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Innovative 24-Note Numerical System: Computational Method and Relationship Model Between Notes, Elements, and Planets

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Abstract

This study introduces a 24-note numerical framework and computational method to explore connections among music, mathematics, the theory of four temperaments, and zodiac constellation. While earlier research has acknowledged these links, it lacked formal mathematical structures and systematic validation. The 24-note system is significant in two respects. First, it validates the existing 17-note system. Second, it establishes a crucial theoretical foundation for microtones. By providing this solid groundwork, it challenges the conventional Western musical framework, which is confined to tones and semitones, and paves the way for the practical application of microtones in fields such as music therapy, where their potential has remained largely untapped. Despite differences in numbers, results show complete agreement between the 24-note and 17-note systems in analyzing scales within Iran's seven main musical modes (Dastgāh). The study reveals that structural symmetry can identify symmetrical scales, but only when the sum of the "Valued Numbers" (Manzeli) within one tetrachord equals the sum within the other tetrachord. It also introduces an entirely novel and invaluable relational model that connects musical notes to the four classical elements and planets. Research demonstrates that note the "LA" functions as the foundational reference, with the position of all other notes calibrated relative to it. Finally, it presents the Cyclic Boundary Theorem (CBT), a novel mathematical concept that offers two complementary computational approaches for the systematic numerical evaluation of partitioned cyclic systems.

Keywords: Keywords: 24-Note Numerical System, 17-Note Numerical System, Four Temperaments (Music), Iranian Radif, Music Therapy.

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1. Introduction

Greek philosophers considered music part of the mathematical sciences. The Greek scientist Pythagoras believed that all things result from the combination of numbers and their ratios. In his writings, he considered the combination of sounds in melody production as a function of numerical equations [13]. In medieval culture, Boethius (6th century) reintroduced the theory of numerical balance between music, the cosmos, and the human body. This philosopher proposed that connections between the world and celestial planets could be conceptualized as a form of "Earthly music" [16]. Persian philosophers established twelve musical modes corresponding to the twelve zodiac signs, which formed their foundational system [21]. Historical

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sources about the zodiac originate primarily from various cuneiform texts (not exclusively astronomical) dating to the first millennium BC [18].

In his work Behjat ol-Rooh (Happiness of the Soul), Safi al-Din Ormavi (d.1294) pointed out the astro-musical correlations between the twelve Iranian Maqāms and zodiac constellations.[1]He related each Maqām to a quote from a past musician in a diagram. Also specifies the seasons and months suitable for the authorities of Iranian music. For example, what should be played in spring, when the sun is in Aries, Taurus, or Gemini, and in summer when the sun is in Cancer, Leo, and Virgo, what tunes are used, and in the same way, he listed other seasons and Maqāms [24]. According to natal astrology, the position of the Sun, Moon, and planets at the time of birth can be used to determine the personality, temperament, and behavior of a subject [25].

In Persian, "temperament" (mizaj) means blending, and in traditional Iranian medicine, it refers to the unique quality formed by the mixture and interaction of the four elements water, earth, air, and fire within a composite body [17]. These elements, which constitute the foundation of all beings (human and non-human), are indivisible and cannot be separated into parts with distinct natures or temperaments. Their defining qualities are as follows: fire (warm and dry), air (warm and moist), water (cold and moist), and earth (cold and dry) [7]. Followers of Hippocrates, aligning with Empedocles, held that all things, including humans, are composed of these four elements. They further mapped each element to a bodily humor: earth to black bile, air to blood, fire to yellow bile, and water to phlegm. Health depends on equilibrium among these humors, while imbalance precipitates illness [11]. Mirroring the Greeks, Iranian philosophers also explored music's psychological and therapeutic effects through the framework of the four temperaments, supported by medical reasoning [20].

The ancients said that the sound below (soprano) is yellow bile, the middle sound (alto) is blood, the third part (tenor) is phlegm, and the bass is black bile [1]. Adapt your music to listeners' temperaments: play tenor for sanguine (ruddy-faced) individuals, soprano for choleric (yellowish) types, alto for melancholic (dark, thin) listeners, and bass for phlegmatic (pale, heavy) audiences reflecting music's ancient alignment with the four humors [19]. In fall, when the sun is in the house of Libra, Scorpio, and Sagittarius, one must sing several melodies that are appropriate to their nature of dirt and cold, and dryness: like Oshshāq and Chāhārgāh and Būsalik and Bozorg and Kuchak and Nayriz and Homāyun and Ozzāl and Hosayni [12].

Currently, there is incomplete information about past scales, and today's Radif, including seven Dastgāhs and five Āvāzes, differs from historical scales in classification and Gūshehs. Thus, the effect of current Dastgāhs and Āvāzes on listeners' temperament and spirit requires study, experimentation, and psychological interpretation [20].With the Safavid dynasty's rise (1501 CE) and its socio-political transformations, Maqām music gradually evolved. While no documents detail this process precisely, scattered writings suggest Dastgāh music emerged in the early 13th century AH (1790 CE)[14].

The current Iranian Radif musical system consists of seven Dastgāhs (Māhur, Shūr, Homāyun, Segāh, Chāhārgāh, Navā, Rāst-Panjgāh) and five Āvāzes (Abu-Atā, Bayāt-e Tork, Afshāri, Dashti, Esfahān)[9]. A Dastgāh is a musical system comprising multiple Gushehs, starting from a core "mother mode" that forms its basis. The performance then modulates through a series of modes in a set order, with each section usually ending in a Forud (cadence), returning the melody to the mother mode[14].The results of the GLM[2] analysis shows that temperament type is related to neuroticism. This relationship is greater in people with a black bile temperament and those who prefer the Dastgāh-e Shūr[4].

At the time of Farabi (d.950) [10], the two main intervals in scale (whole step, half step) attributed to Pythagoras were accepted, and their performance was practiced on the musical instruments of that time, and his contemporaries did not know any other principle except that for intervals on the instruments [22]. Villoteau [23], the pioneering Western musicologist of Napoleon's Egyptian campaign, identified two primary approaches to Eastern scale systems in his research: First, a theoretical 24-tone octave division employing quarter-tones, sometimes extended to 48 microtonal divisions; second, a practical 18-tone system

[1]. Please see appendices I.
[2]. The general linear model (GLM) is a statistical analysis method.

that partitioned each whole tone into three approximately equal parts (roughly 1/3-tone intervals). He conclusively demonstrated that this 18-tone division more accurately reflected the actual performance practice on traditional Eastern instruments [6], despite the theoretical possibility of finer microtonal subdivisions.

Innovative numerical orders of musical scales, and their computational methods based on common intervals of different scales in an octave will be introduced in twelve, seventeen, and twenty-four notes that have been developed to determine the innate and acquired temperaments and the relationship of the celestial zodiac constellation and planets of each scale. This article presents the first study to examine the intricate relationship between the four human temperaments and the zodiac constellations with musical scales. While earlier scholars acknowledged this connection, they failed to establish any logical, systematic, or computational frameworks to elucidate it. Moreover, their interpretations often diverged significantly, leaving room for ambiguity [3]. In a computational number system with 17 notes, the initial note "Do" and the final note "Si" the ascending process (in total, 17 notes or 16 unequal consecutive intervals) is introduced in two models: Makāni and Manzeli (place-value notation and positional) with the same ordering but different numerical values with common computational rules. As shown in Table 1 the order of notes, Makāni numbers, and as in Table 2 based on valuation, is the Manzeli numbers values [3].

Table 1: 17-note numerical Makāni Model

Note symbols	C	D♭	D♯	D	E♭	E♯	E	F	G♭	G♯	G	A♭	A♯	A	B♭	B♯	B
Note names	Do	Re♭	Re♯	Re	Mi♭	Mi♯	Mi	Fa	Sol♭	Sol♯	Sol	La♭	La♯	La	Si♭	Si♯	Si
Makāni numbers	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17

Table 2: 17-note numerical Manzeli Model

Note symbols	C	D♭	D♯	D	E♭	E♯	E	F	G♭	G♯	G	A♭	A♯	A	B♭	B♯	B
Note names	Do	Re♭	Re♯	Re	Mi♭	Mi♯	Mi	Fa	Sol♭	Sol♯	Sol	La♭	La♯	La	Si♭	Si♯	Si
Manzeli numbers	1	2	3	4	5	6	7	8	9	2	3	4	5	6	7	8	9

Research Objectives:

1. Primary Research Objective: To introduce and validate a 24-tone numerical system along with its computational rules for scientific music analysis.
2. Secondary Objectives:
   - To establish the relationship between musical scales and:
     a) The four humors (temperaments) in traditional medicine
     b) Zodiac constellations
   - To present a novel model connecting:
     a) The four classical elements
     b) Celestial bodies with musical notes
   - To test scale symmetry criteria within the proposed system.

2. Introducing the 24-notes number system (a novel theory)

First and foremost, it is crucial to emphasize that the proposed model, within both Iranian and Western music theories, has no prior precedent for the specific application outlined in this study. The 24-note numerical system is a theoretical construct in which the octave is divided into 24 equal intervals, each representing a quarter tone.[3] In this system, each note has a specific number, element, and corresponding

[3]. It is suggested to read the 17-Notes article for a better understanding. Please see reference [3].

planet, and it has characteristics according to the effects of elements and planets. One of the reasons for the difference in the modes of similar scales with different tonalities is the elemental character and the planetary nature of the "Shāhed" or Tonic [4] of the scale. For example, when a melody is played in C major, the listener experiences a different feeling than in F major. Because, despite sharing the same intrinsic scale temperament, the tonic note C major is under the influence of the planet Mercury, and the tonic F major is under the influence of the planet Jupiter, as shown in the following section. This is the same for all scales and modes. Of course, changing the mental states of one scale may be related to other reasons, such as changing the number of commas in the intervals, the mental states of the musician, the gender of the Shāhed (masculine, feminine, or neutral),[5] and the sound quality of the musical instrument.

2.1. 24-notes table approach

To obtain the elements and planets of the notes, the 28-cell table should be used, because the multiplication of four elements in seven planets is 28; however, since the table accommodates only 24 notes, four vacant positions must first be identified. Therefore, as indicated in Table 5, a table with 24 cells should be designed. Then the number 9 is given to the first cell.[6]of the table, and from then on, the number of each cell is multiplied by 9. In other words, it is added nine to nine, and the obtained number of each cell is divided by 4. The remainder is the notation of the element according to Table3.

Table 3: The Elements and Corresponding Notation[3]

Element	**Fire**	**Wind**	**Water**	**Earth**
Notations	1	2	3	4

Note: When the remainder is zero, represent Earth, but the ancient people considered the number 4 as its symbol.

The resulting product of 9 is then divided by 7, with the remainder determining the planetary[7] notation as illustrated in Table 4.

Table 4: The Planets and Corresponding Notation

Planets	**Saturn**	**Jupiter**	**Mars**	**Sun**	**Venus**	**Mercury**	**Moon**
Notations	1	2	3	4	5	6	7

Note: When the remainder is zero, represent the moon, but the ancient people considered the number 7 as its symbol.

[4]. The Shāhed note serves as the tonal center of the melody, the note to which melodic passages constantly return [6].
[5]. The gender of the planets determines the gender of the notes.
[6]. For the reason for choosing the number 9, refer to Appendix II.
[7].While modern astronomy recognizes nine planets (including the Sun), this article adheres to the ancient cosmological framework, which considered only seven celestial bodies. To maintain historical accuracy in presenting traditional rules and formulas, the original seven-planet system.

Table 5: 24-part Table for the Extraction of Elements and Planets of Note

Row	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
Multiple9	9	18	27	36	45	54	63	72	81	90	99	108
Element	Fire	Wind	Water	Earth	Fire	Wind	Water	Earth	Fire	Wind	Water	Earth
Plt.Nos	2	4	6	1	3	5	0	2	4	6	2	3
Row	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**
Multiple9	117	126	135	144	153	162	171	180	189	198	207	216
Element	Fire	Wind	Water	Earth	Fire	Wind	Water	Earth	Fire	Wind	Water	Earth
Plt.Nos	5	0	1	4	6	1	3	5	0	2	4	6

Therefore, considering the order of the elements and planets,[8] the position of the four empty cells is determined according to Table 6.

Table 6: Location table of four empty cells

Names	**Saturn**	**Jupiter**	**Mars**	**Sun**	**Venus**	**Mercury**	**Moon**
Fire	Empty	1	5	9	13	17	21
Wind	18	22	Empty	2	6	10	14
Water	15	11	19	23	Empty	3	7
Earth	4	8	12	16	20	24	Empty

Note: The numbers in Table 6 are the row numbers of Table 5.

The above table shows four empty cells that are 8 cells apart. Therefore, the distance between each full cell is equal to 9 cells. Then, the note "La", which is the origin of the other notes, is placed in the fourteenth cell in the center of the 28-cell table, so that the point of unity of all notes is placed. Subsequently, the remaining notes are arranged in Table 8 (the 28-cell) according to the sequence specified in Table 7.

Table 7: Note Order Table

Number	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
Note	La	La𝄪	Si♭	Si𝄭	Si	Si𝄪	Do	Do𝄪	Re♭	Re𝄭	Re	Re𝄪
Number	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**
Note	Mi♭	Mi𝄭	Mi	Mi𝄪	Fa	Fa𝄪	Sol♭	Sol𝄭	Sol	Sol𝄪	La♭	La𝄭

Note1: A flat note (♭) lowers the sound of the note by a half step (semitone).
Note2: A Koron (𝄭) lowers the sound of the note by approximately a quarter tone.
Note3: A Sori (𝄪) uppers the sound of the note by approximately a quarter tone.

[8].The order of elements in the vertical column and the planets in the horizontal row is adapted from Reference [8] which itself draws upon the science of Abjad (the knowledge governing the relationships between letters and numbers).

Table 8: Position of notes in the 28-cell Makāni structure

Names	Saturn	Jupiter	Mars	Sun	Venus	Mercury	Moon
Fire	Empty	Fa	Sol	La♭	Si♭	Do	Re
	1	5	9	13	17	21	25
Wind	Mi♭	Fa♯	Empty	**La**	Si	Do♯	Re♯
	2	6	10	14	18	22	27
Water	Mi	Sol♭	Sol♯	La♯	Empty	Re♭	Mi♭
	3	7	11	15	19	23	27
Earth	Mi♯	Sol♭	La♭	Si♭	Si♯	Re♭	Empty
	4	8	12	16	20	24	28

If any note other than "A" (La) is placed in the central cell (the fourteenth cell) of the 28-cell table, the resulting outcome becomes unstable, and the overall harmonic coherence of the musical system within the table is lost. This instability serves as proof that the note "A" functions as the only possible point of equilibrium, forming the basis for determining the position of all other notes, such that the location of each note is defined relative to this fixed center.

Table 9: The 9-part table of 28-cell Makāni of notes

Names	Saturn	Jupiter	Mars	Sun	Venus	Mercury	Moon
Fire	Empty	Fa	Sol	La♭	Si♭	Do	Re
	1	5	9	4	8	3	7
Wind	Mi♭	Fa♯	Empty	**La**	Si	Do♯	Re♯
	2	6	1	5	9	4	8
Water	Mi	Sol♭	Sol♯	La♯	Empty	Re♭	Mi♭
	3	7	2	6	1	5	9
Earth	Mi♯	Sol♭	La♭	Si♭	Si♯	Re♭	Empty
	4	8	3	7	2	6	1

The Makāni numbers are based on the order of notes, while the Manzeli numbers are based on valuation [3]. Table 9 shows that all eight notes are separated by one space, which makes a total of nine spaces. Table 10 also specifies the Manzeli numbers of each note. It should be noted that the author-researcher developed the fundamental logic of this numerical system through extensive trial and error, having tested multiple methods to derive all those as mentioned earlier.

Table 10: Manzeli notes numbers in 24-note system

Names	Saturn	Jupiter	Mars	Sun	Venus	Mercury	Moon
Fire		Fa 5	Sol 9	La♭ 5	Si♭ 9	Do 5	Re 9
Wind	Mi♭ 2	Fa♯ 6		La 6	Si 2	Do♯ 6	Re♯ 2
Water	Mi 3	Sol♭ 7	Sol♯ 3	La♯ 7		Re♭ 7	Mi♭ 3
Earth	Mi♯ 4	Sol♭ 8	La♭ 4	Si♭ 8	Si♯ 4	Re♭ 8	

The order of the note cells table is Makāni numbers and, based on valuation, Manzeli numbers. The logic governing Manzeli numbers in the 24-note system (analogous to the 17-note system) inherently avoids two-digit numbers through numerical reduction. The Manzeli numbering system operates on a 28-cell matrix partitioned into 9 sections, where each cell's value is derived via a cyclic digit-addition process that handles two-digit emergence systematically. This work proposes a novel mathematical formalization of this algorithm – the Cyclic Boundary Theorem (CBT) – which, to the best of current knowledge, has not been previously described in the literature. CBT provides a rule for handling numbers in repeating 9-count cycles. When reaching 9 (cycle end), it converts $10 \to 2$ (instead of the conventional $10 \to 1$), creating predictable boundary behavior, as shown in Table 11.

Cyclic Boundary Theorem (CBT)

A mathematical framework for periodic numerical systems defined as:

Notation:

- $k \in \mathbb{Z}^+$: Cycle index (each cycle contains 9 consecutive numbers)
- $P_k = 9$: Base value at cycle boundary Manzeli numbers (constant for all complete cycles)
- M_k: Final computed value after transformations
- $\mathrm{dr}(x)$: Digital root of x (e.g., $\mathrm{dr}(123) = 1+2+3 = 6$)

Core Algorithm:

$$M_k = \begin{cases} k & \text{if } k = 1 \quad \text{(first cycle)} \\ \mathrm{dr}(P_k + 1) + 1 = 2 & \text{if } k \geqslant 2 \quad \text{(subsequent cycles)} \end{cases}$$

Key Mechanism:

At each cycle boundary (when $P_k = 9$):

$$9 \to 9+1 = 10 \to \mathrm{dr}(10) = 1 \to 1+1 = 2 \to M_k = 2$$

In other words, when reaching Manzeli number 9 (cycle boundary), the system calculates the new cycle's starting value through consecutive digit summation: first summing $9+1$'s digits (since the tenth cell corresponds to the Manzeli number), then, it adds 10 to 1, resulting in 11, and the sum of its digits $(1+1)$ equals 2.

Table 11: Manzeli Number Valuation in the 28-Cell Matrix

Names	Saturn	Jupiter	Mars	Sun	Venus	Mercury	Moon
Fire	1	Fa 5	Sol 9	La♭ 5	Si♭ 9	Do 5	Re 9
Wind	Mi♭ 2	Fa♯ 6	10+1=11 1+1=2	La 6	Si 10+1=11 1+1=2	Do♯ 6	Re♯ 10+1=11 1+1=2
Water	Mi 3	Sol♭ 7	Sol♯ 3	La♯ 7	3	Re♭ 7	Mi♭ 3
Earth	Mi♯ 4	Sol♭ 8	La♭ 4	Si♭ 8	Si♯ 4	Re♭ 8	4

3. Features of the table 24-notes numerical system

The originality of each table can be identified from its structural features, which for the Makāni and Manzeli tables of the 24-notes numerical system are as follows:

1. The position of the note "La" in the 14th-cell Makāni model causes each of the seven natural notes to be assigned to a planet. In other words, there would not be a planet with two natural notes.
2. The numbers of the note "La" in Makāni tables of 28 cells of notes (number 14 in the Makāni Table 8 and number 5 in the Makāni Table 9) in the hidden numerical structure are the same as with the number of the Manzeli note "La", i.e., the number 6. In the Makāni model, the number "14", which is the number 5 in the sum of the components $(1+4=5)$, is the same as the number of the nine-part Makāni model. The number 5 also reaches 15 in the sum of natural numbers $(1+2+3+4+5=15)$. Finally, the sum of the digits of the number 15, i.e.,. $(1+5=6)$, is equal to the Manzeli number of the note "La".

$$\begin{aligned} &\text{Initial number (Table 8)} : 14 \\ &\text{Digital root of 14} \ : \mathrm{dr}(14) = 1+4 = 5 \\ &\text{Sum of natural numbers} \left(\sum_{i=1}^{n} i = \frac{n(n+1)}{2} \right) : \sum_{i=1}^{5} i = \frac{5\times 6}{2} = 15 \\ &\text{Digital root of 15} \ : \mathrm{dr}(15) = 1+5 = 6 \end{aligned}$$

3. In the nine-part Makāni Table 9, the distances of the empty cells are equal to each other. That is, when the cells are placed around a circle, the first and last cell will have a common position, in this context, the sum of the digits of the resulting number of summations of all the numbers in Table 9, assuming that the first and last cell are the same, is equal to 9 as shown in Table 12.

Table 12: Sum of numbers in 9-part Makānī model using Digital Root

Sum of all numbers	Digital Root
$1+2+\cdots+9+1+2+\cdots+9+1+2+\cdots+9=135$	$\mathrm{dr}(135)=9$

The characteristic of the number 135 is that the remainder of the division by its component numbers (1, 3, and 5) will be the numbers 9 and 0.

4. As illustrated in Table 13, the sum of the corresponding numbers from each cell in Table 10 (representing Manzeli notes) yields 136. Furthermore, the sum of the digits of 136 $(1+3+6)$ equals 10, which sum of the component numbers to 1 $(1+0)$. This numerical harmony suggests that all musical notes share a single, unified origin.

Table 13: Sum of Manzeli Numbers

Sum of all numbers	Digital Root
$3 \times \left(\sum_{i=1}^{9} i\right) + 1 = 3 \times \left(\frac{9(9+1)}{2}\right) + 1 = 136$	$dr(136) = 10$ $dr(10) = 1$

5. The distance between empty cells is nine in Table 9 So, for every empty cell, the number 1 is considered. By matching the numbers table with the Abjad letters system,[9] the numbers of empty cells will be 1, 10, 100, and 1000, respectively, all of which will be equal to 1 without considering 0 (the sum of the digits becomes 1).
6. In the Manzeli Table 10, by assigning the number 1 to empty cells, the sum of the numbers in each column of the planet will be a two-digit number, which can be reduced to a single-digit number by summing their digits as shown in Table 10. Again, the number one is obtained by reducing the sum of either the final two-digit or single-digit numbers as indicated in tables 14 and 15.

Table 14: Numerical Values in 24-note Manzeli System

Names	Saturn	Jupiter	Mars	Sun	Venus	Mercury	Moon
Fire	1	5	9	4	8	3	7
Wind	2	6	1	5	9	4	8
Water	3	7	2	6	1	5	9
Earth	4	8	3	7	2	6	1
sum	10	26	17	26	16	26	15
components	1	8	8	8	7	8	6

Table 15: Digital Root Calculations for Numerical Components

Sum of all numbers	Digital Root
$10 + 26 + 17 + 26 + 16 + 26 + 15 = 136$	$dr(136) = 10$ $dr(10) = 1$
$1 + 8 + 8 + 8 + 7 + 8 + 6 = 46$	$dr(46) = 10$ $dr(10) = 1$

7. By carefully looking at the common numbers of the four elements in the seven planets of the Manzeli number Table 10 of the notes, it is clear that each element contains two numbers. Fire (5 and 9), Wind (2 and 6), Water (3 and 7), and Earth (4 and 8). By subtracting the larger number from the smaller number, the number 4 is obtained, i.e, $(9-5=4)$ for Fire, $(6-2=4)$ for Wind, $(7-3=4)$ for Water, and $(8-4=4)$ for Earth, which is equal to the number of elements.
8. When the sum of the common numbers of each element of Table 10 of Manzeli numbers of notes together, it becomes 44, and the sum of its digits is equal to 8. Interestingly, the distance between each empty cell of the 28-Manzeli table is also 8 cells. This is shown in Table 16 using the fact that Fire $(5+9=14)$, Wind $(6+2=8)$, Water $(3+7=10)$, and Earth $(4+8=12)$.

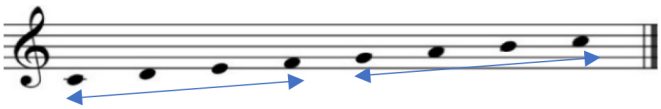
[9]. Please see reference[8].

Table 16: Sum of four element numbers

Sum of all numbers	Digital Root
$12+10+8+14=44$	$\mathrm{dr}(44)=8$

4. Symmetrical and asymmetrical scales based on 24-note numerical systems

As established in its previous study [3], when analyzing symmetrical and asymmetrical scales, structural similarity between two tetrachords alone proves insufficient for classification. Crucially, the sum of notes values calculated using the Manzeli numbering system for the 24-note must also be identical for both tetrachords. This requirement accounts for cases where apparent structural similarities may nonetheless yield different numerical values. Classification models are presented in Figure 1 with Table 17 (for symmetrical scales) and in Figure 2 with Table 18 (for asymmetrical scales), following this methodology.[10]

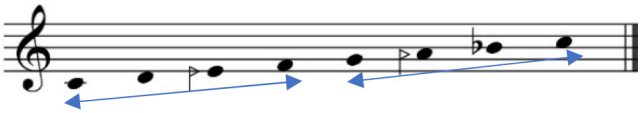


W, W, H- W -W, W, H

Figure 1: Māhūr - C (Do)

Table 17: Symmetrical scale, Māhūr

C	D	E	F	Notes	G	A	B	C
5	9	3	5	Numbers	9	6	2	5
$5+9+3+5=22$				Total	$9+6+2+5=22$			



W, M, M- W -M, M, W

Figure 2: c: Segāh - $E_{\flat}$ ($Mi_{\flat}$)

Table 18: Asymmetrical scale, Segāh

C	D	$E_{\flat}$	F	Notes	G	$A_{\flat}$	$B_{\flat}$	C
5	9	2	5	Numbers	9	5	8	5
$5+9+2+5=21$				Total	$9+5+8+5=27$			

Note1: "M" is Mojannab = Half step ± quarter-tone (ascending or descending).

The significance of specific notes within a scale warrants special attention. A crucial point to emphasize is:[11]

Shāhed Note: Functioning as the melodic centerpiece, the Shāhed note serves as the primary tonal anchor to which musical phrases consistently resolve [15]. Notably, these Shāhed notes are systematically determined according to the ascending degrees of their respective scales[6]. Accordingly, the Shāhed notes of the seven Iranian musical Dastgāhs are presented in Table 19.

[10]. For more information about the intervals of the primary modes of Iranian musical Dastgāhes, refer to article [3].
[11]. For more information, refer to reference [15].

Table 19: Specifications of scale Modes[3]

Number	Mode	Shāhed note
1	Māhūr	Tonic
2	Rāst-Panjgāh	Tonic, then Supertonic
3	Chāhārgāh	Tonic
4	Segāh	Mediant
5	Shūr	Supertonic
6	Navā	Supertonic
7	Homāyun	Supertonic

Note: The degrees of the modes in Table 19 are based on their innate scale, not practical or acquired scale.

5. Calculation notation

The notations used in this paper are presented in Table 20.

Table 20: Calculation notation of 17-note numerical system

Explanation	Notation
Symmetrical scale with a Shāhed note	St_1
Symmetrical scale with two Shāhed notes	St_2
Asymmetric scale with a Shāhed note	$\ddot{S}t_1$
Sum of the notes	Sn_7
First Shāhed note	t_1
Second Shāhed note	t_2
The first tetrachord	x_1
The second tetrachords	x_2
The difference in the sum of the Manzeli numbers of two tetrachords	X
Extracted number	e
Answers to calculations in the first, second	a_1, a_2
Obtained number from total calculations	TC
Final innate element of scale (temper notation)	Tes
Scale zodiac constellation	Sz

6. Computational rules of the 24-notes numerical system.

To obtain the number of the element and the zodiac constellation of the scale, the same calculation approach of the 17-notes numerical system is used. However, the method of obtaining the extracted number is different. It should be mentioned that with the help of the old system in cases and III below and with a tedious process of trial-and-error calculation in cases I and IV, the following rules have been achieved:

I. The result of calculations in both numerical musical systems (17 and 24 notes) must be the same.

II. Obtaining the corresponding scale element (innate temper scale): divide the last number in the total calculation by 4, and the remainder will be the scale element, i.e.,

$$TC \overset{4}{\equiv} Tes.$$

In which the notation $a \overset{c}{\equiv} b$ means that b is the remainder of a divided by c.
Each element is also associated with a temperament. The numerical notation of each element and their temperaments are indicated in Table 21[2].

Table 21: Numerical notations of each element and their temperament[2]

Element	Fire	Wind	Water	Earth
Temper	Warm and dry	Warm and moist	Cold and moist	Cold and dry
Notation	1	2	3	4

Note: When the remainder is zero,"Tes" and "es" represent earth, but the ancient people considered the number 4 as its symbol.

III. Obtaining the corresponding scale zodiac constellation: the remainder of dividing the total number of calculations by the number 12 will be the corresponding scale zodiac constellation, i.e.,

$$\mathsf{TC} \overset{12}{\equiv} \mathsf{Sz}.$$

The numerical notation of each zodiac constellation is shown in Table 22[3].

Table 22: Numerical notations of zodiac constellation[3].

Zodiac	Aries	Taurus	Gemini	Cancer	Leo	Virgo
Notation	1	2	3	4	5	6
Zodiac	Libra	Scorpio	Sagittarius	Capricorn	Aquarius	Pisces
Notation	7	8	9	10	11	12

Note: When the remainder is zero, "Sz" represents Pisces, but ancient people considered the number 12 as its symbol.

IV. Instructions for the extracted number in the 24-note numerical system for symmetrical and asymmetric scales are given below. Note that the extracted number is required for the accurate calculation of the scale zodiac constellation.

- Symmetrical scale: No extracted number (e) is needed. Because The difference (X) between the sum of Manzeli numbers of the two tetrachords in the scale will be zero.

$$\begin{aligned} X &:= x_2 - x_1 \quad (\text{where } X = 0) \\ e &:= X \quad (\text{thus, } e = 0) \end{aligned}$$

- Asymmetric scale with a Shāhed note:[12]The difference between the sum of the Manzeli numbers of the two tetrachords of a scale (X) is calculated as follows:
  - If $X = |x_2 - x_1| > 4$, reduce X by 4 repeatedly until it falls within the range $1 \leqslant X \leqslant 4$ (i.e., $X \bmod 4$).
  - The final extracted number (e) is obtained by multiplying the result by 4.

$$X = |x_2 - x_1|, \quad e := 4 \times \begin{cases} X \bmod 4 & \text{if } X > 4 \\ X & \text{otherwise} \end{cases}$$

Note: The difference (X) must be between 1 and 4. If $X > 4$, subtract 4 repeatedly until the result is within this range (e.g., if $X = 11$, then $11 - 4 - 4 = 3$, and $e = 4 \times 3 = 12$).

[12]. The rules and computational formulas for other types of asymmetrical scales, including those with two Shāhed notes, a variable note, and derivatives of Dastgāh, will be presented in a separate article.

7. Computational method of the 24-note numerical system

Before discussing the computational method, it is worth noting that the author arrived at this numerical system and computational method through trial and error and testing various methods.

1. Seven notes are counted in the sum of scale notes, and there is no need for octave Sn_7.
2. In symmetric and asymmetric scales that have one Shāhed note (the calculation method is the same), the sum of seven notes of a scale is multiplied by the number of the Shāhed note; But the last answer in the asymmetric scale is added with the extracted number as shown below:
   A. Symmetrical scale with a Shāhed note $TCSt_1 := Sn_7 \times t_1$.
   B. Asymmetric scale with a Shāhed note $TC\ddot{S}t_1 := (Sn_7 \times t_1) + e$.
3. Symmetrical scale with two Shāhed notes.
   A. The sum of the scale numbers (which are seven notes) is multiplied by the first Shāhed note and denoted as
   $$a_1 := Sn_7 \times t_1.$$
   B. The second Shāhed note is multiplied by the number 4 and added to the result of step "A"
   $$TCSt_2 := (t_2 \times 4) + a_1.$$

Note: As it is said in article 17-note in appendices VI, "Twelve is a complete cycle or the end of a repetition," and the number 24 also has the same characteristic with repetition. Therefore, the 12 and 24-note systems are the same, but the 17-note system is an incomplete cycle, so its calculations will be different compared to the other two systems in two respects.

1. Calculating the octave number in the sum of the numbers of notes for each scale (7 notes are calculated in the 24 and 12 note systems, and 8 notes are calculated in the 17-note system by considering the octave number).
2. The scale element calculations (es_1 and es_2), while irrelevant in 12- and 24-note systems, become essential for resolving the incomplete cycle inherent to the 17-note system.[13]

8. Findings

Māhūr - C (Do): Symmetrical scale with a Shāhed note as shown in Table 23.

Table 23: The Shāhed note is C =5

C	D	E	F	G	A	B	Sn_7
5	9	3	5	9	6	2	39
$TC = 39 \times 5 = 195.$							

A[14]: The remaining number is 3 and its temperament is cold and moist.

$$195 \overset{4}{\equiv} 3$$

B[15]: The remaining number 3 and the constellation scale is Gemini.

$$195 \overset{12}{\equiv} 3$$

Chāhārgāh - C (Do): Symmetrical scale with a Shāhed note as as given in Table 24

[13]. Please see reference [3].
[14]. Corresponding scale temper: The intended meaning is the innate temperament of the scale.
[15]. Corresponding scale zodiac constellation: The intended meaning is zodiac constellation of the scale.

Table 24: The Shāhed note is C =5

C	$D_\flat$	E	F	G	$A_\flat$	B	Sn_7
5	8	3	5	9	5	2	37
$a_1 = 37 \times 5 = 185.$							

A : The remaining number is 1 and its temperament is warm and dry.

$$185 \overset{4}{\equiv} 1$$

B : The remaining number 5 and the constellation scale is Leo.

$$185 \overset{12}{\equiv} 5$$

Rāst Panjgāh - C (Do): Symmetrical scale with two Shāhed notes as indicated in Tables 25

Table 25: The Shāhed note are C =5 and D=9

C	**D**	E	F	G	A	B	Sn_7
5	9	3	5	9	6	2	39
$a_1 = 39 \times 5 = 195,$ $a_2 = 4 \times 9 = 36,$ $TC = 195 + 36 = 231.$							

A : The remaining number is 3 and its temperament is cold and moist.

$$231 \overset{4}{\equiv} 3$$

B : The remaining number 3 and the constellation scale is Gemini.

$$231 \overset{12}{\equiv} 3$$

Segāh - $E_\flat$ ($Mi_\flat$): Asymmetric scale with a Shāhed note as shown in Tables 26 and 27

Table 26: The extracted number of the scale's Segāh - $E_\flat$

C	**D**	**$E_\flat$**	**F**	**Notes**	**G**	**$A_\flat$**	**$B_\flat$**	**C**
5	9	2	5	**Numbers**	9	5	8	5
$5+9+2+5=21$				**Total**	$9+5+8+5=27$			
27-21=6, 6-4=2, $e = 2 \times 4 = 8.$								

Table 27: The Shāhed note is $Mi_\flat$ =2

C	D	**$E_\flat$**	F	G	$A_\flat$	$B_\flat$	Sn_7
5	9	2	5	9	5	8	43
$a_1 = 43 \times 2 = 86,$ $TC = 86 + 8 = 94.$							

A : The remaining number is 2 and its temperament is warm and moist.

$$94 \overset{4}{\equiv} 2$$

B : The remaining number 10 and the constellation scale is Capricorn.

$$94 \overset{12}{\equiv} 10$$

Homāyun - $A_\flat$ ($La_\flat$): Asymmetric scale with a Shāhed notes as indicated in tables 28 and 29.

Table 28: The extracted number of the scale's Homāyun - $A_\flat$

G	**$A_\flat$**	**B**	**C**	**Notes**	**D**	**$E_\flat$**	**F**	**G**
9	5	2	5	**Numbers**	9	3	5	9
$9+5+2+5=21$				**Total**	$9+3+5+9=26$			
26-21=5, 5-4=1, $e=1\times 4=4$.								

Table 29: (Manzeli) - The Shāhed note is $A_\flat$ =5

G	**$A_\flat$**	B	C	D	$E_\flat$	F	Sn_7
9	5	2	5	9	3	5	38
$a_1=38\times 5=190$, $TC=190+4=194$.							

A : The remaining number is 2 and its temperament is warm and moist.

$$194 \overset{4}{\equiv} 2$$

B : The remaining number 2 and the constellation scale is Taurus.

$$194 \overset{12}{\equiv} 2$$

Navā - A (LA): Asymmetric scale with a Shāhed note as given in tables 30 and 31.

Table 30: The extracted number of the scale's Navā - A

G	**A**	**B**	**C**	**Notes**	**D**	**E**	**F♯**	**G**
9	6	2	5	**Numbers**	9	3	6	9
$9+6+2+5=22$				**Total**	$9+3+6+9=27$			
27 -22=5, 5 -4=1, $e=1\times 4=4$.								

Table 31: The Shāhed note is A =6

G	**A**	B	C	D	E	F♯	Sn_7
9	6	2	5	9	3	6	40
$a_1=40\times 6=240$, $TC=240+4=244$.							

A : The remaining number is 0 and its temperament is cold and dry.

$$244 \overset{4}{\equiv} 0$$

B : The remaining number 4 and the constellation scale is Cancer.

$$244 \overset{12}{\equiv} 4$$

Shūr - D (Re): Asymmetric scale with a Shāhed note as indicated in tables 32 and 33.

Table 32: The extracted number of the scale's Shūr -D

C	**D**	**E♭**	**F**	**Notes**	**G**	**A**	**B♭**	**C**
5	9	2	5	**Numbers**	9	6	8	5
$5+9+2+5=21$				**Total**	$9+6+8+5=28$			
28-21=7, 7-4=3, $e=3\times 4=12.$								

Table 33: (Manzeli) - The Shāhed note is D =9

C	**D**	E♭	F	G	A	B♭	Sn_7
5	9	2	5	9	6	8	44
$a_1=44\times 9=396,$ $TC=396+12=408.$							

A : The remaining number is 0 and its temperament is cold and dry.

$$408 \overset{4}{\equiv} 0$$

B : The remaining number 0 and the constellation scale is Pisces.

$$408 \overset{12}{\equiv} 0$$

The results of calculation steps for seven Dastgāhs are shown in Tables 34 to 36

Table 34: Symmetrical scale to a Shāhed note

Mode	Sn_7	t_1	TC	Tes	Sz
Māhūr	39	5	195	3	3
Chāhārgāh	37	5	185	1	5

Table 35: Symmetrical scale with two Shāheds note

Mode	Sn_7	t_1	a_1	t_2	a_2	TC	Tes	Sz
Rāst-Panjgāh	5	39	195	9	36	231	3	3

Table 36: Asymmetric scale to a Shāhed note

Mode	t_1	Sn_7	e	a_1	TC	Tes	Sz
Segāh	2	43	8	86	94	2	10
Homāyun	5	38	4	190	194	2	2
Navā	6	40	4	240	244	0	4
Shūr	9	44	12	396	408	0	0

9. Discussion

One of the important and influential factors on the balance of the psychological characteristics of the four human temperaments is music, so knowing the temperament and effects of melody can create a new way for useful and targeted research to solve physical diseases and mental disorders.

Iranian scientists have mentioned some Maqāms and their influences in their books regarding the relationship between music, temperaments, and zodiac constellations. However, there are two problems in this regard; First, no formula or rule to address this issue has been presented; for this reason, some consider those opinions as imagination.[16] Second, due to the difference in the names of the scales and Maqāms and

[16]. They attributed these Maqams to the zodiac constellation according to their imagination [21].

the uncertainty of what is used in Iranian music today, it is impossible to verify their opinions.[17] So, this article and the musical scales number systems articles[3] are introduced as the first research in the field of numbering and note calculation to investigate this important issue.

Innovative numerical orders of musical aim to explain the psychological effects of music through an interdisciplinary approach. By employing mathematical analysis of musical structures, they establish a foundation for attributing temperamental qualities (based on the theory of the four humors) and astrological characteristics (zodiac) to music. Given the predefined psychological associations in humorism and astrology, this framework facilitates the extraction of patterns that predict and elucidate the psychological impacts of music. The empirical details and practical applications of this model will be presented in future articles.

Innovative numerical orders of musical scales and their rules are a new approach to understanding and recognizing the theoretical relationships in music therapy, and they benefit from logical and numerical reasoning and calculations. In this method, numbers and calculations are used to express the result of the problem. Therefore, many subjective opinions based on personal perspectives can be modified [3]. The 24-note system is more complete than the 17-note and 12-note systems; its presence is necessary to validate and verify the calculation method. Additionally, there are some important points to consider, some of which were mentioned in the article on the 17-notes numerical system [3], while others are as follows:

1. Without having three calculation tables (two models of 17 numerical notes and a Manzeli model of 24 numerical notes), the possibility of obtaining the calculation method in scales with quarter-tone intervals may be associated with the error.
2. 24-note numerical calculation rules have slight differences compared to 17-note numerical calculations, and calculations are evaluated in each system with its method.
3. The names of the notes in both systems have been proposed to facilitate understanding of the subject and to map them onto well-known intervals. However, undoubtedly, the precise frequency position of the notes can vary, depending on the state or context of the melody. In other words, the interval defined in terms of commas, savarts, or cents for each whole step is not always fixed; it differs according to the auditory culture of each nation and ethnic group. Consequently, equal temperament is not suitable for monodic (single-voiced), melodic, and modal music, such as Iranian music, because, in performance, melodies would sound rigid and lifeless. An exception can be made for polyphonic music types, where some subtleties may be overlooked. For a skilled musician in solo performance, careful attention to the precise intonation of notes and intervals is paramount; the slightest change in the pitch of a note within any musical mode (gousheh or maqam) is of great importance.
   For a master musician performing a solo, carefully controlling the sounds of the notes and intervals, the smallest change in the bass and the treble of a note of any mode or Maqām is important. For example, the note “Mi” in the primary mode Māhūr of “Do”, is almost “1” comma less than note“Mi”, the natural major scale. This makes the interval between notes“Mi” and “Fa” approximately 5 commas. However, it is not used in the calculations because the interval between the notes “Mi” and ”Fa” is inherently Diatonic.
4. The intervals of the musical notes of each nation are defined in certain intervals (comma differences in the intervals), which, in the calculations, is considered the closest frequency to the specified criterion; because the subject of calculations is numerical, and the subject of little differences in commas is the perception.
5. In its practical application, the 24-note numerical system does not necessitate the use of frets with very close spacing. On the contrary, it provides more precise information for calculating the position of each note for specified purposes. Consequently, the presence of all possible frets on stringed instruments is not essential. Also, the 24-note system establishes a crucial theoretical foundation for microtones. By providing this solid groundwork, it challenges the conventional Western musical framework, which

[17]. For example, some scholars have linked Oshshāq to the zodiac sign Scorpio, while others have attributed it to Gemini. For further details.

is confined to tones and semitones, and paves the way for the practical application of microtones in fields such as music therapy, where their potential has remained largely untapped.

6. In the table of the 24-note system, each note is attributed to an element that expresses the temperamental quality of that note. This would happen when it is based on the frequency of the "A" note of the diapason.
7. The innate scale reflects mental states, just as the mental state of the primary mode of the Dastgāh-e Māhūr (equivalent to the natural major scale) has the qualities of dignity and calmness.[18]Dignity and stability are characteristics of a cold and moist temperament. However, the influence of the Gemini constellation's temperament, which is warm and moist, enhances courage, a trait characteristic of a warm and moist temperament. Of course, this holds only when the musical rhythm is not taken into account, and the melody is in free meter, because the mood (or character) of the rhythm also has a very significant influence on the mood of the melody.[19]
8. This study's analysis revealed that the absence of ascending quarter-tone intervals (soris) in the 17-note system [3], compared to the 24-note system, prevents a full identification of the intrinsic scale of certain Iranian musical modes across different degrees and tonalities unless a method and framework are established to compare results from both systems. For example, in the mode "Navā's Mi", the missing note "Do♯", and in "Navā's La", the missing note "Fa♯", make it impossible to precisely compare the 17- and 24-note systems.
   Consequently, the calculation of the primary mode of Dastgāh-e Navā in the article on the 17-note system[3] is incorrect because there is no comparative framework or method related to the 24-note system. Although the argument that "two scales are harmonious within one zodiac constellation" presented in the 17-note article for the modes Navā and Segāh applies to Navā and Bayāt-e Tork, more detailed studies indicate that the intrinsic scale of Navā is cold and dry and linked to the Cancer constellation. This aligns with the emotional character of Navā, which is suited for advice and cautionary messages delivered calmly, fitting the cold and moist temperament associated with Cancer. The melancholy and introspective nature of this Dastgāh strongly relates to its innate cold and dry temperament.
9. Following the review of calculations for the Navā mode in the 24-note system, the corresponding calculations for the 17-note system are presented below. A crucial point and rule, explained here with logical justification, pertains to the quarter-tone notes (soris), which are absent in the 17-note system. Since the 17-note system tables represent an incomplete model of the 24-note system and lack specific numerical assignments for sori notes, the rule for assigning them numerical values is as follows: Subtract the numerical value of the note preceding the sori from that of the note following it (typically resulting in a value of 1); this difference becomes the numerical value for the sori note. It is important to note that in the final step, the constant number 4 is added to the extracted value as the normalization constant for the sori.

   Note: This rule has thus far been applied only once, for the analysis of Dastgāh-e Navā. To verify its accuracy, generalizability, or potential need for modification, it must be further tested and evaluated in future research on analogous musical scales, as given in table 37 and 40.

   Navā - A (La): Asymmetric scale with a Shāhed note as given in tables 37 and 40.

Table 37: (Makāni) - The Shāhed note is A =14

G	**A**	B	C	D	E	F♯	G	Total
11	14	17	1	4	7	1	11	66
$a_1 = 66 \times 14 = 924, \quad es_1 : 924 \overset{4}{\equiv} 4, \quad a_2 = 4 \times 4 = 16,$ $TC = 924 + 16 + 8 + 4 = 952.$								

[18]. Please see the number 1 from the discussion and conclusion reference [3].
[19]. Please see reference [2].

A . The remaining number is 0 and its temperament is cold and dry.

$$952 \overset{4}{\equiv} 0$$

B . The remaining number 4 and the constellation scale is Cancer.

$$952 \overset{12}{\equiv} 4$$

Table 38: (Manzeli) - The Shāhed note is A =6

G	**A**	B	C	D	E	F♯	G	Total
3	6	9	1	4	7	1	3	34
$a_1 = 34 \times 6 = 204, \quad es_1 : 204 \overset{4}{\equiv} 4, \quad a_2 = 4 \times 4 = 16,$ $TC = 204 + 16 + 8 + 4 = 232.$								

A . The remaining number is 0 and its temperament is cold and dry.

$$232 \overset{4}{\equiv} 0$$

B . The remaining number 4 and the constellation scale is Cancer.

$$232 \overset{12}{\equiv} 4$$

10. As shown in Figure 3, the scale of Maqām Oshshāq is based on the fifth degree of the C major scale,[20] which is the main mode of Dastgāh-e Mahūr. By adding a quarter-tone (sori) to the seventh degree of this Oshshāq- G scale, the main modal scale of Maqām Navā is created. This finding closely resembles the way the harmonic minor scale is formed in Western music.

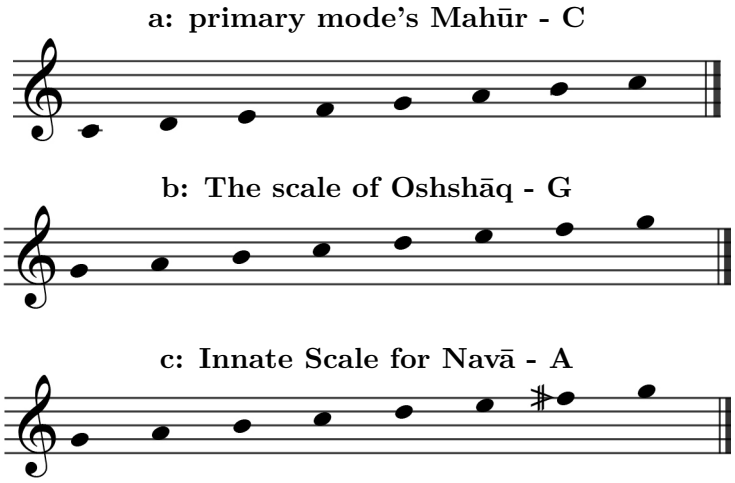


Figure 3: The order of genesis of the primary mode's Navā a to c.

Although the 17 and 24-note number systems are in line with the same goal, the 24-note number system is more complete for the following reasons:

1. It is possible to access the knowledge of temperaments and the planets of the notes in the 24-note numerical table, but this is not possible in the 17-note numerical table.

[20]. This discovery is not aligned with the view of some scholars who equate Mahūr with the older Oshshāq (see reference[9]). However, it does correspond with the views of others who have documented the intervals of Oshshāq from historical sources[6].

2. Achieving the rule of simultaneous notes in consonance and dissonance in the 24-note system is numerically provable, but there is no rule in the 17-note system (Will be explained in future articles).

Research also shows that note the "La" functions as the foundational reference, with the position of all other notes calibrated relative to it. These two systems share common points regarding the note "La", which is considered the most important note, including:

1. In both systems, the 14th cell is the place of "La".
2. In both systems, the Manzeli number of the note "La" is 6. (The note "La" is also the sixth of the seven main notes.)

10. Conclusion

To validate and verify the findings of the 17-note system study[3], it reanalyzed the Iranian music Radif using the 24-note system.The computational results indicated that, despite fundamental differences in note values and the structural division of the octave, the 24-note system and its formulas produce results that are fully consistent with the 17-note system. This consistency holds in the evaluation of the main modal scales of the seven Dastgāhs of Iranian music, which accurately identify the innate temperament (Mizāj) and the associated zodiac sign of each scale. As a result, the significant numerical differences between the two systems do not influence the final analytical outcomes. This finding supports the research of Aslani and Karimi (2018), who reported correlations between human temperaments and musical preferences within the Dastgāhs. Specifically, they observed that individuals with a cold and dry temperament preferred Dastgāh-e Shūr, those with a warm and dry temperament favored Dastgāh-e Chāhārgāh, and individuals with a cold and moist temperament preferred Dastgāh-e Māhur[5].

The results indicate that, through calculations based on the 17- and 24-note numerical systems, it is possible to determine the innate temperament of each musical scale and examine its correlations with the zodiac signs. As shown in Table 41, each zodiac sign exhibits a unique temperament, which may differ from the innate temperament of its associated scale. This divergence fosters distinct interactions between the zodiac sign and the scale's temperament. The innate temperament of the scale shapes the psychological states evoked by the melody, while the zodiac sign's temperament contributes additional effects. When the temperaments of the zodiac sign and the scale align, their combined influence on the overall temperament is intensified, leading to harmonized mental states and a more profound temperamental impact.

Table 39: Innate temperament and zodiac constellation table 7-Dastgāh classical Iranian music.

Mode	**Innate temperament**	**Zodiac sign**	**zodiac temperament**
Homāyun	Warm and moist	Taurus	Cold and dry
Māhūr	Cold and moist	Gemin	Warm and moist
Rāst-Panjgāh	Cold and moist	Gemini	Warm and moist
Chāhārgāh	Warm and dry	Leo	Warm and dry
Segāh	Warm and moist	Capricorn	Cold and dry
Navā	Cold and dry	Cancer	Cold and moist
Shūr	Cold and dry	Pisces	Cold and moist

Despite the apparent similarity in intervals between Dastgāh-e Navā and Shūr, studies have revealed that their fundamental modes are entirely distinct. This difference has historically been considered the primary reason for the independence of Dastgāh-e Navā from Shūr from the perspective of ancient musicologists, thereby dispelling the doubt of some Iranian musicians concerning the derivation of Navā and Shūr.

Investigations show that while structural similarity between tetrachords may suggest symmetry, true classification requires identical sums of Manzeli values in the 24-note system. This numerical criterion reveals fundamental differences between symmetrical scales (characterized by equal tetrachord sums) and

asymmetrical scales (displaying unequal sums), offering a precise quantitative method to distinguish scale types beyond mere structural observation.

As a new achievement, the model of the relationship between elements and planets with each note justifies why the notes are consonant and dissonant (as will be discussed in the next article) and the reasons for the difference in the states of the same scales with different tonalities. Additionally, understanding the elemental relationship and zodiac constellation through the 24-note numerical table, based on established rules and evaluation, will be useful for future research in therapeutic fields.

This research develops a comprehensive mathematical model of the 4-cycle Manzeli numbering system within the 28-cell matrix, revealing fundamental insights about cyclic modular arithmetic. The system's core innovation lies in its automatic digit-collapse mechanism, which activates when intermediate sums reach 10 through a nested digit summation process ($\mathrm{dr}(\mathrm{dr}(9+1)+1)=2$). This boundary-sensitive transformation embodies a profound computational principle. This approach guarantees three fundamental properties: (1) Cyclic Consistency through strict periodicity maintenance, (2) Deterministic Transitions via predictable modular arithmetic at boundaries, and (3) Emergent Simplicity as complex patterns reliably collapse to single-digit solutions. Specifically applied to the 24-bill system, the CBT provides a unified computational basis for handling both regular cyclic patterns and exceptional boundary behavior, with direct applications in cryptography and discrete dynamical systems where controlled phase transitions are critical. The theorem's elegance stems from its dual capability to maintain global system stability while permitting local rule redefinition at cycle edges.

The Cyclic Boundary Theorem (CBT) offers two mathematically equivalent formulations. The original version explicitly captures the musically meaningful $9 \to 2$ transition through a fixed operational sequence, providing clear pedagogical insight into boundary reset mechanisms. In contrast, the generalized version employs cyclic indexing (parameterized by cycle length) to accommodate arbitrary periodic systems. Both formulations guarantee single-digit convergence via iterative digital roots and maintain deterministic boundary transitions through controlled digit collapse. The original formulation's structural transparency makes it particularly valuable for analyzing the 28-cell Manzeli system, In contrast, the generalized version extends the theorem's utility to diverse domains, including discrete mathematics (graph theory, modular arithmetic), cryptography, and dynamical systems, demonstrating the framework's adaptability to position-dependent transformations in abstract algebraic structures. The generalized form is expressed as:

Let $k \in \mathbb{Z}^+$ be the cell's positional index.

- $n(k)$ indicates the cycle number containing the cell k.
- M_k represents the final Manzeli number for cell k, computed through cycle-based digital root transformation.
- The ∞ symbol in dr indicates iterative digital root computation until a single-digit number is obtained.

$$n(k) = \left\lfloor \frac{k-10}{8} \right\rfloor + 1.$$

$$M_k = \begin{cases} k & \text{if } 1 \leqslant k \leqslant 9, \\ \mathrm{dr}^{\infty}(k + n(k)) & \text{if } k \geqslant 10. \end{cases}$$

This study not only provides scientific validation of historical hypotheses regarding the connection between music and mathematics, temperament theory, and astronomy, but also establishes a foundation for interdisciplinary research in music psychology, music therapy, and cultural studies through its precise computational models. The proposed system, with its enhanced accuracy in representing musical intervals, will serve as an effective tool for future investigations into the profound effects of music on human psychology and physiology.

11. Appendices

I. Zodiac Constellation and corresponding planet: Each of the twelve regions of the zodiac Constellation,

where the Sun is located in one of them each month, is roughly equivalent to solar months. Each of the zodiac signs has a constellation (planet) as follows: 1- Aries: Mars 2- Taurus: Venus 3- Gemini: Mercury 4- Cancer: Moon 5- Leo: Sun 6- Virgo: Mercury 7- Libra: Venus 8- Scorpio: Mars 9- Sagittarius: Jupiter 10- Capricorn: Saturn 11- Aquarius: Saturn 12- Pisces: Jupiter.

II. It is assumed that there is a small circle inside a larger circle, both concentric, which is divided into 12 equal parts (see Figure 4). The large circle represents equal half-step (semitone) intervals, while the small circle represents equal quarter-tone intervals, resulting in a total of 24 notes within a 360° angular space. Since the base number in the 24-part extraction of elements and planets' table must be a single digit, 360 is reduced to a single digit by summing its digits ($3+6+0=9$).

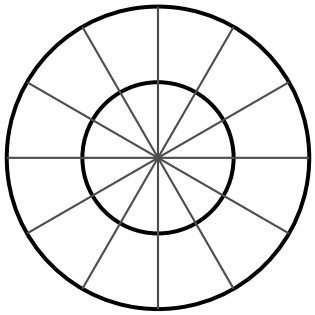


Figure 4: Nested circles diagram showing 12-fold symmetry

It also seems that the ancients associated the number 9 with music. They divided the interval of one whole step into 9 equal parts and called it a comma.
Even in the numerical value of the Latin letters (Gematria) According to Table 40, the word (comma) is equal to the number 144, which becomes 9 in the sum of the numerical digits, which is shown below through the numerical values of the letters in the so-called Gematria of the Latin alphabet table and proves this argument as indicated in table 41.

Table 40: Letter-Number Correspondences

Letter	A	B	C	D	E	F	G	H	I	J	K	L	M
Number	1	2	3	4	5	6	7	8	9	10	20	30	40
Letter	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
Number	50	60	70	80	90	100	200	300	400	500	600	700	800

Table 41: Letter Values and Calculations

A	M	M	O	C	Letter Sum	Digit Sum
1	40	40	60	3	$3+60+40+40+1=144$	$\mathrm{dr}(144)=9$

III. Innate scale: A scale whose interval structure, from a specific degree to its octave, is identical to that of other scales of the same type, thereby defining it as the foundational (innate) form relative to them. Consequently, in a 17-note system, a single scale can be expressed in 17 different modal rotations, yet only one of these 17 interval patterns serves as the reference (innate) scale [3].

IV. AAcquired scale: A scale is a scale that begins on a different degree but retains the identical interval structure of the innate scale. For example, the acquired scale Māhūr on Fa is obtained by playing the distance pattern of the innate Māhūr on Do starting from Fa[3].

V. Practical scale: It is a scale whose performance from a given note defines the personality and emotional state of that Māqam. For instance, the natural minor scale (A minor) originates from the natural major scale (C major), but the practical scale of A minor - the one actually performed - starts on the ascending sixth degree of C major, and it is this specific starting point that establishes the Māqam's identity[3].

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