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Balanced Fermatean quadripartitioned neutrosophic fuzzy graphs and its application

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Abstract

This article presents and investigates the novel model of a balanced Fermatean Quadripartitioned Neutrosophic Fuzzy Graphs, based on density functions. We examine the intrinsic properties of these graphs, focusing on the necessary conditions for Fermatean Quadripartitioned Neutrosophic Fuzzy Graphs (FQNFG) to achieve balance, particularly under self-complementary, complete, and strong graph characteristics. Additionally, we analyze the properties of FQNFG complements, providing insights into their structural relationships and transformations. Finally, we apply this theoretical foundation to model student performance, addressing uncertainties and complexities in educational data. Leveraging balanced FQNFGs provides clearer and fairer insights into student performance, enabling targeted interventions and supporting more effective with equitable educational practices.

Keywords: Fermatean neutrosophic graph (FNG), Density of FNG, Balanced FNG, Fermatean quadripartitioned neutrosophic graph.

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1. Introduction

The Fuzzy Set Theory was first proposed by Zadeh (1965) [32]. Graphs are visual tools used to represent relationships among objects. In many real-world situations, these relationships are not precise, making Fuzzy graph (FG) models more suitable than regular graphs, even though both share a similar structural foundation. Kaufmann introduced FGs based on Zadeh's fuzzy relation. Rosenfeld later introduced and expanded several foundational concepts for FGs, such as cycles, connectedness, and bridges.

Karunambigai and Parvathi [23] introduced Intuitionistic Fuzzy graphs (IFG), which were later extended into Intuitionistic Fuzzy hypergraphs along with investigations into their applications [5].The notion of Pythagorean fuzzy graphs (PFG) was incorporated into the FG framework in [22].More recently, the combination of FG with Pythagorean Neutrosopic set has resulted in the formulation of Pythagorean neutrosophic fuzzy graphs [1, 2, 3, 4, 10, 14, 15]. The Fermatean Fuzzy Set [26] was initially proposed by Senapati et al. Fermatean Neutrosophic Fuzzy sets (FNFS), initially presented in [7], integrate the ideas

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of Fermatean frameworks with neutrosophic concepts. Subsequently, Said Broumi et al. utilized this new concept to develop a framework for Fermatean Neutrosophic Graphs and their applications [11].

Belnap [9] introduced a four-valued logical system to address incomplete and inconsistent information, which inspired the development of Quadripartitioned Neutrosophic Sets, where the indeterminacy component is divided into contradiction and ignorance. Building on this concept, Satham Hussain et al. [18, 19] further extended the framework, enhancing its ability to represent complex forms of uncertainty. Bimal Shil et al. [27] further strengthened this theory by examining various structural properties. More relevant works on quadripartitioned neutrosophic graphs are provided in [21, 25, 23]. Following this, V. Divya and J. Jesintha Rosline [16, 17] introduced *FQNFG*, providing enhanced flexibility for modeling complex uncertain information.

Balanced fuzzy graphs are highly important and having a significant impact across many fields and disciplines. They are particularly useful for modeling balanced networks, where strong structural connectivity can facilitate network advancements. In general, they offer a flexible framework for modeling uncertainty and balance in complex systems. Talal Al-Hawary is credited with the initial development of balanced fuzzy graphs [30].Subsequently, Karunambigai et al. pioneered the concept of a balanced Intuitionistic fuzzy graph [20]. Following this, Rashmanlou and Pal investigated Balanced Interval-valued fuzzy graphs [24], while Sankar and Akram proposed balanced bipolar fuzzy graphs [6]. Further studies have explored various extensions, including balanced picture fuzzy graphs, neutrosophic graphs and Fermatean neutrosophic graphs [8, 12, 13]. Notably, Fermatean neutrosophic graphs are particularly effective in modeling complex network topologies. Related works in the literature have also been reviewed to support the development of the proposed approach [28, 29, 31].

Inspired by these advancements and the expanding concept of balanced graphs, we have focused on introducing balanced and strictly balanced graphs within FQNFG. The present study investigates the key characteristics of balanced *FQNFGs*. Section 2 presents the preliminaries. Section 3 delves into the characteristics of the balanced FQNFG. Finally, Section 4 presents a model application designed to optimize student academic performance and enhance interactions within educational systems.

2. Preliminaries

Definition 2.1. [11] A *FNG* is defined on a universal set X is a pair $G = (P, Q)$ where P is Fermatean neutrosophic set on X and Q is a Fermatean Neutrosophic relation on X so that: $T_Q(u,v) \leqslant T_p(u) \wedge T_p(v)$
$I_Q(u,v) \geqslant I_p(u) \vee I_p(v))$
$F_Q(u,v) \geqslant F_p(u) \vee F_p(v)$
and $0 \leqslant T_Q^3(u,v) + I_Q^3(u,v) + F_Q^3(u,v) \leqslant 2\ \forall u,v \in X$, where, $T_Q : X \times X \to [0,1]$, $I_Q : X \times X \to [0,1]$ and $F_Q : X \times X \to [0,1]$ indicates degree of membership, degree of indeterminacy-membership and degree of non-membership of Q, accordingly. Here, P and Q are the Fermatean Neutrosophic vertex and edge set of G.

Definition 2.2. [16] A *FQNFG* is defined on a Universal Set X is a pair $G_{FQ} = (\sigma_{FQ}, \mu_{FQ})$, $\sigma_{FQ} : X \to [0,1]$ is a Fermatean Quadripartitioned Neutrosophic Set on X and $\mu_{FQ} : X \times X \to [0,1]$ is a Fermatean Quadripartitioned Neutrosophic mapping on $X \times X$ so that

$$\begin{aligned} T\mu_{FQ}(uv) &\leqslant \min(T_{\sigma_{FQ}}(u), T_{\sigma_{FQ}}(v)) \\ C_{\mu_{FQ}}(uv) &\leqslant \min(C_{\sigma_{FQ}}(u), C_{\sigma_{FQ}}(v)) \\ I_{\mu_{FQ}}(uv) &\geqslant \max(I_{\sigma_{FQ}}(u), I_{\sigma_{FQ}}(v)) \\ F_{\mu_{FQ}}(uv) &\geqslant \max(F_{\sigma_{FQ}}(u), F_{\sigma_{FQ}}(v)) \end{aligned}$$

with $0 \leqslant T^3_{\mu_{FQ}}(uv) + C^3_{\mu_{FQ}}(uv) + I^3_{\mu_{FQ}}(uv) + F^3_{\mu_{FQ}}(uv) \leqslant 3\ \forall u,v \in X$, where $T_{\mu_{FQ}} : X \times X \to [0,1]$, $C_{\mu_{FQ}} : X \times X \to [0,1]$, $I_{\mu_{FQ}} : X \times X \to [0,1]$, $F_{\mu_{FQ}} : X \times X \to [0,1]$, indicates degree of truth, contradiction, ignorance and false membership of μ_{FQ}. σ_{FQ} and μ_{FQ} is the Fermatean Quadripartitioned Neutrosophic vertex and edge set of G_{FQ}.

Definition 2.3. [16] A *FQNFG* $G_{FQ} = (\sigma_{FQ}, \mu_{FQ})$ is said to be complete if the following hold:

$$\begin{aligned} T_{\mu_{FQ}}(uv) &= (T_{\sigma_{FQ}}(u) \wedge T_{\sigma_{FQ}}(v)) \\ C_{\mu_{FQ}}(uv) &= (C_{\sigma_{FQ}}(u) \wedge C_{\sigma_{FQ}}(v)) \\ I_{\mu_{FQ}}(uv) &= (I_{\sigma_{FQ}}(u) \vee I_{\sigma_{FQ}}(v)) \\ F_{\mu_{FQ}}(uv) &= (F_{\sigma_{FQ}}(u) \vee F_{\sigma_{FQ}}(v)) \end{aligned}$$

where $u, v \in \sigma_{FQ}$.

Definition 2.4. [16] Let $G_{FQ} = (\sigma_{FQ}, \mu_{FQ})$ be a *FQNFG*. The Complement of *FQNFG* is $\overline{G_{FQ}}= (\overline{\sigma_{FQ}},\overline{\mu_{FQ}})$, where $\overline{\sigma_{FQ}}$=($\overline{T_{\sigma_{FQ}}}$,$\overline{C_{\sigma_{FQ}}}$, $\overline{I_{\sigma_{FQ}}}$, $\overline{F_{\sigma_{FQ}}}$) and $\overline{\mu_{FQ}}$=($\overline{T_{\sigma_{FQ}}}$,$\overline{C_{\sigma_{FQ}}}$, $\overline{I_{\sigma_{FQ}}}$, $\overline{F_{\sigma_{FQ}}}$) defined by

$$\begin{aligned} \overline{T_{\mu_{FQ}}} &= |T_{\sigma_{FQ}}(u) \wedge T_{\sigma_{FQ}}(v) - T_{\mu_{FQ}}(uv)| \\ \overline{C_{\mu_{FQ}}} &= |C_{\sigma_{FQ}}(u) \wedge C_{\sigma_{FQ}}(v) - C_{\mu_{FQ}}(uv)| \\ \overline{I_{\mu_{FQ}}} &= |I_{\sigma_{FQ}}(u) \vee I_{\sigma_{FQ}}(v) - I_{\mu_{FQ}}(uv)| \\ \overline{F_{\mu_{FQ}}} &= |F_{\sigma_{FQ}}(u) \vee F_{\sigma_{FQ}}(v) - F_{\mu_{FQ}}(uv)| \end{aligned}$$

$\forall\ u, v \in \sigma_{FQ}$ and $uv \in \mu_{FQ}$.

Definition 2.5. [16] A *FQNFG* $G_{FQ} = (\sigma_{FQ}, \mu_{FQ})$ is said to be strong, if

$$T_{\mu_{FQ}}(u, v) = (T_{\sigma_{FQ}}(u) \wedge T_{\sigma_{FQ}}(v))$$

$$C_{\mu_{FQ}}(u, v) = (C_{\sigma_{FQ}}(u) \wedge C_{\sigma_{FQ}}(v))$$
$$I_{\mu_{FQ}}(u, v) = (I_{\sigma_{FQ}}(u) \vee I_{\sigma_{FQ}}(v))$$
$$F_{\mu_{FQ}}(u, v) = (F_{\sigma_{FQ}}(u) \vee F_{\sigma_{FQ}}(v))$$

where $u, v \in \sigma_{FQ}$.

Definition 2.6. [28] The density of a *FNG* $G = (P, Q)$ associated with the underlying crisp graph $G^* = (V, E)$ is defined as $D(G) = (D_T(G), D_I(G), D_F(G))$, where

$$D_T(G) = \frac{2 \sum\limits_{u,v \in P} T_Q(u, v)}{\sum\limits_{u,v \in Q} T_P(u) \wedge T_P(v)}$$

$$D_I(G) = \frac{2 \sum\limits_{u,v \in P} I_Q(u, v)}{\sum\limits_{u,v \in Q} I_P(u) \vee I_P(v)}$$

$$D_F(G) = \frac{2 \sum\limits_{u,v \in P} F_Q(u, v)}{\sum\limits_{u,v \in Q} F_P(u) \vee F_P(v)}$$

for $(u, v) \in P$

Definition 2.7. [28] A *FNG* is said to be balanced if $D(H) \leqslant D(G)$, i.e. $D_T(H) \leqslant D_T(G)$, $D_I(H) \leqslant D_I(G)$, $D_F(H) \leqslant D_F(G)$ for all subgraphs H of G.

3. Balanced Fermatean Quadripartitioned Neutrosophic Fuzzy Graphs

Definition 3.1. The density of a *FQNFG* $G_{FQ} = (\sigma_{FQ}, \mu_{FQ})$ is defined as $D(G_{FQ}) = (D_T(G_{FQ}), D_C(G_{FQ}), D_I(G_{FQ}), D_F(G_{FQ}))$, where

$$D_T(G_{FQ}) = \frac{2\sum_{u,v\in\sigma_{FQ}} T_{\mu_{FQ}}(u,v)}{\sum_{(u,v)\in\mu_{FQ}} T_{\sigma_{FQ}}(u)\wedge T_{\sigma_{FQ}}(v)}$$

$$D_C(G_{FQ}) = \frac{2\sum_{u,v\in\sigma_{FQ}} C_{\mu_{FQ}}(u,v)}{\sum_{(u,v)\in\mu_{FQ}} C_{\sigma_{FQ}}(u)\wedge C_{\sigma_{FQ}}(v)}$$

$$D_I(G_{FQ}) = \frac{2\sum_{u,v\in\sigma_{FQ}} I_{\mu_{FQ}}(u,v)}{\sum_{(u,v)\in\mu_{FQ}} I_{\sigma_{FQ}}(u)\vee I_{\sigma_{FQ}}(v)}$$

$$D_F(G_{FQ}) = \frac{2\sum_{u,v\in\sigma_{FQ}} F_{\mu_{FQ}}(u,v)}{\sum_{(u,v)\in\mu_{FQ}} F_{\sigma_{FQ}}(u)\vee F_{\sigma_{FQ}}(v)}$$

Definition 3.2. A *FQNFG*, $G_{FQ} = (\sigma_{FQ}, \mu_{FQ})$ is said to be balanced if $D(H_{FQ}) \leqslant D(G_{FQ})$, that is $D_T(H_{FQ}) \leqslant D_T(G_{FQ})$, $D_C(H_{FQ}) \leqslant D_C(G_{FQ})$, $D_I(H_{FQ}) \leqslant D_I(G_{FQ})$, $D_F(H_{FQ}) \leqslant D_F(G_{FQ})$, for all subgraphs H_{FQ} of G_{FQ}.

Example 3.3. Consider a *FQNFG*, $G_{FQ} = (\sigma_{FQ}, \mu_{FQ})$, such that $\sigma_{FQ} = (v_1, v_2, v_3)$, $\mu_{FQ} = (v_1v_2), (v_1v_3), (v_2v_3)$

$D_T(G_{FQ}) = \frac{2(0.28+0.225+0.225)}{0.5+0.4+0.4} = 1.1$

$D_C(G_{FQ}) = \frac{2(0.18+0.18+0.18)}{0.3+0.3+0.3} = 1.2$

$D_I(G_{FQ}) = \frac{2(0.53+0.74+0.73)}{0.5+0.7+0.7} = 2.1$

$D_F(G_{FQ}) = \frac{2(0.64+0.63+0.42)}{0.6+0.6+0.4} = 2.1$

$D(G_{FQ}) = (D_T(G_{FQ}), D_C(G_{FQ}), D_I(G_{FQ}), D_F(G_{FQ})) = (1.1, 1.2, 2.1, 2.1)$

Let $H_{1_{FQ}} = (v_1, v_2)$, $H_{2_{FQ}} = (v_1, v_3)$, $H_{3_{FQ}} = (v_2, v_3)$, $H_{4_{FQ}} = (v_1, v_2, v_3)$ be non-empty subgraphs of G_{FQ}.
Density $(D_T(H_{FQ}), D_C(H_{FQ}), D_I(H_{FQ}), D_F(H_{FQ}))$ is $D(H_{1_{FQ}}) = (1.1, 1.2, 2.1, 2.1)$, $D(H_{2_{FQ}}) = (1.1, 1.2, 2.1, 2.1)$, $D(H_{3_{FQ}}) = (1.1, 1.2, 2.1, 2.1)$, $D(H_{4_{FQ}}) = (1.1, 1.2, 2.1, 2.1)$. So $D(H_{FQ}) \leqslant D(G_{FQ})$ for all subgraphs H_{FQ} of G_{FQ}.
Hence G_{FQ} is balanced *FQNFG*.

Definition 3.4. A *FQNFG*, $G_{FQ} = (\sigma_{FQ}, \mu_{FQ})$ is said to be strictly balanced if for $u, v \in \sigma_{FQ}$, $D(H_{FQ}) = D(G_{FQ})$ for all subgraphs H_{FQ} of G_{FQ}.

Example 3.5. Consider a *FQNFG*, $G_{FQ} = (\sigma_{FQ}, \mu_{FQ})$ such that $\sigma_{FQ} = (v_1, v_2, v_3)$, $\mu_{FQ} = (v_1v_2), (v_1v_3), (v_2v_3)$

$D_T(G_{FQ}) = \frac{2(0.49+0.425+0.425)}{0.7+0.6+0.6} = 1.4$

$D_C(G_{FQ}) = \frac{2(0.28+0.27+0.28)}{0.4+0.4+0.4} = 1.4$

$D_I(G_{FQ}) = \frac{2(0.73+0.94+0.93)}{0.7+0.9+0.9} = 2.1$

$D_F(G_{FQ}) = \frac{2(0.85+0.83+0.53)}{0.8+0.8+0.5} = 2.1$

$D(G_{FQ}) = (D_T(G_{FQ}), D_C(G_{FQ}), D_I(G_{FQ}), D_F(G_{FQ})) = (1.4, 1.4, 2.1, 2.1)$

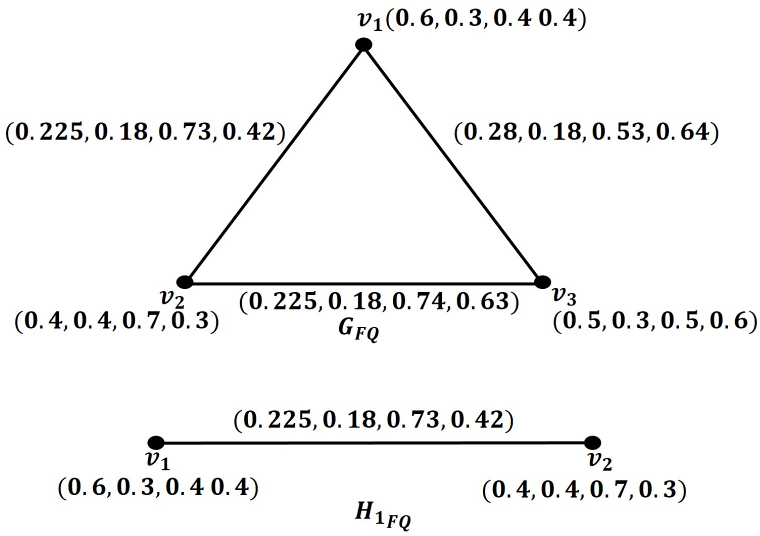

Figure 1: Balanced *FQNFG*

Let $H_{1_{FQ}} = (v_1, v_2)$, $H_{2_{FQ}} = (v_1, v_3)$, $H_{3_{FQ}} = (v_2, v_3)$, $H_{4_{FQ}} = (v_1, v_2, v_3)$ be non-empty subgraphs of G_{FQ}. Density $(D_T(H_{FQ}), D_C(H_{FQ}), D_I(H_{FQ}), D_F(H_{FQ}))$ is $D(H_{1_{FQ}}) = (1.4, 1.4, 2.1, 2.1)$, $D(H_{2_{FQ}}) = (1.4, 1.4, 2.1, 2.1)$, $D(H_{3_{FQ}}) = (1.4, 1.4, 2.1, 2.1)$, $D(H_{4_{FQ}}) = (1.4, 1.4, 2.1, 2.1)$. So $D(H_{FQ}) = D(G_{FQ})$ for all subgraphs H_{FQ} of G_{FQ}.
Hence G_{FQ} is strictly balanced *FQNFG*.

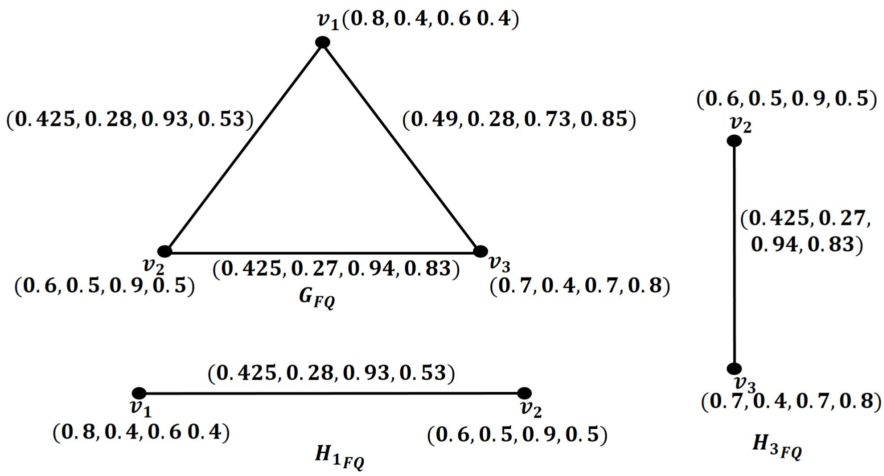

Figure 2: Balanced *FQNFG*

Theorem 3.6. *Every Complete FQNFG is balanced.*

Proof. Consider a complete *FQNFG* $G_{FQ} = (\sigma_{FQ}, \mu_{FQ})$. By definition,

$$
\begin{aligned}
T_{\mu_{FQ}}(u,v) &= T_{\sigma_{FQ}}(u) \wedge T_{\sigma_{FQ}}(v) \\
C_{\mu_{FQ}}(u,v) &= C_{\sigma_{FQ}}(u) \wedge C_{\sigma_{FQ}}(v) \\
I_{\mu_{FQ}}(u,v) &= I_{\sigma_{FQ}}(u) \vee I_{\sigma_{FQ}}(v) \\
F_{\mu_{FQ}}(u,v) &= F_{\sigma_{FQ}}(u) \vee F_{\sigma_{FQ}}(v), \text{for every } u, v \in \sigma_{FQ}
\end{aligned}
$$

$$\therefore \sum_{u,v\in\sigma_{FQ}} T_{\mu_{FQ}}(u,v) = \sum_{u,v\in\mu_{FQ}} T_{\sigma_{FQ}}(u)\wedge T_{\sigma_{FQ}}(v)$$
$$\sum_{u,v\in\sigma_{FQ}} C_{\mu_{FQ}}(u,v) = \sum_{u,v\in\mu_{FQ}} C_{\sigma_{FQ}}(u)\wedge C_{\sigma_{FQ}}(v)$$
$$\sum_{u,v\in\sigma_{FQ}} I_{\mu_{FQ}}(u,v) = \sum_{u,v\in\mu_{FQ}} I_{\sigma_{FQ}}(u)\vee I_{\sigma_{FQ}}(v)$$
$$\sum_{u,v\in\sigma_{FQ}} F_{\mu_{FQ}}(u,v) = \sum_{u,v\in\mu_{FQ}} F_{\sigma_{FQ}}(u)\vee F_{\sigma_{FQ}}(v)$$

Now,

$$D(G_{FQ}) = \left(\frac{2\sum_{u,v\in\sigma_{FQ}} T_{\mu_{FQ}}(u,v)}{\sum_{(u,v)\in\mu_{FQ}} T_{\sigma_{FQ}}(u)\wedge T_{\sigma_{FQ}}(v)}, \frac{2\sum_{u,v\in\sigma_{FQ}} C_{\mu_{FQ}}(u,v)}{\sum_{(u,v)\in\mu_{FQ}} C_{\sigma_{FQ}}(u)\wedge C_{\sigma_{FQ}}(v)}, \frac{2\sum_{u,v\in\sigma_{FQ}} I_{\mu_{FQ}}(u,v)}{\sum_{(u,v)\in\mu_{FQ}} I_{\sigma_{FQ}}(u)\vee I_{\sigma_{FQ}}(v)}, \frac{2\sum_{u,v\in\sigma_{FQ}} F_{\mu_{FQ}}(u,v)}{\sum_{(u,v)\in\mu_{FQ}} F_{\sigma_{FQ}}(u)\vee F_{\sigma_{FQ}}(v)}\right)$$

$$D(G_{FQ}) = \left(\frac{2\sum_{(u,v)\in\mu_{FQ}} T_{\sigma_{FQ}}(u)\wedge T_{\sigma_{FQ}}(v)}{\sum_{(u,v)\in\mu_{FQ}} T_{\sigma_{FQ}}(u)\wedge T_{\sigma_{FQ}}(v)}, \frac{2\sum_{(u,v)\in\mu_{FQ}} C_{\sigma_{FQ}}(u)\wedge C_{\sigma_{FQ}}(v)}{\sum_{(u,v)\in\mu_{FQ}} C_{\sigma_{FQ}}(u)\wedge C_{\sigma_{FQ}}(v)}, \frac{2\sum_{(u,v)\in\mu_{FQ}} I_{\sigma_{FQ}}(u)\vee I_{\sigma_{FQ}}(v)}{\sum_{(u,v)\in\mu_{FQ}} I_{\sigma_{FQ}}(u)\vee I_{\sigma_{FQ}}(v)}, \frac{2\sum_{(u,v)\in\mu_{FQ}} F_{\sigma_{FQ}}(u)\vee F_{\sigma_{FQ}}(v)}{\sum_{(u,v)\in\mu_{FQ}} F_{\sigma_{FQ}}(u)\vee F_{\sigma_{FQ}}(v)}\right)$$

$D(G_{FQ}) = (2,2,2,2)$
Let H_{FQ} be a non-empty subgraph of G_{FQ} then, $D(H_{FQ}) = (2,2,2,2)$ for every $H_{FQ} \subseteq G_{FQ}$.
Thus, G_{FQ} is balanced. □

Note: The previous theorem's converse need not be true. Each balanced FQNFG does not have to be complete.

Example 3.7. Let $G_{FQ} = (\sigma_{FQ}, \mu_{FQ})$ be a *FQNFG* such that $\sigma_{FQ} = (v_1, v_2, v_3)$, $\mu_{FQ} = \{v_1v_2, v_1v_3, v_2v_3\}$
$D(G_{FQ}) = (D_T(G_{FQ}), D_C(G_{FQ}), D_I(G_{FQ}), D_F(G_{FQ})) = (1.8, 1.7, 2.0, 2.2)$
Let $H_{1_{FQ}} = (v_1, v_2)$, $H_{2_{FQ}} = (v_1, v_3)$, $H_{3_{FQ}} = (v_2, v_3)$, $H_{4_{FQ}} = (v_1, v_2, v_3)$ be non-empty subgraphs of G_{FQ}.
Their densities are $D(H_{1_{FQ}}) = (1.8, 1.7, 2.0, 2.2)$, $D(H_{2_{FQ}}) = (1.8, 1.7, 2.0, 2.2)$, $D(H_{3_{FQ}}) = (1.8, 1.7, 2.0, 2.2)$, $D(H_{4_{FQ}}) = (1.8, 1.7, 2.0, 2.2)$.
Therefore $D(H_{FQ}) \leqslant D(G_{FQ})$ for all subgraphs H_{FQ} of G_{FQ}. Hence, G_{FQ} is a balanced *FQNFG*.
Figure 3 shows that:
$T_{\mu_{FQ}}(u,v) \neq T_{\sigma_{FQ}}(u)\wedge T_{\sigma_{FQ}}(v)$, $C_{\mu_{FQ}}(u,v) \neq C_{\sigma_{FQ}}(u)\wedge C_{\sigma_{FQ}}(v)$, $I_{\mu_{FQ}}(u,v) = I_{\sigma_{FQ}}(u)\vee I_{\sigma_{FQ}}(v)$ and $F_{\mu_{FQ}}(u,v) \neq F_{\sigma_{FQ}}(u)\vee F_{\sigma_{FQ}}(v)$, for every $u, v \in \sigma_{FQ}$.
Thus, G_{FQ} is balanced but not complete.

Corollory 1: Every strong *FQNFG* is balanced.

Theorem 3.8. *Let* $G_{FQ} = (\sigma_{FQ}, \mu_{FQ})$ *be a self-complementary FQNFG, then* $D(G_{FQ}) = (1,1,1,1)$

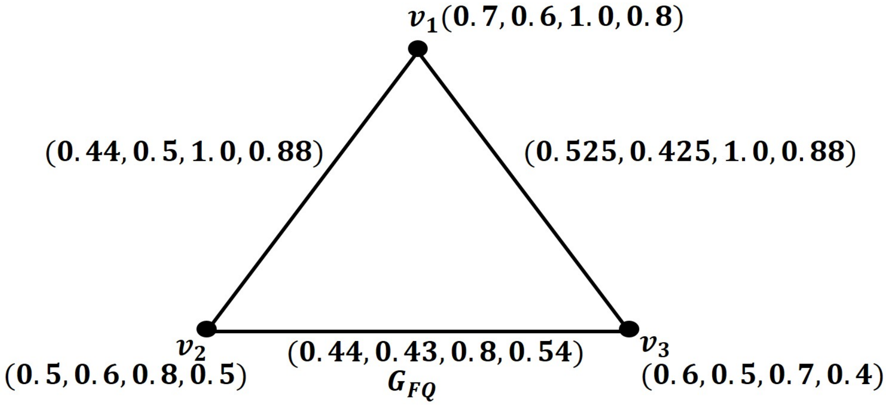

Figure 3: Balanced *FQNFG*

Proof. Consider a self-complementary *FQNFG* $G_{FQ} = (\sigma_{FQ}, \mu_{FQ})$. Then

$$\begin{aligned}
\sum_{u,v\in\sigma_{FQ}} T_{\mu_{FQ}}(u,v) &= \frac{1}{2}\sum_{(u,v)\in\mu_{FQ}} T_{\sigma_{FQ}}(u)\wedge T_{\sigma_{FQ}}(v)\\
\sum_{u,v\in\sigma_{FQ}} C_{\mu_{FQ}}(u,v) &= \frac{1}{2}\sum_{(u,v)\in\mu_{FQ}} C_{\sigma_{FQ}}(u)\wedge C_{\sigma_{FQ}}(v)\\
\sum_{u,v\in\sigma_{FQ}} I_{\mu_{FQ}}(u,v) &= \frac{1}{2}\sum_{(u,v)\in\mu_{FQ}} I_{\sigma_{FQ}}(u)\vee I_{\sigma_{FQ}}(v)\\
\sum_{u,v\in\sigma_{FQ}} F_{\mu_{FQ}}(u,v) &= \frac{1}{2}\sum_{(u,v)\in\mu_{FQ}} F_{\sigma_{FQ}}(u)\vee F_{\sigma_{FQ}}(v)
\end{aligned}$$

Now,

$$D(G_{FQ}) = \left(\frac{2\sum\limits_{u,v\in\sigma_{FQ}} T_{\mu_{FQ}}(u,v)}{\sum\limits_{(u,v)\in\mu_{FQ}} T_{\sigma_{FQ}}(u)\wedge T_{\sigma_{FQ}}(v)}, \frac{2\sum\limits_{u,v\in\sigma_{FQ}} C_{\mu_{FQ}}(u,v)}{\sum\limits_{(u,v)\in\mu_{FQ}} C_{\sigma_{FQ}}(u)\wedge C_{\sigma_{FQ}}(v)}, \frac{2\sum\limits_{u,v\in\sigma_{FQ}} I_{\mu_{FQ}}(u,v)}{\sum\limits_{(u,v)\in\mu_{FQ}} I_{\sigma_{FQ}}(u)\vee I_{\sigma_{FQ}}(v)}, \frac{2\sum\limits_{u,v\in\sigma_{FQ}} F_{\mu_{FQ}}(u,v)}{\sum\limits_{(u,v)\in\mu_{FQ}} F_{\sigma_{FQ}}(u)\vee F_{\sigma_{FQ}}(v)}\right)$$

$$D(G_{FQ}) = \left(\frac{2\sum\limits_{(u,v)\in\mu_{FQ}} T_{\sigma_{FQ}}(u)\wedge T_{\sigma_{FQ}}(v)}{2\sum\limits_{(u,v)\in\mu_{FQ}} T_{\sigma_{FQ}}(u)\wedge T_{\sigma_{FQ}}(v)}, \frac{2\sum\limits_{(u,v)\in\mu_{FQ}} C_{\sigma_{FQ}}(u)\wedge C_{\sigma_{FQ}}(v)}{2\sum\limits_{(u,v)\in\mu_{FQ}} C_{\sigma_{FQ}}(u)\wedge C_{\sigma_{FQ}}(v)}, \frac{2\sum\limits_{(u,v)\in\mu_{FQ}} I_{\sigma_{FQ}}(u)\vee I_{\sigma_{FQ}}(v)}{2\sum\limits_{(u,v)\in\mu_{FQ}} I_{\sigma_{FQ}}(u)\vee I_{\sigma_{FQ}}(v)}, \frac{2\sum\limits_{(u,v)\in\mu_{FQ}} F_{\sigma_{FQ}}(u)\vee F_{\sigma_{FQ}}(v)}{2\sum\limits_{(u,v)\in\mu_{FQ}} F_{\sigma_{FQ}}(u)\vee F_{\sigma_{FQ}}(v)}\right)$$

Hence $D(G_{FQ}) = (1, 1, 1, 1)$ □

Theorem 3.9. *Let* $G_{FQ} = (\sigma_{FQ}, \mu_{FQ})$ *be a strictly balanced FQNFG and* $\overline{G_{FQ}} = (\overline{\sigma_{FQ}}, \overline{\mu_{FQ}})$ *be its complement then* $D(G_{FQ}) + D(\overline{G_{FQ}}) = (2, 2, 2, 2)$

Proof. Let $G_{FQ} = (\sigma_{FQ}, \mu_{FQ})$ be a strictly balanced *FQNFG* and $\overline{G_{FQ}} = (\overline{\sigma_{FQ}}, \overline{\mu_{FQ}})$ be its complement. Let H_{FQ} be a subgraph of G which is non-empty. $D(G_{FQ}) = D(H_{FQ})$ for all $H_{FQ} \subseteq G_{FQ}$ and $u, v \in \sigma_{FQ}$. Since G is strictly balanced.

In $\overline{G_{FQ}}$,

$$\overline{T_{\mu_{FQ}}(u,v)} = T_{\sigma_{FQ}}(u) \wedge T_{\sigma_{FQ}}(v) - T_{\mu_{FQ}}(u,v) \tag{3.1}$$
$$\overline{C_{\mu_{FQ}}(u,v)} = C_{\sigma_{FQ}}(u) \wedge C_{\sigma_{FQ}}(v) - C_{\mu_{FQ}}(u,v) \tag{3.2}$$
$$\overline{I_{\mu_{FQ}}(u,v)} = I_{\sigma_{FQ}}(u) \vee I_{\sigma_{FQ}}(v) - I_{\mu_{FQ}}(u,v) \tag{3.3}$$
$$\overline{F_{\mu_{FQ}}(u,v)} = F_{\sigma_{FQ}}(u) \vee F_{\sigma_{FQ}}(v) - F_{\mu_{FQ}}(u,v) \ \forall \ (u,v) \in \mu_{FQ} \tag{3.4}$$

Dividing (3.1) by $T_{\sigma_{FQ}}(u) \wedge T_{\sigma_{FQ}}(v)$

$$\frac{\overline{T_{\mu_{FQ}}(u,v)}}{T_{\sigma_{FQ}}(u) \wedge T_{\sigma_{FQ}}(v)} = 1 - \frac{T_{\mu_{FQ}}(u,v)}{T_{\sigma_{FQ}}(u) \wedge T_{\sigma_{FQ}}(v)}$$

Similarly, Dividing (3.2) by $C_{\sigma_{FQ}}(u) \wedge C_{\sigma_{FQ}}(v)$

$$\frac{\overline{C_{\mu_{FQ}}(u,v)}}{C_{\sigma_{FQ}}(u) \wedge C_{\sigma_{FQ}}(v)} = 1 - \frac{C_{\mu_{FQ}}(u,v)}{C_{\sigma_{FQ}}(u) \wedge C_{\sigma_{FQ}}(v)}$$

Dividing (3.3) by $I_{\sigma_{FQ}}(u) \vee I_{\sigma_{FQ}}(v)$

$$\frac{\overline{I_{\mu_{FQ}}(u,v)}}{I_{\sigma_{FQ}}(u) \vee I_{\sigma_{FQ}}(v)} = 1 - \frac{I_{\mu_{FQ}}(u,v)}{I_{\sigma_{FQ}}(u) \vee I_{\sigma_{FQ}}(v)}$$

Dividing (3.4) by $F_{\sigma_{FQ}}(u) \vee F_{\sigma_{FQ}}(v)$

$$\frac{\overline{F_{\mu_{FQ}}(u,v)}}{F_{\sigma_{FQ}}(u) \vee F_{\sigma_{FQ}}(v)} = 1 - \frac{F_{\mu_{FQ}}(u,v)}{F_{\sigma_{FQ}}(u) \vee F_{\sigma_{FQ}}(v)}$$

then

$$\sum_{u,v \in \sigma_{FQ}} \frac{\overline{T_{\mu_{FQ}}(u,v)}}{T_{\sigma_{FQ}}(u) \wedge T_{\sigma_{FQ}}(v)} = 1 - \sum_{u,v \in \sigma_{FQ}} \frac{T_{\mu_{FQ}}(u,v)}{T_{\sigma_{FQ}}(u) \wedge T_{\sigma_{FQ}}(v)}$$
$$\sum_{u,v \in \sigma_{FQ}} \frac{\overline{C_{\mu_{FQ}}(u,v)}}{C_{\sigma_{FQ}}(u) \wedge C_{\sigma_{FQ}}(v)} = 1 - \sum_{u,v \in \sigma_{FQ}} \frac{C_{\mu_{FQ}}(u,v)}{C_{\sigma_{FQ}}(u) \wedge C_{\sigma_{FQ}}(v)}$$
$$\sum_{u,v \in \sigma_{FQ}} \frac{\overline{I_{\mu_{FQ}}(u,v)}}{I_{\sigma_{FQ}}(u) \vee I_{\sigma_{FQ}}(v)} = 1 - \sum_{u,v \in \sigma_{FQ}} \frac{I_{\mu_{FQ}}(u,v)}{I_{\sigma_{FQ}}(u) \vee I_{\sigma_{FQ}}(v)}$$
$$\sum_{u,v \in \sigma_{FQ}} \frac{\overline{F_{\mu_{FQ}}(u,v)}}{F_{\sigma_{FQ}}(u) \vee F_{\sigma_{FQ}}(v)} = 1 - \sum_{u,v \in \sigma_{FQ}} \frac{F_{\mu_{FQ}}(u,v)}{F_{\sigma_{FQ}}(u) \vee F_{\sigma_{FQ}}(v)}$$

Multiply the above equations by 2 on both sides

$$2\sum_{u,v \in \sigma_{FQ}} \frac{\overline{T_{\mu_{FQ}}(u,v)}}{T_{\sigma_{FQ}}(u) \wedge T_{\sigma_{FQ}}(v)} = 2 - 2\sum_{u,v \in \sigma_{FQ}} \frac{T_{\mu_{FQ}}(u,v)}{T_{\sigma_{FQ}}(u) \wedge T_{\sigma_{FQ}}(v)}$$
$$2\sum_{u,v \in \sigma_{FQ}} \frac{\overline{C_{\mu_{FQ}}(u,v)}}{C_{\sigma_{FQ}}(u) \wedge C_{\sigma_{FQ}}(v)} = 2 - 2\sum_{u,v \in \sigma_{FQ}} \frac{C_{\mu_{FQ}}(u,v)}{C_{\sigma_{FQ}}(u) \wedge C_{\sigma_{FQ}}(v)}$$
$$2\sum_{u,v \in \sigma_{FQ}} \frac{\overline{I_{\mu_{FQ}}(u,v)}}{I_{\sigma_{FQ}}(u) \vee I_{\sigma_{FQ}}(v)} = 2 - 2\sum_{u,v \in \sigma_{FQ}} \frac{I_{\mu_{FQ}}(u,v)}{I_{\sigma_{FQ}}(u) \vee I_{\sigma_{FQ}}(v)}$$
$$2\sum_{u,v \in \sigma_{FQ}} \frac{\overline{F_{\mu_{FQ}}(u,v)}}{F_{\sigma_{FQ}}(u) \vee F_{\sigma_{FQ}}(v)} = 2 - 2\sum_{u,v \in \sigma_{FQ}} \frac{F_{\mu_{FQ}}(u,v)}{F_{\sigma_{FQ}}(u) \vee F_{\sigma_{FQ}}(v)}$$

for every $u, v \in \sigma_{FQ}$

$$\begin{aligned} D_T(\overline{G_{FQ}}) &= 2 - D_T(G_{FQ}) \\ D_C(\overline{G_{FQ}}) &= 2 - D_C(G_{FQ}) \\ D_I(\overline{G_{FQ}}) &= 2 - D_I(G_{FQ}) \\ D_F(\overline{G_{FQ}}) &= 2 - D_F(G_{FQ}) \end{aligned}$$

Now,
$D(G_{FQ}) + D(\overline{G_{FQ}}) = (D_T(G_{FQ}), D_C(G_{FQ}), D_I(G_{FQ}), D_F(G_{FQ})) + (D_T(\overline{G_{FQ}}), D_C(\overline{G_{FQ}}), D_I(\overline{G_{FQ}}), D_F(\overline{G_{FQ}}))$
$D(G_{FQ}) + D(\overline{G_{FQ}}) = ((D_T(G_{FQ} + D_T(\overline{G_{FQ}})), (D_C(G_{FQ}) + D_C(\overline{G_{FQ}})), (D_I(G_{FQ}) + D_I(\overline{G_{FQ}})), (D_F(G_{FQ}) + D_F(\overline{G_{FQ}})))$
Hence $D(G_{FQ}) + D(\overline{G_{FQ}}) = (2, 2, 2, 2)$ □

Theorem 3.10. *The complement of a strictly balanced FQNFG is also strictly balanced.*

Proof. Let $G_{FQ} = (\sigma_{FQ}, \mu_{FQ})$ be a strictly balanced *FQNFG* and $\overline{G_{FQ}} = (\overline{\sigma_{FQ}}, \overline{\mu_{FQ}})$ be its complement.
Let H_{FQ} be a subgraph of G_{FQ} which is non-empty.
$D(G_{FQ}) = D(H_{FQ})$ for all $H_{FQ} \subseteq G_{FQ}$ and $u, v \in \sigma_{FQ}$.
Since G is strictly balanced.
As G is strictly balanced by theorem 3.6, $D(G_{FQ}) + D(\overline{G_{FQ}}) = (2, 2, 2, 2)$
Since $D(H_{FQ}) + D(\overline{H_{FQ}}) = (2, 2, 2, 2)$ for every $H_{FQ} \subseteq G_{FQ}$ which implies $D(\overline{H_{FQ}}) = D(\overline{G_{FQ}})$
Hence $\overline{G_{FQ}}$ is strictly balanced. □

Theorem 3.11. *The complement of strong FQNFG is balanced.*

Proof. Let $G_{FQ} = (\sigma_{FQ}, \mu_{FQ})$ be a strong *FQNFG* and $\overline{G_{FQ}} = (\overline{\sigma_{FQ}}, \overline{\mu_{FQ}})$ be its complement.
Since G_{FQ} is strongly *FQNFG*, then

$$\begin{aligned} T_{\mu_{FQ}}(u, v) &= T_{\sigma_{FQ}}(u) \wedge T_{\sigma_{FQ}}(v) \\ C_{\mu_{FQ}}(u, v) &= C_{\sigma_{FQ}}(u) \wedge C_{\sigma_{FQ}}(v) \\ I_{\mu_{FQ}}(u, v) &= I_{\sigma_{FQ}}(u) \vee I_{\sigma_{FQ}}(v) \\ F_{\mu_{FQ}}(u, v) &= F_{\sigma_{FQ}}(u) \vee F_{\sigma_{FQ}}(v) \\ & \forall (u, v) \in \mu_{FQ} \end{aligned} \tag{3.5}$$

In $\overline{G_{FQ}}$,

$$\begin{aligned} \overline{T_{\mu_{FQ}}(u, v)} &= T_{\sigma_{FQ}}(u) \wedge T_{\sigma_{FQ}}(v) - T_{\mu_{FQ}}(u, v) \\ \overline{C_{\mu_{FQ}}(u, v)} &= C_{\sigma_{FQ}}(u) \wedge C_{\sigma_{FQ}}(v) - C_{\mu_{FQ}}(u, v) \\ \overline{I_{\mu_{FQ}}(u, v)} &= I_{\sigma_{FQ}}(u) \vee I_{\sigma_{FQ}}(v) - I_{\mu_{FQ}}(u, v) \\ \overline{F_{\mu_{FQ}}(u, v)} &= F_{\sigma_{FQ}}(u) \vee F_{\sigma_{FQ}}(v) - F_{\mu_{FQ}}(u, v) \\ & \forall (u, v) \in \mu_{FQ} \end{aligned} \tag{3.6}$$

As G is strong, then

$$\begin{aligned} (3.6) \Rightarrow \overline{T_{\mu_{FQ}}(u, v)} &= T_{\sigma_{FQ}}(u) \wedge T_{\sigma_{FQ}}(v) - T_{\mu_{FQ}}(u, v) \\ &= (T_{\sigma_{FQ}}(u) \wedge T_{\sigma_{FQ}}(v)) - (T_{\sigma_{FQ}}(u) \wedge T_{\sigma_{FQ}}(v)) \quad \text{from (3.5)} \\ \overline{T_{\mu_{FQ}}(u, v)} &= 0 \end{aligned}$$

Similarly, $\overline{C_{\mu_{FQ}}(u, v)} = 0$, $\overline{I_{\mu_{FQ}}(u, v)} = 0$, $\overline{F_{\mu_{FQ}}(u, v)} = 0 \ \forall (u, v) \in \mu_{FQ}$ and

$$\begin{aligned} \overline{T_{\mu_{FQ}}(u, v)} &= T_{\sigma_{FQ}}(u) \wedge T_{\sigma_{FQ}}(v) \\ \overline{C_{\mu_{FQ}}(u, v)} &= C_{\sigma_{FQ}}(u) \wedge C_{\sigma_{FQ}}(v) \\ \overline{I_{\mu_{FQ}}(u, v)} &= I_{\sigma_{FQ}}(u) \vee I_{\sigma_{FQ}}(v) \\ \overline{F_{\mu_{FQ}}(u, v)} &= F_{\sigma_{FQ}}(u) \vee F_{\sigma_{FQ}}(v) \ \ \forall \ (u, v) \in \mu_{FQ} \end{aligned}$$

$\Rightarrow\ \overline{G_{FQ}}$ is a strong FQNFG.
From Corollary 1, $\overline{G_{FQ}}$ is balanced. □

Theorem 3.12. *Let* $G_{FQ} = (\sigma_{FQ}, \mu_{FQ})$ *be a FQNFG and* $\overline{G_{FQ}} = (\overline{\sigma_{FQ}}, \overline{\mu_{FQ}})$ *be its complement, then* $\overline{\overline{G_{FQ}}} = G_{FQ}$.

Proof. Let $\overline{G_{FQ}}$, By the definition of complement,

$$\overline{T_{\mu_{FQ}}(u,v)} = T_{\sigma_{FQ}}(u) \wedge T_{\sigma_{FQ}}(v) - T_{\mu_{FQ}}(u,v) \tag{3.7}$$

Applying complement again,

$$\overline{\overline{T_{\mu_{FQ}}(u,v)}} = T_{\sigma_{FQ}}(u) \wedge T_{\sigma_{FQ}}(v) - \overline{T_{\mu_{FQ}}(u,v)} \tag{3.8}$$

Substituting (3.7) in (3.8),
$\overline{\overline{T_{\mu_{FQ}}(u,v)}} = T_{\sigma_{FQ}}(u) \wedge T_{\sigma_{FQ}}(v) - [T_{\sigma_{FQ}}(u) \wedge T_{\sigma_{FQ}}(v) - T_{\mu_{FQ}}(u,v)]$
$\overline{\overline{T_{\mu_{FQ}}(u,v)}} = T_{\mu_{FQ}}(u,v)$
Similarly,
$\overline{\overline{C_{\mu_{FQ}}(u,v)}} = C_{\mu_{FQ}}(u,v)$
$\overline{\overline{I_{\mu_{FQ}}(u,v)}} = I_{\mu_{FQ}}(u,v)$
$\overline{\overline{F_{\mu_{FQ}}(u,v)}} = F_{\mu_{FQ}}(u,v)$
Hence $\overline{\overline{G_{FQ}}} = G_{FQ}$. □

4. Application

4.1. Modeling Student Performance with Balanced Fermatean Quadripartitioned Neutrosophic Fuzzy Graphs

Traditional student performance assessments often fail to capture partial understanding, inconsistent performance, and unobserved contributions. To address these limitations, we employ Balanced Fermatean Quadripartitioned Neutrosophic Fuzzy Graphs to model student performance and evaluate educational equity. The technique has been illustrated in the steps following:
Step 1: Define Vertices and Edges
Vertices (Students): Each student is represented as a vertex with a quadripartitioned neutrosophic fuzzy set. Edges (Interactions): Each edge represents academic interactions between students.
Step 2: Compute Densities
The density of the FQNFG is computed using the density formula defined in Definition 3.1. The density values for the full graph G_{FQ} and its subgraphs H_{FQ} are obtained by substituting the corresponding vertex and edge values into this formula.
Step 3: Check Balance Condition
The FQNFG is balanced if $D(H_{FQ}) \leqslant D(G_{FQ})$ for all subgraphs H_{FQ}.

4.2. Illustrative Example

To demonstrate the proposed model, we consider three students (Student A, Student B, Student C) and their academic interactions. In the graph, each vertex represents a student, and each edge represents an interaction between two students. Each student is characterized by four membership values in the interval [0,1]: truth, contradiction, ignorance, and falsity. These values are derived from performance indicators such as quiz scores, assignments, class participation, and self-assessment. Truth represents the level of demonstrated understanding, contradiction reflects inconsistency in performance, ignorance denotes unobserved or incomplete information, and falsity indicates misconceptions or lack of understanding. Similarly, each edge is assigned four membership values to describe the quality of interaction between students, capturing its beneficial, inconsistent, unobserved, or detrimental effects on learning. The vertex and edge values are presented in Figure 4. This example illustrates how the model identifies differences in individual performance and interaction quality. The aim is to examine whether the learning network is balanced, where balance implies that academic support, opportunities, and outcomes are fairly distributed to promote effective learning for all students.

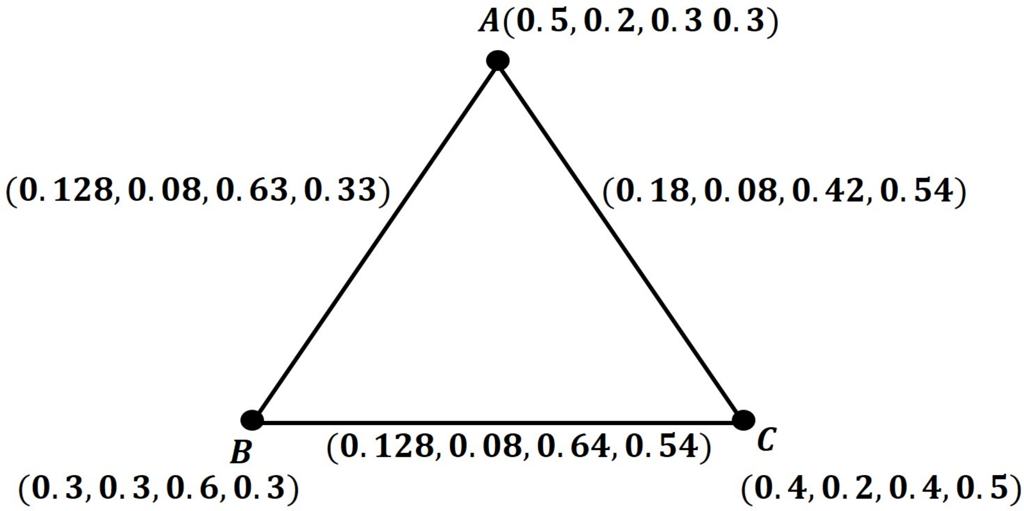

Figure 4: A FQNFG for 3 students

Step 1: From Figure 4, consider the graph with three vertices V=(A,B,C) and three edges E=(AB, AC, BC). The vertex membership values are: A = (0.5,0.2,0.3,0.3), B = (0.3,0.3,0.6,0.3), C = (0.4,0.2,0.4,0.5). The edge membership values are: AB = (0.128,0.08,0.63,0.33), AC = (0.18,0.08,0.42,0.54), BC = (0.128,0.08,0.64,0.54).
Step 2: The densities of the graph are as follow:
$D(G_{FQ}) = (D_T(G_{FQ}), D_C(G_{FQ}), D_I(G_{FQ}), D_F(G_{FQ})) = (0.9, 0.8, 2.1, 2.2)$
The density of the subgraph
$D(H_{FQ}) = (D_T(H_{FQ}), D_C(H_{FQ}), D_I(H_{FQ}), (D_F(H_{FQ}) = (0.9, 0.8, 2.1, 2.2)$

Step 3: $D(H_{FQ}) \leqslant D(G_{FQ})$ for all subgraphs H_{FQ} of G_{FQ}.
Thus, G_{FQ} is a balanced *FQNFG*. This result suggests that variations in student performance and interactions are systematically distributed within the learning network. A balanced structure supports fair assessment and enables precise, data-driven academic interventions. Consequently, the model provides a reliable analytical approach for monitoring student progress and identifying areas requiring support. The method can be extended to n students, offering a scalable tool for equitable and comprehensive performance evaluation in educational systems.

5. Conclusion

FQNFG provides a powerful, flexible, and accurate framework for modeling complex, uncertain, and inconsistent real-world information, surpassing traditional and even basic fuzzy methods. Our work introduces the novel concept of a balanced *FQNFG*, further enhancing this analytical power. The achievement of a balanced *FQNFG* within an educational context, mathematically verified by its density conditions, represents a quantifiable benchmark for excellence in learning facilitation. It directly signifies a more equitable and effective learning ecosystem where resources and pedagogical strategies consistently impact student outcomes. This state minimizes disparities and areas of unknown performance, ensuring all students receive the necessary support and opportunities for success.

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