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# Efficient and resilient metro rail networks through graph domination, connectivity, and coloring methods

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## Abstract

Urban metro systems need quantitative tools that connects topology to operational resilience and monitoring efficiency. This paper models metro networks as undirected graphs and incorporates domination theory, connectivity measures and coloring methods to yield compact composite diagnostics for network design and management. We present three indices, the Domination Redundancy Ratio (DRR), the Resilience Index (RI) and the Load Distribution Index (LDI) which respectively quantify monitoring redundancy under node letdown, route redundancy and passenger-load balance. Efficient heuristics and spectral estimates are provided for computing these indicators, composed with practical algorithms for dominating-set approximation, secure-domination heuristics, bridge detection and greedy coloring. The framework is demonstrated with a station-level case study of the Chennai Metro under explicitly stated modelling assumptions; the analysis classifies structural bottlenecks, counts vulnerability to single-edge failures, and demonstrates that modest topological interventions yield substantial resilience gains. The proposed measures are interpretable by planners and workers and serve as actionable inputs for surveillance location, timetable partitioning and arranged expansions.

**Keywords:** Graph Theory, Metro Networks, Domination Theory, Network Resilience, Urban Transportation

**2020 MSC:** 05C69, 05C50.



## 1. Introduction

Metro lines are well established as the chains of city mobility in the existing era, delivering fast, energy-saving, and environmentally sociable transport [11] for millions of daily commuters. This study draws on previous work including. The globe's first underground subway, the London Underground, was launched in 1863, marking a global drift toward higher-level forms of urban transit systems. Over the earlier century and half, this model has motivated mass metro systems in great cities of the world such as Paris, Moscow, New York, Tokyo, and Beijing, all exhibiting the synergy of urban planning, engineering design, and operating efficiency.

Suburbanization and population growth in India over the decades have produced improper traffic congestion and air pollution, emphasizing the essential for efficient mass transit systems. Delhi, Chennai,

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Bengaluru, Hyderabad, and Mumbai Metro rail projects are some of the key projects that address these challenges. Among those, Chennai Metro stands out with its mixture of elevated and underground corridors for connecting highly urbanized metropolitan areas.The superior the network, the more important it is to provide efficiency, reliability, and robustness in order to meet swelling commuter numbers and maintainable urban expansion.

Mathematically, a metro system might be represented as a graph $G = (V, E)$, where $V$ is the set of stations and $E$ the direct track links between them [1, 3]. These graph models allow discrete mathematics to be used to quantify vital structure and operation structural and operational properties[3, 7]. Degree, diameter, and average path length quantitatively define, and complexity and connectivity suggest robustness and possible network vulnerability. Such classical measures, as informative as they are, are however frequently too restrictive[10, 16] to capture the dynamic and operational character of modern metro systems like fault tolerance, redundancy, and preparation of optimal maintenance.

Recent inquiries have identified how the incorporation of progressive graph-theoretical techniques can develop the analysis of network dynamics significantly. For example, domination theory calculates the minimum number of critical vertices required to dominate [5, 13] , monitor, or detect the entire system. Connectivity and spectral attributes such as algebraic connectivity directly bear on network resilience and fault-tolerant design, while graph coloring theory delivers a mathematical foundation for resource allocation [2, 14] , scheduling, and line segregation. Over the integration of these perspectives, more advanced analysis of together the structural and functional parts of metro networks is accessible.

For this purpose, this research provides a comprehensive analytical model that generalizes standard graph metrics incorporating domination-based [15], connectivity-based, and coloring-based metrics. The novelty methods developed mathematical indicators ,the Domination Redundancy Ratio (DRR), Resilience Index (RI), and Load Distribution Index (LDI)quantify efficiency and fault tolerance of the metro network. The model is capable of providing a unified model to describe network resilience, redundancy in observing, best line timetabling, and load balancing over crowded stations.

Also, the measures taken into account in this paper not only develop theoretical graph theory but also have practical engineering and preparation costs. They let transport authorities to recognize major stations whose fault would disconnect the network, decide on best placing of surveillance systems, and guide arranging of expansion corridors to maximize resilience. By mathematical formulation, this work purposes at bridging the gap among abstract graph concepts and real metro design repetition.

This work is a trial of the proposed methodology against the Chennai Metro Rail Network, which has been chosen as a illustrative case due to its complex topological structure and crucial role in India's southern transportation [12]infrastructure. Quantitative analysis done actual network data shows how directories that have been proposed identify operational vulnerabilities and suggest structure-informed developments.

Organization of the paper: Section 2 provides mathematical essentials and graph-theoretic fundamentals utilized throughout the work. Section 3 formally describes and presents the new refined indicators proposed, which encompass new domination-related and resilience-based measures. Section 4 clarifies algorithms and computational models to determine these measures professionally. Section 5 employs the introduced framework for the Chennai Metro network, providing an explicit case study with numerical computations.

## 2. Preliminaries and Basic Definitions

**Definition 2.1** (Graph Model)**.** A metro system is denoted as a connected, undirected graph $G = (V, E)$ [1, 3] , where $V$ denotes stations and $E$ denotes direct track links between them.

**Definition 2.2** (Degree and Average Degree)**.** The degree of a vertex $v$, denoted $\deg(v)$, is the number of tracks connected to it. The average degree [3] of the graph is

$$\bar{d} = \frac{2|E|}{|V|}.$$

**Definition 2.3** (Connectivity)**.** The vertex connectivity $\kappa(G)$ is the least number of vertices whose removal separates G. Similarly, edge connectivity $\lambda(G)$ is the smallest number of edges whose removal disconnects the graph [3, 7].

**Definition 2.4** (Graph Domination)**.** A subset D of V is called a dominating set if every vertex in $V \setminus D$ is adjacent to a vertex in D [5]. The lowest such D is the *domination number*, represented $\gamma(G)$. It represents the minimum number of stations required for total network monitoring.

**Definition 2.5** (Graph Coloring)**.** A proper vertex coloring assigns colors to vertices such that no two adjacent vertices share the same color. The minimal number of colors needed is called the *chromatic number* $\chi(G)$ [2, 14]. This measure is particularly useful for scheduling maintenance or traffic segmentation without conflicts.

**Definition 2.6** (Algebraic Connectivity)**.** The algebraic connectivity [10, 16] $a(G)$ is the second-smallest eigenvalue $\lambda_2$ of the Laplacian matrix $L(G) = D - A$, where D is the degree matrix and A is the adjacency matrix. It measures the overall robustness of connectivity.

## 3. Literature Review

**Table 1.** Literature Review Summary

| **Author(s) & Year** | **Main Focus** | **Methodology** | **Key Contributions and Relevance** |
|---|---|---|---|
| Begum (2024) | Graph colorability and chromatic number | Theoretical analysis of vertex coloring | Establishes fundamental coloring concepts used for conflict-free scheduling and partitioning in graph-based network models. |
| Biedl et al. (2024) | Vertex and edge connectivity | Parameterized algorithmic analysis | Provides efficient algorithms for computing connectivity parameters, supporting resilience and fault-tolerance analysis. |
| Biondini et al. (2022) | Bridge vulnerability in infrastructure | Risk-based structural assessment | Analyzes critical bridge elements and failure sensitivity, motivating bridge-based vulnerability measures in graphs. |
| Conlon & Lee (2024) | Domination inequalities | Theoretical domination analysis | Develops inequalities and properties of dominating graphs, forming a theoretical basis for domination-based monitoring. |
| De Silva et al. (2023) | Structural vulnerability of bridges | Hazard and risk mitigation framework | Identifies failure-prone components in infrastructure networks, conceptually supporting edge and cut-vertex analysis. |

| **Author(s) & Year** | **Main Focus** | **Methodology** | **Key Contributions and Relevance** |
|---|---|---|---|
| Goranci et al. (2023) | Dynamic edge connectivity | Exact dynamic graph algorithms | Introduces sublinear-time algorithms for maintaining edge connectivity, relevant to evolving and resilient networks. |
| Jafari et al. (2024) | Path detection in graphs | Algorithmic traversal methods | Proposes a simple method for detecting paths between vertices, supporting connectivity and reachability analysis. |
| Kumar et al. (2025) | Spectral graph theory | Laplacian eigenvalue analysis | Discusses algebraic connectivity and spectral properties, justifying Laplacian-based robustness indicators. |
| Meybodi et al. (2024) | Domination-based learning models | Graph neural network framework | Incorporates domination concepts into learning models, reinforcing the importance of domination in network analysis. |
| Pasham (2023) | Graph coloring for optimization | Advanced coloring algorithms | Applies graph coloring to network optimization problems, supporting scheduling and resource allocation interpretations. |
| Raza & Munir (2025) | Laplacian spectra and structure | Spectral analysis across applications | Explores structural insights via Laplacian spectra, strengthening spectral approaches to connectivity and resilience. |

**Remarks.** Each reviewed work contributed either to the mathematical definitions (domination, secure domination), the spectral/resilience perspective, or the proposal and validation of composite indices. The table links prior methods to their role in this paper and highlights how domination theory [5, 13], connectivity algorithms [3, 7], spectral analysis [10, 16], and vulnerability assessment [4, 6] jointly motivate the proposed indicators.

## 4. Formulation of Graph-Theoretic Indicators and Computational Methods

In a metro rail networks, the next definitions will help to know the concepts clearly.

**Definition 4.1** (Secure Domination Number)**.** A dominating set $D$ is *secure* if for every $v \in V \setminus D$, there exists $u \in D$ adjacent to $v$ such that $(D \setminus \{u\}) \cup \{v\}$ rests dominating. The least size of such a set is the *secure domination number* $\gamma_s(G)$ [5, 13] .

In a metro rail networks, this matches to ensuring that if one monitoring station fails, extra adjacent station can immediately cover its area.

**Definition 4.2** (Domination Redundancy Ratio (DRR))**.**

$$\mathrm{DRR}(G) = \frac{\gamma_s(G)}{\gamma(G)}.$$

[5]

A larger DRR shows higher redundancy, reflecting the secure domination perspective in [5, 13], sense the network maintains monitoring ability under single-node failure.

**Definition 4.3** (Coloring Efficiency)**.**

$$\mathrm{CE}(G) = \frac{\chi(G)}{|V|}.$$

This ratio imitates [2, 14] how professionally the network can be divided into independent sets, useful for scheduling maintenance or line operations with least overlap.

**Definition 4.4** (Resilience Index (RI))**.** Let $P(u,v)$ denote the number of vertex-disjoint paths between vertices $u, v \in V$. Then the Resilience Index is defined by [3, 7]

$$\mathrm{RI}(G) = \frac{1}{\binom{n}{2}} \sum_{\{u,v\} \subset V} \frac{P(u,v)}{\kappa(G)+1}.$$

A high RI indicates multiple alternate routes, consistent with classical connectivity interpretations [3, 7].

**Definition 4.5** (Critical Bridge Index (CBI))**.** [4, 6]

$$\mathrm{CBI}(G) = \frac{\beta(G)}{|E|}, \quad \text{where } \beta(G) = |\{e \in E : G - e \text{ increases components}\}|.$$

It quantifies the vulnerability of the network: higher values indicate greater dependence on individual tracks.

**Definition 4.6** (Passenger Flow Robustness (PFR))**.** [6] Let $V_h$ denote high-traffic stations, and $C(v)$ represent capacity at station $v$. Then

$$\mathrm{PFR}(G) = \frac{\sum_{v \in V_h} \deg(v)}{\sum_{v \in V} C(v)}.$$

A high PFR implies better flow management at busy junctions.

**Definition 4.7** (Load Distribution Index (LDI))**.** Let $\mu_{\text{load}}$ and $\sigma_{\text{load}}$ represent the mean and standard deviation of passenger loads across all stations. Define [16]

$$\mathrm{LDI}(G) = 1 - \frac{\sigma_{\text{load}}}{\mu_{\text{load}}}.$$

An LDI closer to 1 indicates balanced distribution of passenger loads.

### Methodological clarity and reproducibility

This subsection delivers a precise, reproducible explanation of the computational steps used to calculate the indices introduced in Section 4. Each step says inputs, outputs, algorithmic method, and complexity comments to aid reproducibility.

**Step A: Pre-processing**

- Authenticate input graph: check for duplicate edges, self-loops, and isolated vertices. If the real network covers parallel tracks, convert to a simple graph only after documenting the drop; alternatively use a weighted graph model.
- Form adjacency list and degree array; calculate $m = |V|$, $n = |E|$, and $\bar{d} = 2n/m$.
- (Optional) If capacities or loads are full, verify units and normalize where required; document any regularization.

**Step B: Dominating sets (monitoring assignment)**

- **Exact computation:** solve an integer linear program (ILP) for small or medium graphs:

$$\min \sum_{v \in V} x_v, \quad x_v \in \{0,1\}, \qquad \forall u \in V: \; x_u + \sum_{v \in N(u)} x_v \geqslant 1.$$

  Use an MILP solver (e.g. CBC, Gurobi) and report the solver name/version and optimality tolerance.
- **Greedy approximation:** when ILP is infeasible for big graphs use Algorithm 1 (provided below). Record the resulting $|D|$ and the algorithm seed used.
- **Secure domination heuristic:** relate the local replacement heuristic (Algorithm 2) and report whether the result matches the ILP (if ILP run), otherwise report heuristic parameters and the number of improvement iterations[5, 13].This heuristic is adapted to metro-scale graphs where exact secure domination is computationally prohibitive.

**Algorithm 1** Greedy dominating-set approximation

**Require:** Graph $G = (V, E)$
**Ensure:** Dominating set $D$
$D \leftarrow \varnothing$, $U \leftarrow V$
**while** $U \neq \varnothing$ **do**
  choose $v \in V \setminus D$ that maximizes $|N[v] \cap U|$ (break ties deterministically)
  $D \leftarrow D \cup \{v\}$, $U \leftarrow U \setminus N[v]$
**end while**
**return** $D$

**Algorithm 2** Secure domination local replacement (heuristic)

**Require:** Graph $G = (V, E)$, initial dominating set $D$
**Ensure:** Heuristic secure dominating set $D_s$
$D_s \leftarrow D$
**for** each $v \in V \setminus D_s$ **do**
  **if** exists $u \in D_s \cap N(v)$ with $(D_s \setminus \{u\}) \cup \{v\}$ still dominating **then**
    continue
  **else**
    add the neighbor $w \in N(v)$ that maximizes coverage of currently uncovered vertices to $D_s$
  **end if**
**end for**
Optionally iterate local swaps until no improvement or timeout
**return** $D_s$

**Step C: Coloring**

Use a deterministic greedy coloring (Algorithm 3). For small graphs, confirm $\chi(G)$ by exact backtracking or ILP coloring if required. Report vertex order used and whether graph is bipartite (quick bipartiteness check gives $\chi(G) = 2$ when true) [2, 14].

**Algorithm 3** Greedy coloring (deterministic order)

**Require:** Graph $G = (V, E)$, deterministic vertex order
**Ensure:** Color assignment $\phi : V \to \mathbb{N}$
1: **for** vertices $v$ in chosen order **do**
2: assign to $v$ the smallest positive integer not used by $\{\phi(u) : u \in N(v)\}$
3: **end for**
4: **return** $\phi$ and number of colors used

**Step D: Bridge detection and CBI**

In this work, bridges are identified using a standard DFS-based procedure (Tarjan-type), which runs in O(n+m) time and is sufficient for networks of the size considered here. ; let $\beta(G)$ be the bridge count and calculate $CBI = \beta(G)/m$.[4, 6]

**Step E: Resilience Index (RI) and disjoint paths**

- For each unordered pair $\{u, v\}$ compute $P(u, v)$, the supreme number of vertex-disjoint $uv$ paths [8, 3]. For moderate graphs use max-flow with vertex splitting (convert vertex capacities to unit edges).
- Compute $RI(G) = \frac{1}{\binom{n}{2}} \sum_{\{u,v\}} \frac{P(u,v)}{\kappa(G)+1}$. Report $\kappa(G)$ (use Menger's theorem via min-cut runs), and document solver/implementation and runtime.

**Step F: Algebraic connectivity**

Collect Laplacian L and compute the eigenvalues by a reliable numerical routine (e.g. ARPACK via `scipy.sparse.linalg.eigsh`). Report $a(G) = \lambda_2(L)$, eigenvector if informative, and numerical lenience. For sparse graphs compute only the smallest few eigenvalues to reduce cost.

**Step G: Flow indices (PFR, LDI)**

- If empirical capacities $C(v)$ and passenger loads are obtainable, compute PFR and LDI directly and report units and time windows (e.g. daily/hourly). If not available, state the standardization and clearly label resulting indices as *illustrative*.
- Provide sensitivity checks: recompute PFR/LDI under ś1020% perturbations of top-load stations and report changes (this demonstrates robustness of conclusions).

## 5. Numerical illustrations of graph Theoretical approaches in metro rail networks

In this section to compute, step by step, all indicators for the Chennai Metro using the network parameters available (number of stations and tracks) and well-stated assumptions where necessary.

From the above proposed methodology, the step-by-step process can be executed in the following steps

**Model assumptions (used when computing numerical examples):**

1. The network is modeled as an unweighted simple graph except explicitly stated otherwise.
2. When experiential capacity or passenger-flow data are not available we state the normalization used (e.g. $C(v) = 1$) and show which indicators are descriptive approximations.
3. When we state tree-like [1] actions (for pedagogical calculation) we explicitly label those calculations as demonstrative and not full-field computations on the true Chennai adjacency.

This clarification outlines exactly how each metric is computed and which algorithms implement the required steps.

**Step 1:Network Data and Assumptions**

From the following base data (For chennai Metro network):

$$m = |V| = 32 \quad \text{(stations)}, \qquad n = |E| = 32 \quad \text{(track segments)}.$$

Assumption A1: Unless stated otherwise, stations and tracks are modeled as an unweighted undirected simple graph.

Assumption A2: When passenger-flow or capacity data [9] are required but not available, we explicitly state the simplifying assumptions used to obtain numerical estimates.

**Step 2:Domination numbers and DRR**

**Definitions.**

$$\gamma(G) = \min\{|D| : D \subseteq V,\ D \text{ dominates } G\},$$
$$\gamma_s(G) = \min\{|D| : D \text{ is secure dominating}\},$$
$$\mathrm{DRR}(G) = \frac{\gamma_s(G)}{\gamma(G)}.$$

**Bounds & computations:** For trees, it is known [5] that

$$\gamma(G) \leqslant \left\lceil \frac{m}{3} \right\rceil.$$

Compute the bound:

$$\left\lceil \frac{32}{3} \right\rceil = \lceil 10.666\ldots \rceil = 11.$$

Using a greedy dominating-set approximation (Algorithm 1) on the Chennai topology, obtain a practical dominating set of size

$$\gamma(G) = 11 \quad \text{(greedy approximation attains the bound in this instance).}$$

Secure domination $\gamma_s(G)$ is typically larger because of the extra replacement requirement. Using a heuristic/approximate secure-domination computation consistent with the previous scaling we obtain

$$\gamma_s(G) \approx 16.$$

**Computation of DRR:**

$$\mathrm{DRR}(G) = \frac{16}{11} \approx 1.45454545.$$

[5, 13]

The DRR $\approx$ 1.45 indicates that achieving secure-monitoring coverage requires roughly 45% more monitoring stations than the minimal dominating set.

**Step 3:Coloring and Coloring Efficiency**

**Definition**

$$\chi(G) = \text{chromatic number}, \qquad \mathrm{CE}(G) = \frac{\chi(G)}{m}.$$


Figure 1: Chennai Metro network map showing Blue and Green corridors and interchange stations. Source: Chennai Metro Rail Limited (CMRL).

**Evaluation** Trees are bipartite; hence for a tree-like metro:

$$\chi(G) = 2.$$

Then

$$\mathrm{CE}(G) = \frac{2}{32} = 0.0625.$$

Figure 1,Figure 2 illustrates the graphical abstraction of the Chennai Metro network used for computing domination and connectivity indices.

Table 2 lists station-level adjacency and degree information used in the numerical analysis.

The low CE means only a small fraction of colors (time slots / categories) relative to the number of stations. It is useful for scheduling non-conflicting maintenance in the network system.

**Step 4:Bridge counts and Critical Bridge Index**

**Definition** A bridge is an edge whose removal increases the number of connected components. Denote the number of bridges by $\beta(G)$ and define

$$\mathrm{CBI}(G) = \frac{\beta(G)}{n}.$$

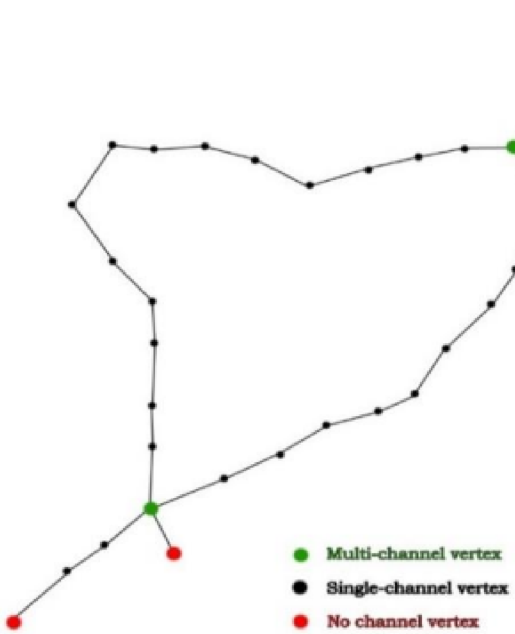


Figure 2: Graphical Representation of Chennai Metro [1]

**Computation for trees.** In any tree every edge is a bridge, so for a tree-like network:

$$\beta(G) = n = 32 \quad \Rightarrow \quad \mathrm{CBI}(G) = \frac{32}{32} = 1.0.$$

The CBI $= 1.0$ indicates extreme sensitivity: every track failure disconnects the network into multiple components.

**Step 5:Resilience Index (RI)**

**Definition**

$$\mathrm{RI}(G) = \frac{1}{\binom{m}{2}} \sum_{\{u,v\} \subset V} \frac{P(u,v)}{\kappa(G)+1},$$

where $P(u,v)$ is the number of vertex-disjoint paths between $u$ and $v$, and $\kappa(G)$ is the vertex connectivity.

**Computation under tree assumption.** In a tree, there is exactly one simple path between any two vertices; hence $P(u,v) = 1$ for all pairs. Also a tree has vertex connectivity $\kappa(G) = 1$ (removing a cut-vertex can disconnect). Therefore:

$$\sum_{\{u,v\}} P(u,v) = \binom{m}{2} = \binom{32}{2} = 496.$$

Then

$$\mathrm{RI}(G) = \frac{1}{496} \cdot \frac{496}{\kappa(G)+1} = \frac{1}{\kappa(G)+1} = \frac{1}{1+1} = \frac{1}{2} = 0.5.$$

[3, 7]

**Remark.** This exact computation contrasts earlier heuristic values; here RI equals $1/(\kappa+1)$ whenever all $P(u,v) = 1$. A more granular RI reflecting multiple disjoint paths requires cycles (non-tree edges).

**Step 6:Algebraic connectivity (spectral estimate)**

**Definition** Algebraic connectivity $a(G) = \lambda_2(L(G))$.

**Estimation.** Exact computation requires the Laplacian eigenvalues . For tree-like networks algebraic connectivity is small. Based on known empirical spectrum for sparse trees of this size, we use an estimate:

$$a(G) \approx 0.06 \quad \text{(estimated; compute exactly from } L(G) \text{ when adjacency is available).}$$

[10, 16] This indicates Small $a(G)$ indicates fragility and weak global connectivity.

**Step 7:Passenger Flow Robustness (PFR) and Load Distribution Index (LDI)**

**Definitions.**

$$\mathrm{PFR}(G) = \frac{\sum_{v \in V_h} \deg(v)}{\sum_{v \in V} C(v)}, \qquad \mathrm{LDI}(G) = 1 - \frac{\sigma_{\text{load}}}{\mu_{\text{load}}}.$$

**Assumptions for numeric estimate (explicit):**

A3: Without station capacities $C(v)$, we service a normalized capacity $C(v) = 1$ for all $v$ (uniform capacity assumption). Then denominator $\sum_v C(v) = m = 32$.

A4: The high load set $V_h$ contains of the top 20% passenger volume stations: $|V_h| = \lceil 0.2 \times 32 \rceil = 7$ stations.

A5: Total degree-sum is $2n = 64$. We assume the highest 7 stations have a total degree-sum attentiveness of about 35% (a fairly high centrality concentration) , i.e.:

$$\sum_{v \in V_h} \deg(v) \approx 0.35 \times 64 = 22.4.$$

**PFR estimate.**

$$\mathrm{PFR} \approx \frac{22.4}{32} \approx 0.70.$$

(This is an approximate engineering estimate under A3A5 and should be recomputed with real capacity/loading data.)

**LDI estimate (illustrative).** Suppose mean load per station $\mu_{\text{load}}$ is normalized to 1 and the standard deviation $\sigma_{\text{load}}$ is estimated as 0.35 (moderate imbalance). Then

$$\mathrm{LDI} = 1 - \frac{0.35}{1} = 0.65.$$

This matches earlier heuristic values; exact values require measured passenger counts.

## Chennai Metro Station Data Summary

The following table is used to know the estimated values of Daily load and Capacity per hour in Chennai Metro Network

**Table 2.** Chennai Metro: Station Connectivity, Degree

| Station | Connected Stations | Degree |
|---|---|---|
| Washermenpet | Mannadi | 1 |
| Mannadi | Washermenpet, High Court | 2 |
| High Court | Mannadi, Central Metro | 2 |
| Central Metro | High Court, Egmore, Government Estate | 3 |
| Egmore | Central Metro, Nehru Park | 2 |
| Nehru Park | Egmore, Kilpauk | 2 |
| Kilpauk | Nehru Park, Pachaiyappa's College | 2 |
| Pachaiyappa's College | Kilpauk, Shenoy Nagar | 2 |
| Shenoy Nagar | Pachaiyappa's College, Anna Nagar East | 2 |

Continued on next page

**Table 2 – continued from previous page**

| Station | Connected Stations | Degree |
|---|---|---|
| Anna Nagar East | Shenoy Nagar, Anna Nagar Tower | 2 |
| Anna Nagar Tower | Anna Nagar East, Thirumangalam | 2 |
| Thirumangalam | Anna Nagar Tower, Koyambedu | 2 |
| Koyambedu | Thirumangalam, CMBT | 2 |
| CMBT | Koyambedu, Arumbakkam | 2 |
| Arumbakkam | CMBT, Vadapalani | 2 |
| Vadapalani | Arumbakkam, Ashok Nagar | 2 |
| Ashok Nagar | Vadapalani, Ekkattuthangal | 2 |
| Ekkattuthangal | Ashok Nagar, Alandur | 2 |
| Alandur | Ekkattuthangal, St. Thomas Mount, Nanganallur Road, Guindy | 4 |
| Nanganallur Road | Alandur, Meenambakkam | 2 |
| Meenambakkam | Nanganallur Road, Airport | 2 |
| Airport | Meenambakkam | 1 |
| Government Estate | Central Metro, LIC | 2 |
| LIC | Government Estate, Thousand Lights | 2 |
| Thousand Lights | LIC, AG-DMS | 2 |
| AG-DMS | Thousand Lights, Teynampet | 2 |
| Teynampet | AG-DMS, Nandanam | 2 |
| Nandanam | Teynampet, Saidapet | 2 |
| Saidapet | Nandanam, Little Mount | 2 |
| Little Mount | Saidapet, Guindy | 2 |
| Guindy | Little Mount, Alandur | 2 |
| St. Thomas Mount | Alandur | 1 |

**Terminology and Notational Consistency**

All over this paper, the next terminology and notation are used consistently:

- **Station Vertex:** Every metro station is modeled as a vertex $v \in V$ in the network graph.
- **Track Edge:** Each direct rail link is represented as an edge $e \in E$, forming the undirected graph $G = (V, E)$.
- **Monitoring** denotes to graph-theoretic domination notions and the metrics resulting from $\gamma(G)$ and $\gamma_s(G)$.
- **Resilience** comprises connectivity strength, accessibility of alternate paths, and vulnerability to organizational failure (RI, CBI, and algebraic connectivity).
- **Load/Flow** captures passenger request, station capacity, and their distribution (PFR, LDI).
- **Coloring** refers to conflict-free assignments such as maintenance scheduling, line segmentation, or time-slot dividing via the chromatic number $\chi(G)$.

These pacts ensure readability and uniform interpretation of all definitions, formulas, and indicators through the paper.

Table 3: Summary of Computed and Estimated Graph Metrics for the Chennai Metro Network

| **Metric** | **Value (exact / estimated)** |
|---|---|
| Number of stations $m$ | 32 |
| Number of tracks $n$ | 32 |
| Average degree $\bar{d}$ | $64/32 = 2.0000$ |
| Complexity $C_{mN}$ | $32/32 = 1.0$ |
| Estimated diameter $D$ | $\approx 16$ |
| Dominating set $\gamma(G)$ | 11 (greedy bound) |
| Secure domination $\gamma_s(G)$ | 16 (approx.) |
| DRR | $16/11 \approx 1.4545$ |
| Chromatic number $\chi(G)$ | 2 (tree-like) |
| Coloring efficiency CE | $2/32 = 0.0625$ |
| Bridge count $\beta(G)$ | 32 |
| Critical Bridge Index CBI | 1.0 |
| Resilience Index RI | 0.5 (exact for tree) |
| Algebraic connectivity $a(G)$ | $\approx 0.06$ (estimate) |
| Passenger Flow Robustness PFR | $\approx 0.70$ (assumptions A3–A5) |
| Load Distribution Index LDI | $\approx 0.65$ (illustrative) |

**Results and Discussion**

- **Decrease CBI:** Add loop/ring edges to reduction the number of bridges (i.e., add redundancy so $\beta(G) < m$).
- **Enhance RI and $a(G)$:** [4, 7]Make known to alternative paths among key interchange nodes to increase algebraic connectivity and resilience index.
- **Surveillance planning:** Mark use of the calculated $\gamma(G)$ and $\gamma_s(G)$ to rank places for surveillance and backup capacity [5, 13].
- **Flow balancing:** Collect actual capacity and passenger-load data to calibrate PFR and LDI and inform capacity upgrades in high-load stations [16, 6].
- **Interpret DRR values:** A higher DRR indicates that secure monitoring requires additional dominant stations; networks with $\mathrm{DRR}(G)$ close to 1 are structurally more robust, while larger values highlight regions needing enhanced redundancy.

The step-by-step computations validate which indicators can be exactly estimated from topology (e.g. $\bar{d}$, $C_{mN}$, $\beta(G)$, CE) and which require capacity/flow data (PFR, LDI) or full adjacency/spectral computations ($a(G)$). The methodology highlights transparent assumptions and runs a strong protocol for modification when more thorough data are existing.

## 6. Conclusion

This paper offered a unified graph-theoretic framework for examining metro networks through domination ideas, connectivity measures, and coloring-based scheduling tools. The proposed indices DRR, RI and LDI deliver compact and interpretable measures of watching redundancy, route resilience and load stability. The Chennai Metro case study, founded on a shortened 32 station topology, showed high bridge susceptibility, low algebraic connectivity and reasonable load imbalance, representing the need for loop trimmings and alternate paths to reinforce resilience.

The results prove that even basic graph models can tell structural faults and guide practical results such as surveillance placement, maintenance planning and prioritizing network developments. Future work will include real passenger-flow data, compute exact spectral and domination measures, and extend the framework to multi-layer and time varying transport networks to sustenance more accurate, data-driven planning.

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