\delta_4, \delta_5)\}$$

Step 4. The fuzzy support, fuzzy correlation coefficient (r) and t value of testing the fuzzy correlation coefficient of each element of C_2 is given below:

Table 3: Fuzzy support, Fuzzy correlation coefficient and t values of each C_i's

C_2	fsupp	r	t
$(\{\delta_1\}\{\delta_2\})$	0.2482	-0.7216	-2.4950
$(\{\delta_1\}\{\delta_3\})$	0.2285	-0.9706	-8.6455
$(\{\delta_1\}\{\delta_4\})$	0.2703	-0.5285	-1.7133
$(\{\delta_1\}\{\delta_5\})$	0.2397	-0.7615	-2.7526
$(\{\delta_2\}\{\delta_3\})$	0.3309	0.8299	0.8362
$(\{\delta_2\}\{\delta_4\})$	0.3654	0.9589	2.2876
$(\{\delta_2\}\{\delta_5\})$	0.2974	0.6327	0.2423
$(\{\delta_3\}\{\delta_4\})$	0.3314	0.6478	0.2744
$(\{\delta_3\}\{\delta_5\})$	0.3100	0.8473	0.9246
$(\{\delta_4\}\{\delta_5\})$	0.3015	0.4170	-0.1291

Step 5. In the above table, an element whose fsupp is greater than or equal to s_A, fuzzy correlation coefficient is greater than or equal to r_A and the t-value is greater than or equal to $t_{0.80,2}(1.061)$ is considered an element of L_2. Thus, $L_2 = \{(\{\delta_2\}\{\delta_4\})\}$.

Step 6. Since no C_3 can be generated, the process can be terminated with the set C_2.

Step 7. Next, the fuzzy confidences of the candidate fuzzy correlation rules are generated.

Table 4: Fuzzy confidence of the elements of L_2

C_2	Fconf
$\delta_2 \to \delta_4$	6.2605
$\delta_4 \to \delta_2$	5.5863

Both $\{\delta_2\} \to \{\delta_4\}$ and $\{\delta_4\} \to \{\delta_2\}$ are interesting fuzzy correlation rules.

9. Algorithm for Solving MAGDM Problem Using Artificial Neural Network (ANN-Perceptron)

The Perceptron Learning Rule is a supervised learning algorithm used for binary classification. It adjusts the weights of the perceptron based on the errors made during training.

Pseudo-Code For Perceptron-Based Ranking Method

1. **Input Preparation**
   Collect the defuzzified decision matrix $\widetilde{R}_D = \{\delta_1, \delta_2, \ldots, \delta_m\}$, where each δ_i is an n-dimensional vector of attributes.
   Define the target output labels $y = \{y_1, y_2, \ldots, y_m\}$.
   Set learning rate η and the maximum number of epochs E.
   Symbols: $\widetilde{R}_D \in \mathbb{R}^{m \times n},\ y \in \{0,\ 1\}^m$
2. **Initialization**
   Start with weights set to zero: $w = [0, 0, \ldots, 0]$.
   Initialize the bias term: $b = 0$.
   Symbols: $w \leftarrow 0,\ b \leftarrow 0$.
3. **Training Phase**
   Repeat for each epoch until convergence or maximum E:
   For each alternative δ_i:

Compute the activation value: $z_i = w \cdot \delta_i + b$.
Assign the predicted class:

$$\widehat{y}_i = \begin{cases} 1 & \text{if } z_i > 0 \\ 0 & otherwise \end{cases}$$

Compute the error: $e_i = y_i - \widehat{y}_i$.
If $e_i \neq 0$, update the parameters: $w \leftarrow w + \eta e_i \delta_i,\ b \leftarrow b + \eta e_i$.
If no errors are found in the entire epoch, stop training (model has converged).

4. **Testing and Scoring**
   After training, compute a score for each alternative using the learned weights and bias:

$$s_i = w \cdot \delta_i + b.$$

   A higher score corresponds to a stronger preference for that alternative.
5. **Ranking**
   Sort the alternatives in descending order of s_i. Output the ranked list, where the first element is the best alternative.
   Symbols: $\mathrm{Rank}\left(\widetilde{R}_D\right) = \mathrm{argsort}\,(-s_i)\,,\ i = 1, \ldots, m.$

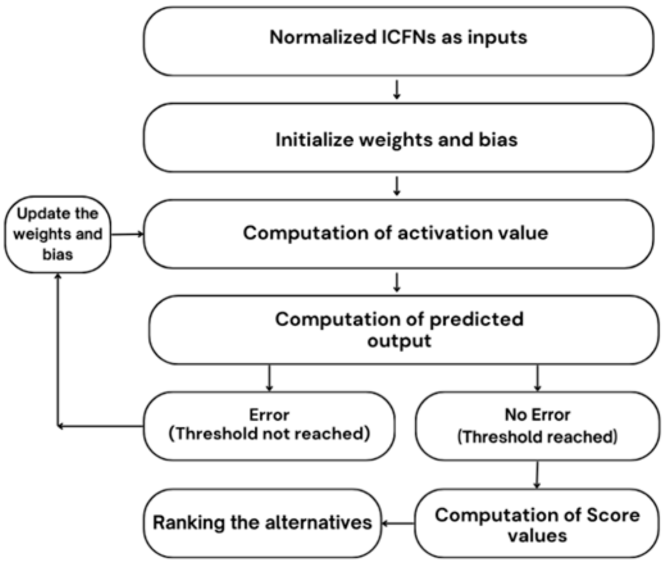

Figure 3: Workflow of the proposed ANN Procedure

9.1. Output from the proposed ANN procedure

Final Weights: [0.0417, -0.0429, 0.0524, -0.0681]
Final Bias: -0.05
Scores: [-0.102095 0.01744 -0.045203 0.00252 -0.065939]
Ranking (best → worst): [2 4 3 5 1]
Best Alternative: A 2

From the output, we can observe that δ_2 is the best alternative. Hence, Azure is preferred to be the best cloud provider.

As shown in the figure, the training error decreases progressively with each epoch, indicating that the perceptron learning rule achieves convergence. The reduction in early epochs followed by stabilization suggests efficient learning and minimal overfitting.

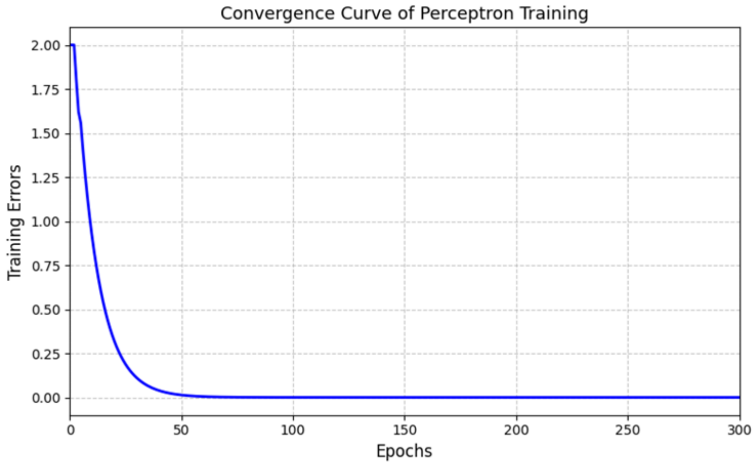

Figure 4: Error Convergence of the proposed ANN method

Table 5: Ranking of the optimal alternative from all proposed methods

	Proposed Method	Ranking of the Alternatives
Method-1	MAGDM with ICFECG operator, Score Function	$\delta_4 > \delta_2 > \delta_5 > \delta_3 > \delta_1$
Method-2	Data Mining with Correlation Rule Mining	δ_2,δ_4
Method-3	ANN with Perceptron	$\delta_2 > \delta_4 > \delta_3 > \delta_5 > \delta_1$

10. Discussion

In this study, we applied the newly proposed Intuitionistic Complex Fuzzy Set (ICFS) framework to a multi-attribute group decision-making (MAGDM) problem involving five alternatives. This approach directly addressed the analytical foundation outlined in the introduction. By representing each alternative with two interacting complex-valued characteristics, the ICFS effectively captured multidimensional and correlated uncertainty which confirmed its improved expressive ability over traditional intuitionistic fuzzy sets. We developed a score function specifically for ICFS and implemented the Intuitionistic Complex Fuzzy Einstein Correlated Geometric (ICFECG) operator. This led to a more accurate aggregation of expert evaluations. The resulting ranking, $\delta_4 > \delta_2 > \delta_5 > \delta_3 > \delta_1$, validates the enhanced discriminative power of the proposed MAGDM model. To assess robustness and consistency, a data mining-based dimensionality reduction technique uncovered strong fuzzy correlations—especially between $\delta_2$and δ_4—and removed less significant alternatives (δ_1, δ_3, and δ_5). Meanwhile, the Perceptron Learning Rule provided further computational validation with a closely matched ranking of $\delta_2 > \delta_4 > \delta_3 > \delta_5 > \delta_1$. Despite slight variations in order, all three methods consistently identified δ_2 and δ_4 as the most influential alternatives. This reinforces the reliability, stability, and interpretability of the ICFS-driven decision-making framework.

11. Conclusion

Overall, this study makes a significant contribution to MAGDM research by introducing a new ICFS-based decision-support framework that combines theoretical ideas with practical computational improvements. The new Intuitionistic Complex Fuzzy Einstein Correlated Geometric (ICFECG) operator acts as a strong aggregation tool that keeps amplitude, phase, membership and non-membership information intact. This provides a clearer and more reliable representation of complex-valued expert assessments which enhances the MAGDM process. Additionally, a data mining-driven dimensionality reduction method helps identify and remove unnecessary or weak attributes, which reduces computational load and improves the accuracy of decision outcomes. Using Artificial Neural Network (ANN) techniques enhances the model by allowing adaptive learning, dynamic optimization, and better performance when handling high-dimensional and uncertain data. Together, these elements create a unified hybrid model that addresses complex, nonlinear, and uncertain decision situations with greater accuracy, understanding, and efficiency. The MAGDM problem was solved using three distinct approaches namely ICFECG aggregation, data-mining–based computation, and an ANN model—and all three methods consistently identified the same best alternative, thereby reinforcing the robustness and reliability of the proposed decision-making framework. Looking ahead, several promising areas for future work include expanding the framework to dynamic or evolving decision situations, integrating deep learning methods for automated feature extraction, and applying the approach to real-world large-scale fields such as healthcare diagnostics, environmental management, finance, supply chain optimization, and smart city planning. Also, exploring hybrid optimization techniques, developing scalable parallel computing methods, and improving explainability will make intelligent ICFS-based decision-support systems more applicable and transparent.

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