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An intuitionistic fuzzy EOQ model with advance payment, deterioration, and preservation technology under inflation

Viyasar Mouly[a,*], A. Thangam[a]

[a]Department of Mathematics, Community College (Lawspet), Pondicherry University, Kalapet-605 008, Pondicherry, India.

Abstract

This study develops an economic order quantity model that accounts for advance payment schemes, deterioration, and preservation technology in an intuitionistic fuzzy environment to capture real-world inventory uncertainty. The proposed model further integrates demand, which depends on the selling price, advertising frequency, and stock levels, along with time-varying holding costs, allowance for partial backorders, and the impact of inflation. First, the crisp model is formulated, and then the triangular intuitionistic fuzzy number is applied to this model. Defuzzification of the fuzzy model is performed using two distinct approaches: the signed distance method and the graded mean integration representation method. A solution procedure is developed to determine optimal solutions, and an algorithm is created by combining the results of all models derived from the analytical study. The numerical example is illustrated for both the crisp and fuzzy models, and 2D and 3D graphs are plotted using MATLAB (R2025a).A sensitivity analysis examines how changes in inventory parameters affect the optimal solution, offering managers valuable insights for decision-making under uncertainty.

Keywords: Advance payment, Triangle intuitionistic fuzzy number, signed distance method, Preservation technology, inflation.

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1. Introduction

Inventory control is a fundamental aspect of the supply chain process that focuses on maintaining an optimal level of stock to meet customer demand. It facilitates the right quantity of goods available at the right time and place, thereby avoiding shortages, overstocking, and unnecessary expense. Effective inventory control is essential for improving customer satisfaction, enhancing operational efficiency, and increasing profitability. Advance Payment(AP) on orders is also common in business and generally refers to paying suppliers in advance for goods delivered. These types of payments typically include an incentive to encourage retailers to purchase larger quantities [37].

Deterioration is an important factor in inventory control, especially for products such as food items, medicines, chemicals, and electronic goods that lose quantity, quality, or value over time. Deteriorating items create additional cost through spoilage, damage, obsolescence, and reduced sales value [22]. To

*Corresponding author

Email addresses: viyasarmouly@pondiuni.ac.in (Viyasar Mouly), thangam.pucc@pondiuni.ac.in (A. Thangam)

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mitigate deterioration, the firm used Preservation Technology (PT), such as refrigeration, temperature-controlled storage, improved packaging, and advanced handling systems. PT helps extend product life, maintain quality, and slow deterioration.

Traditional inventory models assume demand is constant. But demand can be affected by many different factors, such as advertising, inventory levels, price, and seasonality. A lower sales price generally increases customer attraction, while a higher price may reduce purchasing interest. Similarly, effective advertising creates customer awareness, improves product visibility, and stimulates growth. Stock level also significantly affects customer behavior. When a large quantity of goods is displayed, customers often perceive the products as popular and readily available, which can increase sales [33].On the other hand, low stock levels may discourage purchases due to fear of shortage or reduced product appeal.

Inflation is an important economic factor that significantly influences inventory control and business decision- making. It indicates a continuous rise in the overall price level, gradually diminishing money's purchasing power to buy the same amount of goods and services. In the inventory model, inflation affects various cost components [27]. Therefore, incorporating inflation into inventory models helps organizations make effective replenishment and financial decisions under changing economic conditions.

Shortages are an important aspect of an inventory model, occurring when available stock is insufficient to meet customer demand. They may arise from delayed replenishment, unexpected demand increases, or poor inventory planning. Shortages can lead to lost sales, customer dissatisfaction, and reduced goodwill [10].In classical inventory models, holding cost is often assumed to be fixed. But, in real business environments, holding costs may vary over time due to inflation, rising warehouse rents, increased insurance costs, maintenance expenses, and changes in the opportunity cost of capital [15].

In practical inventory models, many cost parameters are often uncertain due to market fluctuations, estimation errors, and incomplete information. The classical crisp model cannot effectively represent such vagueness. To overcome this limitation, Triangular Intuitionistic Fuzzy Numbers (TIFN) are used to describe the uncertain cost parameters. A TIFN extends a triangular fuzzy number by including both membership and non-membership degrees, offering a more comprehensive way to manage hesitation and uncertainty in decision-making.[5].In this study, the Signed Distance Method (SDM) and Graded Mean Integration Representation (GMIR) were employed to defuzzify the TIFN. These methods help to compare results and analyze the effect of uncertainty on the present study.

1.1. Literature Review

Recent research has examined how demand, influenced by pricing, advertising, and inventory levels, impacts inventory and production strategies.[21] investigated the effects of permissible delay payments with advertising, time, and selling price-driven demand. [24] formulated a defective production system with both price and advertising efforts influencing demand. [17] adopted by optimizing imperfect production systems where demand is contingent upon advertising and price. [28] analyzed optimizing a system with three production rates, considering demand influenced by price, stock levels, and advertising. [33] introduced a model for imperfect production systems that optimizes demand in relation to selling price, advertisement frequency, and eco-friendly product features. [19] developed a model featuring ramp-type demand that depends on stock levels.[20] addressed trapezoidal type demand with decay items, [14] analyzed stock-based demand in the context of uncertainty parameter.[29] formulated time driven demand model under compound interest.

[43] studied time-driven holding costs and how they can be applied to make better decisions on price, inventory, and frequency of advertising. [30] proposes a partial backlogging model that considers time-

driven holding costs. [12] develops a pricing model that optimizes over time-varying holding costs. [26] presents an optimized inventory control model for the supply chain that accounts for time-varying holding costs. [15] analyzed an inventory model by integrating time-driven holding costs and employing Maclaurin series approximations.[32] constructed an Economic Order Quantity (EOQ) model with variable holding costs for seasonal products. [41] formulated a sustainable EOQ model with time-driven holding cost. [36] analyzed linear variable holding cost with inflation.

[50] developed a supply chain model that includes suppliers, retailers, and consumers, accounting for both advance and deferred payments.[37] explores inventory optimization using a three-parameter Weibull deteriorating model, which incorporates strategies of prepayment on goods to manage deterioration effectively. [25] proposes a sustainable two-warehouse inventory model that integrates advanced payment policies with trade credit strategies to balance cost and efficiency. [39] and [40] focus on cost minimization in replenishment decisions by AP policy, optimizing inventory through strategic payment timing.[7] proposes high-tech cash credit payment systems intended to expedite financial decision-making in inventory systems.[42] proposed a multi-item inventory model with AP. [3] integrated a two-warehouse model with cash and AP.

[38] discusses an inventory system that uses discrete scheduling to handle backlogging shortages and balance work efficiency.[48] develops a model that examines the costs of backlogging and lost sales during shortages, balancing customer retention and financial impacts. [49] proposes an EOQ model for imperfect items that incorporates shortages to optimize inventory decisions when product quality varies. [2] introduces a multi-item EOQ model that examines how reliability impacts inventory systems with shortages and partial back-ordering.[34] develops a model for perishable goods that integrates AP with PT to balance costs and product freshness. [23] proposes an optimal inventory policy for decaying items that uses the PT approach to minimize losses and optimize stock levels. [22] examines deteriorating items with expiration dates, incorporating PT. [27] formulated an inventory model with an impact of inflation and a decaying item.

[11] introduces a fuzzy inventory model for stochastic inflation, offering a flexible approach to uncertain economic environments. [51] develops an EOQ for decaying items that accounts for inflation to optimize stock decisions while managing perishability. [4] explores a cloudy fuzzy inventory model under inflation. [46] examines an EOQ model with delayed payments, addressing inflations impact on cost efficiency. [10] investigates an inventory model tailored to post-COVID-19 supply chain challenges, incorporating inflation to ensure resilience in disrupted markets. [6] proposes an integrated supply chain model that balances inflation and carbon-emission constraints, thereby promoting sustainable inventory practices. [18] developed intuitionistic complex fuzzy sets in decision support systems. [47] explores an IFN inventory model integrated with AP. [13] introduces fuzzy numbers to optimize a multi-item inventory model, thereby enhancing decision-making under uncertainty.[45] constructed IFN sets with multi-criteria group decision making .[44] investigates an IFN inventory model incorporating waste disposal, balancing cost efficiency with environmental considerations. [31] examines sustainable two-warehouse inventory models under different fuzzy systems. [16] addresses linguistic IFN sets in multi-attribute group decision-making problems.[5] constructed a sustainable EOQ inventory model based on a pythagorean fuzzy environment.[1] analyzed trapezoidal intuitionistic fuzzy numbers with multi-criteria group decision making.[52] developed the inventory model with various fuzzy numbers and AP. Table 1 provides an overview of the significant literature relevant to this study.

Table 1. Review of some major contributions related to the proposed study

Study	**Demand depends upon**	**Time driven Holding cost**	**Advance payment**	**Shortages**	**Deterioration**	**Preservation Technology**	**Inflation**	**Uncertain environment**
[21]	Price,Stock Advertisement				✓			
[22]	Constant	✓	✓		✓	✓		
[30]	Price,Stock	✓		✓				
[51]	Price Advertisement			✓	✓		✓	
[4]	Time			✓	✓		✓	✓
[46]	Price			✓	✓		✓	
[47]	Quadratic time		✓		✓			✓
[52]	Price	✓	✓					✓
[34]	Price,Stock		✓	✓	✓	✓		
Present study	Price,Stock Advertisement	✓	✓	✓	✓	✓	✓	✓

1.2. Research gap and contribution of this model

Prior studies have investigated inventory systems that integrate fuzzy parameters and a prepayment scheme. However, there is still a noticeable gap in addressing how these models can account for demand influenced by advertising, price, and stock levels in an IFN environment with deteriorating items and advance payment. This research addresses this gap by combining financial factors, such as advance payments, with operational factors, such as product decay and shortages.

Significant contributions:

1. This paper presents a fuzzy EOQ model using TIFN to address uncertainty in key cost factors.
2. Demand is influenced by advertising, price, and stock levels.
3. The time-driven holding cost is considered. The model is taken under an inflationary environment.
4. The model considers an advance payment policy in which payment is made before receiving the goods.
5. The model incorporates PT investments to reduce inventory losses and enhance the reliability of perishable products.
6. The SDM and GMIR methods are used for defuzzification of fuzzy numbers.

2. Notations and Assumptions

2.1. Notations

Table 2. Notations used in the model

Symbol	**Description**
Q_1	Maximum inventory level (units)
P	Purchasing cost (cost/unit)
π	Fraction of demand backlogged (constant)
a	Base demand coefficient (units/time)

Symbol	Description
r	Inflation rate (%/year)
D	Demand rate (units/time)
b	Price elasticity parameter (units/(cost/time))
p	Selling price(cost/unit)
A	Advertising intensity (constant)
Q	Quantity ordered (units)
χ	Advertising elasticity factor (constant)
K	Shortage cost(cost/unit/time)
ψ	Sensitivity coefficient for inventory level (1/unit)
Q_2	Maximum shortage level (units)
η	Annual interest rate on stock investment (1/time)
L	Duration of the advance payment period (time)
t_1	Positive inventory stock period (time)
T	Total replenishment cycle length (time)
S	Ordering cost (cost/order)
$I_1(t)$	Inventory level during positive stock period at time t (units)
l	Lost-sales cost (cost/unit)
$I_2(t)$	Inventory level during shortage period at time t (units)
C	Deterioration cost (cost/unit)
TC	Total cost (cost/time)

2.2. *Assumptions*

1. The buyer obtains a price reduction from the supplier by paying the total amount in advance before receiving the items.
2. The demand rate is driven by advertising, stock levels, and the selling price. It is defined as:

$$D(A, p, I(t)) = \begin{cases} A^{\chi}\left(a - bp + \psi I(t)\right), & \text{when } I(t) > 0, \\ A^{\chi}\left(a - bp\right), & \text{when } I(t) \leqslant 0. \end{cases}$$

3. The holding cost is time-dependent $h(t) = \alpha + \sigma t$, where α is the constant component and σ is the time-dependent variable component.
4. The model allows shortages with partial backlogging.
5. The unit selling price differs from the unit procurement cost.
6. The model is developed under an inflationary environment, where cost parameters vary over time.
7. The retailer pays a fixed PT cost to minimize inventory item deterioration.

3. Mathematical model

3.1. *Deterministic EOQ model*

The buyer orders Q units and settles the payment during the lead time L. In return, the supplier grants a discount rate δ on the purchase price P. As the payment is completed before delivery, the buyer incurs an advance payment interest charge at a rate η. At the start of each replenishment cycle, the inventory level is at its highest. Over the period $[0, t_1]$, the stock is gradually depleted because of customer demand and item deterioration. At $t = t_1$, the inventory level becomes zero. Thereafter, shortages arise over the interval $[t_1, T]$. The replenishment cycle is shown in Fig. 1.

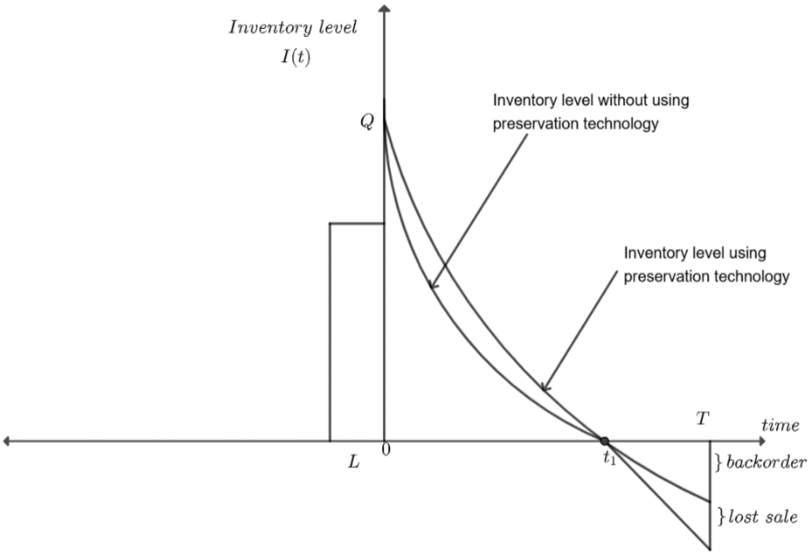

Figure 1: Inventory level

Accordingly, the inventory levels follow the differential equations below.

$$\frac{dI_1(t)}{dt} + \theta I_1(t) = -A^X\left[(a-bp) + \psi I_1(t)\right], \qquad 0 < t \leqslant t_1 \tag{3.1}$$

$$\frac{dI_2(t)}{dt} = -\pi A^X(a-bp), \qquad t_1 < t \leqslant T \tag{3.2}$$

The corresponding boundary conditions are

$$I_1(t_1) = 0, \quad I_1(0) = Q_1, \quad \text{and} \quad I_2(T) = -Q_2.$$

By solving the earlier differential equations, the obtained results are as follows for the inventory levels:

$$I_1(t) = \frac{A^X(a-bp)}{\theta + A^X\psi}\left[e^{(\theta+A^X\psi)(t_1-t)} - 1\right]. \tag{3.3}$$

$$I_2(t) = \pi A^X(a-bp)(t_1-t). \tag{3.4}$$

Maximum inventory level:
The maximum inventory level represents the highest on-hand stock available at the beginning of the replenishment cycle. It is obtained by substituting $t = 0$ into $I_1(t)$. Therefore,

$$Q_1 = I_1(0) = \frac{A^X(a-bp)}{\theta + A^X\psi}\left[e^{(\theta+A^X\psi)t_1} - 1\right]. \tag{3.5}$$

Maximum shortage:
The maximum shortage occurs at the end of the cycle, when $t = T$, at $I_2(t)$.Hence,

$$Q_2 = -I_2(T) = -\pi A^X(a-bp)(t_1-T). \tag{3.6}$$

Total order quantity:
The total order quantity in each cycle must satisfy both the positive inventory demand and the accumulated shortage quantity. Thus, the replenishment cycle consists of the amount required to satisfy positive inventory and the shortage quantity to be filled in the next cycle. Hence,

$$Q = Q_1 + Q_2$$

Hence,

$$Q = \frac{A^{x}(a-bp)}{\theta + A^{x}\psi}\left[e^{(\theta+A^{x}\psi)t_1} - 1\right] - \pi A^{x}(a-bp)(t_1 - T). \tag{3.7}$$

The total cost includes the following cost components:

i)Ordering Cost (OC):

Ordering costs are a recurring expense incurred each time an order is submitted. Since it is incurred in every cycle,

$$OC = S \tag{3.8}$$

ii)Holding Cost (HC):

Holding cost represents the cost of storing items during the positive inventory period $[0, t_1]$. Since the holding cost varies with time and inflation, it is given by

$$\begin{aligned} HC &= \int_0^{t_1} h(t)\, I_1(t)\, e^{-rt}\, dt \\ &= \int_0^{t_1} (\alpha + \sigma t)\, \frac{A^{x}(a-bp)}{\theta + A^{x}\psi}\left(e^{(\theta+A^{x}\psi)(t_1-t)} - 1\right) e^{-rt}\, dt \end{aligned}$$

$$HC = \frac{A^{x}(a-bp)}{\theta + A^{x}\psi}\left[\begin{aligned} &\frac{\alpha}{(\theta + A^{x}\psi + r)^2}\left(e^{(\theta+A^{x}\psi)t_1} - e^{-rt_1}\right) + \frac{\alpha}{r}\left(1 - e^{-rt_1}\right) \\ &+ \sigma\left(\frac{e^{(\theta+A^{x}\psi)t_1} - e^{-rt_1}}{(\theta + A^{x}\psi + r)^2} - \frac{t_1 e^{-rt_1}}{\theta + A^{x}\psi + r}\right) \\ &- \sigma\left(\frac{1 - e^{-rt_1}}{r^2} - t_1 e^{-rt_1}\right) \end{aligned}\right] \tag{3.9}$$

iii)Deteriorating Cost (DC):

Deterioration cost is incurred when stored items spoil or decay during a positive inventory period. Since this cost is influenced by inflation, it is represented by the DC given by:

$$DC = C\theta \int_0^{t_1} I_1(t) e^{-rt}\, dt$$

$$DC = \frac{C\theta A^{x}(a-bp)}{\theta + A^{x}\psi}\left[\frac{e^{(\theta+A^{x}\psi)t_1} - e^{-rt_1}}{\theta + A^{x}\psi + r} + \frac{e^{-rt_1} - 1}{r}\right] \tag{3.10}$$

iv)Shortage Cost (SC):

Shortage cost is incurred for the portion of demand that is backlogged during the shortage period $[t_1, T]$. Considering inflation, the SC is

$$\begin{aligned} SC &= -K \int_{t_1}^{T} I_2(t) e^{-rt}\, dt \\ &= -K\pi \int_{t_1}^{T} A^{x}(a-bp)(t_1 - t) e^{-rt}\, dt \end{aligned}$$

$$SC = -\frac{K\pi A^{x}(a-bp)}{r^2}\left[e^{-rT}(-rt_1 + rT - 1) - e^{-rt_1}\right] \tag{3.11}$$

v)Last Sale Cost (LSC):

A fraction $(1 - \pi)$ of demand during a shortage is permanently lost. Since inflation is considered, the

LSC is

$$
\begin{aligned}
LSC &= l\int_{t_1}^{T}(1-\pi)D(A,p)e^{-rt}\,dt\\
&= l\int_{t_1}^{T}(1-\pi)A^{x}(a-bp)e^{-rt}\,dt\\
LSC &= l(1-\pi)A^{x}(a-bp)\left[\frac{e^{-rT}-e^{-rt_1}}{r}\right]
\end{aligned}
\tag{3.12}
$$

vi)Interest Cost during Prepayment (IP_A):

Since the buyer pays in advance before receiving stock, an interest charge is incurred during the lead time L. Hence,

$$
\begin{aligned}
IP_A &= QP\delta\eta L\\
IP_A &= P\delta\eta LA^{x}(a-bp)\left[\frac{e^{(\theta+A^{x}\psi)}-1}{\theta+A^{x}\psi}-\pi(t_1-T)\right]
\end{aligned}
\tag{3.13}
$$

vii)Interest Cost for Items in Stock (IP_S):

Capital invested in stored inventory also incurs interest during the positive stock period. Therefore,

$$
\begin{aligned}
IP_S &= \eta P\delta\int_{0}^{t_1}I_1(t)dt\\
IP_S &= \frac{P\eta\delta t_1A^{x}(a-bp)}{\theta+A^{x}\psi}\left[\frac{e^{(\theta+A^{x}\psi)}-1}{\theta+A^{x}\psi}-t_1\right]
\end{aligned}
\tag{3.14}
$$

viii)Preservation Technology Cost (PTC):

To reduce deterioration, the retailer invests in PT. Let ω denote the preservation cost. Then,

$$PTC = \omega T \tag{3.15}$$

Total Cost (TC):

By summing all cost components, the total cost per cycle is

$$TC(t_1,T) = \frac{1}{T}\left[OC+HC+DC+IP_A+IP_S+SC+LSC+PTC\right] \tag{3.16}$$

$$
TC(t_1,T)=\frac{1}{T}\left[
\begin{aligned}
&S+\frac{A^{x}(a-bp)}{\theta+A^{x}\psi}\left(\frac{\alpha}{(\theta+A^{x}\psi+r)^2}\left(e^{(\theta+A^{x}\psi)t_1}-e^{-rt_1}\right)+\frac{\alpha}{r}(1-e^{-rt_1})\right.\\
&\left.+\sigma\left(\frac{e^{(\theta+A^{x}\psi)t_1}-e^{-rt_1}}{(\theta+A^{x}\psi+r)^2}-\frac{t_1e^{-rt_1}}{\theta+A^{x}\psi+r}\right)-\sigma\left(\frac{1-e^{-rt_1}}{r^2}-t_1e^{-rt_1}\right)\right)\\
&+\frac{C\theta A^{x}(a-bp)}{\theta+A^{x}\psi}\left(\frac{e^{(\theta+A^{x}\psi)t_1}-e^{-rt_1}}{\theta+A^{x}\psi+r}+\frac{e^{-rt_1}-1}{r}\right)\\
&+P\delta\eta LA^{x}(a-bp)\left(\frac{e^{(\theta+A^{x}\psi)t_1}-1}{\theta+A^{x}\psi}-\pi(t_1-T)\right)\\
&+\frac{P\eta\delta t_1A^{x}(a-bp)}{\theta+A^{x}\psi}\left(\frac{e^{(\theta+A^{x}\psi)t_1}-1}{\theta+A^{x}\psi}-t_1\right)\\
&-\frac{K\pi A^{x}(a-bp)}{r^2}\left(e^{-rT}(-rt_1+rT-1)-e^{-rt_1}\right)\\
&+l(1-\pi)A^{x}(a-bp)\frac{e^{-rT}-e^{-rt_1}}{r}+\omega T
\end{aligned}
\right]
\tag{3.17}
$$

Using Taylor expansions for small θ *and* r*:.* To reduce the mathematical complexity of exponential terms, the Taylor series expansion is used under the assumption that the deterioration rate θ and inflation rate r are small. Retaining terms up to the second order, we have:

$$\begin{aligned} e^{(\theta+A^x\psi)t_1} &\approx 1+(\theta+A^x\psi)t_1+\frac{(\theta+A^x\psi)^2t_1^2}{2} \\ e^{-rt_1} &\approx 1-rt_1+\frac{r^2t_1^2}{2} \\ 1-e^{-rt_1} &\approx rt_1-\frac{r^2t_1^2}{2} \\ e^{(\theta+A^x\psi)t_1}-1 &\approx (\theta+A^x\psi)t_1+\frac{(\theta+A^x\psi)^2t_1^2}{2} \end{aligned}$$

Using these approximations and neglecting higher-order terms, the simplified total cost function is obtained as $TC(t_1,T)$

$$TC(t_1,T)=\frac{1}{T}\left[\begin{aligned} &S+\frac{A^x(a-bp)}{\theta+A^x\psi}\left(2\alpha t_1+\alpha\left(\theta+A^x\psi-\frac{3r}{2}\right)t_1^2-\frac{\sigma t_1^2}{2}+\frac{\sigma r(\theta+A^x\psi)t_1^3}{2(\theta+A^x\psi+r)}+\frac{\sigma}{2}\right) \\ &+\frac{C\theta A^x(a-bp)t_1^2}{2}+P\delta\eta LA^x(a-bp)\left(\frac{(\theta+A^x\psi)t_1^2}{2}+t_1-\pi(t_1-T)\right) \\ &+\frac{P\delta\eta t_1^2A^x(a-bp)}{2} \\ &+\pi KA^x(a-bp)\left(Tt_1+\frac{2}{r^2}+\frac{3T^2}{2}+\frac{rT^2t_1}{2}+\frac{t_1^2}{2}-\frac{2T}{r}-\frac{rT^3}{2}\right) \\ &+l(1-\pi)(a-bp)A^x\left(t_1-T-\frac{rt_1^2}{2}+\frac{rT^2}{2}\right)+\omega T \end{aligned}\right] \tag{3.18}$$

The crisp optimization problem is given by

$$\begin{aligned} \text{Minimize} \quad & TC(t_1,T) \\ \text{Subject to} \quad & 0<t_1<T, \\ & T>0. \end{aligned} \tag{3.19}$$

(Detailed optimization procedure and derivations are available in Appendix I.)

3.2. *Intuitionistic Fuzzy EOQ model*

This section extends the deterministic model to an IFN to manage uncertainty. The uncertain parameters are represented using TIFN, which are denoted as: $(\tilde{S},\tilde{\alpha},\tilde{\sigma},\tilde{C},\tilde{K},\tilde{l},\tilde{P})$
where

$$\tilde{S}=(S_1,S_2,S_3;S_1',S_2,S_3'),\quad \tilde{\alpha}=(\alpha_1,\alpha_2,\alpha_3;\alpha_1',\alpha_2,\alpha_3'),\quad \tilde{\sigma}=(\sigma_1,\sigma_2,\sigma_3;\sigma_1',\sigma_2,\sigma_3')$$
$$\tilde{C}=(C_1,C_2,C_3;C_1',C_2,C_3'),\quad \tilde{K}=(K_1,K_2,K_3;K_1',K_2,K_3'),\quad \tilde{l}=(l_1,l_2,l_3;l_1',l_2,l_3')$$
$$\tilde{P}=(P_1,P_2,P_3;P_1',P_2,P_3')$$

The fuzzy $\widetilde{TC}(t_1, T)$ is formulated by replacing the crisp cost components in Equation (18) with their corresponding fuzzy model:

$$\widetilde{TC}(t_1,T)=\frac{1}{T}\left[\begin{array}{l}\tilde{S}+\frac{A^{x}(a-bp)}{\theta+A^{x}\psi}\left(2\tilde{\alpha}t_1+\tilde{\alpha}\left(\theta+A^{x}\psi-\frac{3r}{2}\right)t_1^2-\frac{\tilde{\sigma}t_1^2}{2}+\frac{\tilde{\sigma}t_1^3 r(\theta+A^{x}\psi)}{2(\theta+A^{x}\psi+r)}+\frac{\tilde{\sigma}}{2}\right)\\ +\frac{\tilde{C}\theta A^{x}(a-bp)t_1^2}{2}+\tilde{P}\delta\eta LA^{x}(a-bp)\left(\frac{(\theta+A^{x}\psi)t_1^2}{2}+t_1-\pi(t_1-T)\right)\\ +\frac{\tilde{P}\delta\eta A^{x}(a-bp)}{2}t_1^2\\ +\pi\tilde{K}A^{x}(a-bp)\left(Tt_1+\frac{2}{r^2}+\frac{3T^2}{2}+\frac{rT^2t_1}{2}+\frac{t_1^2}{2}-\frac{2T}{r}-\frac{rT^3}{2}\right)\\ +\tilde{l}(1-\pi)(a-bp)A^{x}\left(t_1-T-\frac{rt_1^2}{2}+\frac{rT^2}{2}\right)+\omega T\end{array}\right] \tag{3.20}$$

3.2.1. Signed Distance Method (SDM)

Using the SDM, for each Number TIFN

$$\widetilde{\phi}=(\phi_1,\phi_2,\phi_3;\phi_1',\phi_2,\phi_3')$$

where ϕ_2 represents the common modal value, ϕ_1, ϕ_3 are the lower and upper bounds of the membership function, and ϕ_1', ϕ_3' are the lower and upper bounds of the non-membership function. TIFN is defuzzified using the following formula:

$$SDM(\widetilde{\phi})=\frac{1}{8}\left(\phi_1+4\phi_2+\phi_3+\phi_1'+\phi_3'\right) \tag{3.21}$$

The defuzzified total cost using SDM is:

$$\Delta(\widetilde{TC}_{SDM}(t_1,T))=\frac{1}{T}\left[\begin{array}{l}\frac{S_1+2S_2+S_3+S_1'+S_3'}{8}+\frac{A^{x}(a-bp)}{\theta+A^{x}\psi}\left(2\cdot\frac{\alpha_1+2\alpha_2+\alpha_3+\alpha_1'+\alpha_3'}{8}t_1\right.\\ +\frac{\alpha_1+2\alpha_2+\alpha_3+\alpha_1'+\alpha_3'}{8}\left(\theta+A^{x}\psi-\frac{3r}{2}\right)t_1^2\\ \left.+\frac{\sigma_1+2\sigma_2+\sigma_3+\sigma_1'+\sigma_3'}{8}\left(-\frac{t_1^2}{2}+\frac{t_1^3 r(\theta+A^{x}\psi)}{2(\theta+A^{x}\psi+r)}+\frac{1}{2}\right)\right)\\ +\frac{(C_1+2C_2+C_3+C_1'+C_3')\,\theta A^{x}(a-bp)t_1^2}{16}\\ +\frac{P_1+2P_2+P_3+P_1'+P_3'}{8}\delta\eta LA^{x}(a-bp)\left(\frac{(\theta+A^{x}\psi)t_1^2}{2}+t_1-\pi(t_1-T)\right)\\ +\frac{P_1+2P_2+P_3+P_1'+P_3'}{16}\delta\eta t_1^2A^{x}(a-bp)\\ +\pi\frac{K_1+2K_2+K_3+K_1'+K_3'}{8}A^{x}(a-bp)\\ \left(Tt_1+\frac{2}{r^2}+\frac{3T^2}{2}+\frac{rT^2t_1}{2}+\frac{t_1^2}{2}-\frac{2T}{r}-\frac{rT^3}{2}\right)\\ +\frac{l_1+2l_2+l_3+l_1'+l_3'}{8}(1-\pi)(a-bp)A^{x}\left(t_1-T-\frac{rt_1^2}{2}+\frac{rT^2}{2}\right)+\omega T\end{array}\right] \tag{3.22}$$

The intuitionistic fuzzy optimization problem is formulated as follows:

$$\begin{aligned} \text{Minimize} \quad & \Delta\left(\widetilde{TC}_{SDM}(t_1, T)\right) \\ \text{Subject to} \quad & 0 < t_1 < T, \\ & T > 0. \end{aligned} \tag{3.23}$$

(Detailed optimization procedure and derivation are available in Appendix III.)

3.2.2. Graded Mean Integration Method (GMIR)

The GMIR of a TIFN $\widetilde{\phi}$ is defuzzified as:

$$\text{GMIR}(\widetilde{\phi}) = \frac{1}{12}\left(\phi_1 + 4\phi_2 + \phi_3 + \phi_1' + \phi_3'\right) \tag{3.24}$$

The defuzzified total cost using GMIR is:

$$\Delta\left(\widetilde{TC}_{GMIR}(t_1, T)\right) = \frac{1}{T}\left[\begin{aligned} & \frac{S_1 + 2S_2 + S_3 + S_1' + S_3'}{12} + \frac{A^x(a - bp)}{\theta + A^x\psi}\left(2\frac{\alpha_1 + 2\alpha_2 + \alpha_3 + \alpha_1' + \alpha_3'}{12}t_1 \right. \\ & \quad + \frac{\alpha_1 + 2\alpha_2 + \alpha_3 + \alpha_1' + \alpha_3'}{12}\left(\theta + A^x\psi - \frac{3r}{2}\right)t_1^2 \\ & \quad \left. + \frac{\sigma_1 + 2\sigma_2 + \sigma_3 + \sigma_1' + \sigma_3'}{12}\left(-\frac{t_1^2}{2} + \frac{t_1^3 r(\theta + A^x\psi)}{2(\theta + A^x\psi + r)} + \frac{1}{2}\right)\right) \\ & \quad + \frac{(C_1 + 2C_2 + C_3 + C_1' + C_3')\,\theta A^x(a - bp)t_1^2}{2} \\ & \quad + \frac{(P_1 + 2P_2 + P_3 + P_1' + P_3')\,\delta\eta L A^x(a - bp)}{12} \\ & \left(\frac{(\theta + A^x\psi)t_1^2}{2} + t_1 - \pi(t_1 - T)\right) \\ & \quad + \frac{(P_1 + 2P_2 + P_3 + P_1' + O_3')\,\delta\eta t_1^2 A^x(a - bp)}{24} \\ & \quad + \pi\frac{K_1 + 2K_2 + K_3 + K_1' + K_3'}{12}A^x(a - bp) \\ & \left(Tt_1 + \frac{2}{r^2} + \frac{3T^2}{2} + \frac{rT^2t_1}{2} + \frac{t_1^2}{2} - \frac{2T}{r} - \frac{rT^3}{2}\right) \\ & \quad + \frac{l_1 + 2l_2 + l_3 + l_1' + l_3'}{12}(1 - \pi)(a - bp)A^x\left(t_1 - T - \frac{rt_1^2}{2} + \frac{rT^2}{2}\right) + \omega T \end{aligned}\right] \tag{3.25}$$

The intuitionistic fuzzy optimization problem is formulated as follows:

$$\begin{aligned} \text{Minimize} \quad & \Delta\left(\widetilde{TC}_{GMIR}(t_1, T)\right) \\ \text{Subject to} \quad & 0 < t_1 < T, \\ & T > 0. \end{aligned} \tag{3.26}$$

(Detailed optimization procedure and derivation are available in Appendix IV.)

3.3. Solution algorithm

This section presents an algorithm constructed to determine the optimal inventory cost for the proposed model.

Input: Numeric values of all parameters.

Output: Optimum cycle time (T^*), optimal time period of positive stock (t_1^*), and minimum inventory cost $TC^*(t_1^*, T^*)$ among crisp, TIFN (SDM), and TIFN (GMIR) environments.

1. Find t_1^* and T^* for the crisp model by minimizing $TC_{crisp}(t_1, T)$. This is achieved by solving $\frac{\partial TC(t_1,T)}{\partial t_1} = 0$ and $\frac{\partial TC(t_1,T)}{\partial T} = 0$.
2. Check the convexity of $TC_{crisp}(t_1, T)$ for the sufficient condition (Appendix I).
3. Compute the optimum $TC^*_{crisp}(t_1^*, T^*)$ using Eq.(3.19).
4. Apply TIFN with SDM.
5. Find t_1^* and T^* for the SDM model by minimizing $TC_{TIFN(SDM)}(t_1, T)$. This is achieved by solving $\frac{\partial \Delta\left(\widetilde{TC}_{SDM}(t_1,T)\right)}{\partial t_1} = 0$ and $\frac{\partial \Delta\left(\widetilde{TC}_{SDM}(t_1,T)\right)}{\partial T} = 0$.
6. Check the convexity of $TC_{TIFN(SDM)}(t_1, T)$ for the sufficient condition (Appendix III).
7. Compute the optimum $TC^*_{TIFN(SDM)}(t_1^*, T^*)$ using Eq.(3.23).
8. Find $TC^*(t_1^*, T^*) = \min\left\{TC^*_{crisp}(t_1^*, T^*), TC^*_{TIFN(SDM)}(t_1^*, T^*)\right\}$.
9. Apply TIFN with GMIR.
10. Find t_1^* and T^* for the GMIR model by minimizing $TC_{TIFN(GMIR)}(t_1, T)$. This is achieved by solving $\frac{\partial \Delta\left(\widetilde{TC}_{GMIR}(t_1,T)\right)}{\partial t_1} = 0$ and $\frac{\partial \Delta\left(\widetilde{TC}_{GMIR}(t_1,T)\right)}{\partial T} = 0$.
11. Check the convexity of $TC_{TIFN(GMIR)}(t_1, T)$ for the sufficient condition (Appendix IV).
12. Compute the optimum $TC^*_{TIFN(GMIR)}(t_1^*, T^*)$ using Eq.(3.26).
13. If $TC^*(t_1^*, T^*) > TC^*_{TIFN(GMIR)}(t_1^*, T^*)$, then set $TC^*(t_1^*, T^*) = TC^*_{TIFN(GMIR)}(t_1^*, T^*)$.
14. Report the final optimal values (t_1^*, T^*), confirm whether convexity conditions were satisfied for the corresponding case, and specify which environment (crisp / TIFN (SDM) / TIFN (GMIR)) produces the minimum cost TC^*.

4. Result and discussion

4.1. Numerical analysis

The model is evaluated under two cases: crisp parameters and TIFN parameters, with the following values used for numerical analysis:

4.1.1. Crisp model

$S = 1000$, $a = 100$, $b = 1.5$, $p = 40$, $\psi = 0.5$, $A = 2$, $\chi = 0.2$, $P = 30$, $\eta = 0.2$, $\delta = 0.9$, $L = 0.01$, $\alpha = 1.5$, $\sigma = 20$, $k = 4$, $l = 6$, $\pi = 0.5$, $\theta = 0.2$, $C = 10$, $r = 0.1$, $\omega = 5$.

4.1.2. TIFN

$a = 100$, $b = 1.5$, $p = 40$, $\psi = 5$, $A = 9$, $\chi = 0.5, \eta = 0.2$, $\delta = 0.9$, $L = 0.1$, $\pi = 0.5$, $\theta = 0.02, r = 0.1$, $\omega = 5$

Table 3: TIFN parameters and defuzzified values

Parameter	Valid TIFN	SDM	GMIR
$\tilde{S}$	$((920, 1000, 1040), (900, 1000, 1060))$	990.00	993.33
$\tilde{C}$	$((9.2, 10, 10.4), (9.0, 10, 10.6))$	9.90	9.93
$\tilde{P}$	$((27.6, 30, 31.2), (27.0, 30, 31.8))$	29.70	29.80
$\tilde{\alpha}$	$((1.38, 1.5, 1.56), (1.35, 1.5, 1.60))$	1.48	1.49
$\tilde{\beta}$	$((18.4, 20, 20.8), (18.0, 20, 21.0))$	19.77	19.85
$\tilde{K}$	$((3.68, 4, 4.16), (3.6, 4, 4.3))$	3.96	3.97
$\tilde{l}$	$((5.52, 6, 6.24), (5.4, 6, 6.4))$	5.95	5.96

The optimal values of T^*, t_1^*, and their corresponding TC^* for each method are summarized in Table 4.

Table 4: Comparison of cost savings

Models	t_1^*	T^*	TC^*
Crisp model	1.3213	5.7161	2352.27
TIFN–(SDM)	1.3196	5.7148	2329.75
TIFN–(GMIR)	1.3195	5.7152	2335.18

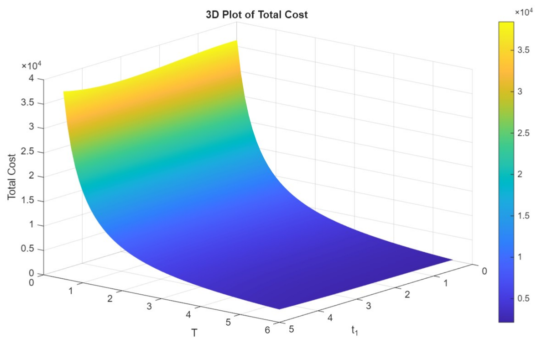

Figure 2: Convexity of graph

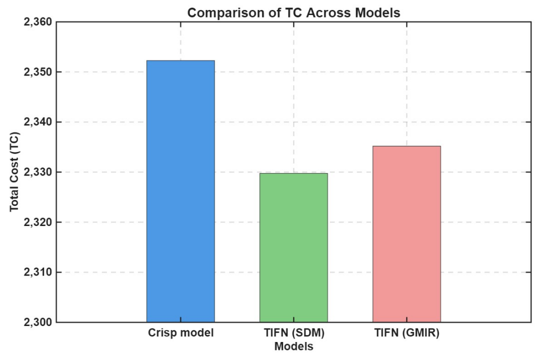

Figure 3: Comparison of total cost

4.2. *Sensitivity Analysis*

This section explores how fluctuations in specific parameter values affect the optimal results. Each variable is adjusted by ±10% and ±20%, while the remaining parameters remain fixed.

Table 5. Crisp model

Parameter	**% change**	**Value**	t_1^*	T^*	TC^*
S	+20	1200	1.3212	5.7346	2377.72
	+10	1100	1.3212	5.7254	2365.01
	−10	900	1.3213	5.7068	2339.52
	−20	800	1.3214	5.6975	2326.74
C	+20	12	1.2601	5.6933	2362.20
	+10	11	1.2907	5.7045	2357.39
	−10	9	1.3520	5.7281	2346.83
	−20	8	1.3826	5.7405	2341.06
P	+20	36	0.9943	5.6071	2388.93
	+10	33	1.1577	5.6564	2374.34
	−10	27	1.4853	5.7882	2322.11
	−20	24	1.6498	5.8756	2283.10
α	+20	1.8	1.2444	5.6543	2389.75
	+10	1.65	1.2828	5.6845	2371.46
	−10	1.35	1.3599	5.7491	2332.16
	−20	1.2	1.3986	5.7838	2311.11
σ	+20	24	1.7575	6.0691	2181.58
	+10	22	1.5578	5.8688	2281.43
	−10	18	1.0346	5.5995	2391.40
	−20	16	0.6782	5.5135	2393.19
K	+20	4.8	1.2624	5.6839	2803.22
	+10	4.4	1.2919	5.6984	2578.26

Parameter	**% change**	**Value**	t_1^*	T^*	TC^*
	−10	3.6	1.3507	5.7380	2125.25
	−20	3.2	1.3802	5.7657	1897.15
l	+20	7.2	1.3282	5.6899	2352.55
	+10	6.6	1.3247	5.7029	2352.44
	−10	5.4	1.3179	5.7295	2352.05
	−20	4.8	1.3145	5.7432	2351.76
θ	+20	0.24	1.1130	5.6413	2376.86
	+10	0.22	1.2174	5.6759	2366.75
	−10	0.18	1.4247	5.7627	2332.98
	−20	0.16	1.5278	5.8170	2308.31
r	+20	0.12	1.1484	5.0371	1924.47
	+10	0.11	1.2269	5.3481	2114.39
	−10	0.09	1.4329	6.1582	2656.27
	−20	0.08	1.5815	6.6994	3054.53
ω	+20	6	1.3213	5.7161	2353.27
	+10	5.5	1.3213	5.7161	2352.77
	−10	4.5	1.3213	5.7161	2351.77
	−20	4	1.3213	5.7161	2351.27
L	+20	0.012	1.3200	5.7165	2352.35
	+10	0.011	1.3207	5.7163	2352.31
	−10	0.009	1.3219	5.7159	2352.23
	−20	0.008	1.3226	5.7157	2352.20
η	+20	0.24	0.9943	5.6071	2388.93
	+10	0.22	1.1577	5.6564	2374.34
	−10	0.18	1.4853	5.7882	2322.11
	−20	0.16	1.6498	5.8756	2283.10

Table 6. TIFN (SDM) model

Parameter	**% change**	**Value**	t_1^*	T^*	TC^*
$\tilde{S}$	+20	1188	1.3168	5.7332	2354.94
	+10	1089	1.3168	5.7240	2342.36
	-10	891	1.3169	5.7055	2317.12
	-20	792	1.3170	5.6962	2304.47
$\tilde{C}$	+20	11.88	1.2556	5.6921	2339.48
	+10	10.89	1.2862	5.7032	2334.77
	-10	8.91	1.3476	5.7267	2324.61
	-20	7.92	1.3783	5.7390	2318.74
$\tilde{P}$	+20	35.64	0.9834	5.6063	2365.57
	+10	32.67	1.1530	5.6554	2351.37
	-10	26.73	1.4866	5.7892	2298.98
	-20	23.76	1.6498	5.8736	2261.76
$\tilde{\alpha}$	+20	1.783	1.2399	5.6531	2366.71
	+10	1.634	1.2785	5.6834	2348.61
	-10	1.337	1.3557	5.7478	2309.85
	-20	1.188	1.3946	5.7825	2289.00
$\tilde{\sigma}$	+20	23.72	1.7533	6.0653	2162.17
	+10	21.74	1.5530	5.8660	2260.47
	-10	17.79	1.0292	5.5987	2367.98

Parameter	% change	Value	t_1^*	T^*	TC^*
	-20	15.81	0.6715	5.5131	2369.13
$\tilde{K}$	+20	4.75	1.2581	5.6829	2774.87
	+10	4.35	1.2878	5.6975	2550.00
	-10	3.56	1.3467	5.7367	2102.82
	-20	3.16	1.3764	5.7646	1874.80
$\tilde{I}$	+20	7.14	1.3241	5.6874	2330.01
	+10	6.54	1.3206	5.7005	2329.92
	-10	5.35	1.3137	5.7272	2329.55
	-20	4.76	1.3086	5.7477	2329.12
θ	+20	0.24	1.1086	5.6393	2353.17
	+10	0.22	1.2131	5.6737	2349.89
	-10	0.18	1.4207	5.7599	2310.88
	-20	0.16	1.5240	5.8139	2286.70
r	+20	0.12	1.1449	5.0350	1905.91
	+10	0.11	1.2231	5.3459	2094.07
	-10	0.09	1.4323	6.1555	2630.97
	-20	0.08	1.5764	6.6963	3025.59
ω	+20	6	1.3172	5.7137	2330.76
	+10	5.5	1.3172	5.7137	2330.26
	-10	4.5	1.3172	5.7137	2329.26
	-20	4	1.3172	5.7137	2328.76
L	+20	0.012	1.3159	5.7140	2329.84
	+10	0.011	1.3165	5.7138	2329.80
	-10	0.009	1.3178	5.7135	2329.73
	-20	0.008	1.3185	5.7133	2329.69
η	+20	0.24	0.9897	5.6052	2365.53
	+10	0.22	1.1533	5.6543	2351.35
	-10	0.18	1.4814	5.7854	2300.17
	-20	0.16	1.6461	5.8723	2261.85

Table 7. TIFN (GMIR) model

Parameter	% change	Value	t_1^*	T^*	TC^*
$\tilde{S}$	+20	1191.99	1.3194	5.7337	2360.46
	+10	1092.66	1.3194	5.7245	2347.83
	-10	893.99	1.3195	5.7059	2322.51
	-20	794.66	1.3196	5.6966	2309.82
$\tilde{C}$	+20	11.91	1.2584	5.6925	2344.97
	+10	10.92	1.2889	5.7037	2340.23
	-10	8.93	1.3504	5.7272	2329.76
	-20	7.94	1.3810	5.7395	2324.06
$\tilde{P}$	+20	35.76	0.9922	5.6063	2371.39
	+10	32.78	1.1557	5.6556	2357.01
	-10	26.82	1.4836	5.7872	2305.32
	-20	23.84	1.6482	5.8745	2266.68
$\tilde{\alpha}$	+20	1.78	1.2446	5.6550	2371.39
	+10	1.63	1.2833	5.6855	2353.08
	-10	1.34	1.3584	5.7484	2315.10
	-20	1.19	1.3974	5.7833	2294.07

Parameter	**% change**	**Value**	t_1^*	T^*	TC^*
$\tilde{\sigma}$	+20	23.82	1.7559	6.0675	2166.17
	+10	21.83	1.5556	5.8671	2265.28
	-10	17.86	1.0318	5.5986	2373.87
	-20	15.88	0.6759	5.5130	2375.37
$\tilde{K}$	+20	4.76	1.2609	5.6833	2780.42
	+10	4.36	1.2906	5.6979	2555.49
	-10	3.57	1.3491	5.7372	2108.17
	-20	3.17	1.3788	5.7650	1880.13
$\tilde{l}$	+20	7.15	1.3264	5.6890	2335.44
	+10	6.55	1.3229	5.7021	2335.34
	-10	5.36	1.3160	5.7287	2334.96
	-20	4.76	1.3126	5.7424	2334.68
θ	+20	0.24	1.1110	5.6405	2359.45
	+10	0.22	1.2155	5.6751	2349.48
	-10	0.18	1.4230	5.7617	2316.10
	-20	0.16	1.5261	5.8158	2291.70
r	+20	0.12	1.1469	5.0363	1910.48
	+10	0.11	1.2252	5.3473	2099.02
	-10	0.09	1.4349	6.1572	2636.98
	-20	0.08	1.5793	6.6982	3032.37
ω	+20	6	1.3195	5.7152	2336.18
	+10	5.5	1.3195	5.7152	2335.68
	-10	4.5	1.3195	5.7152	2334.68
	-20	4	1.3195	5.7152	2334.18
L	+20	0.012	1.3182	5.7156	2335.26
	+10	0.011	1.3188	5.7154	2335.26
	-10	0.009	1.3201	5.7150	2335.14
	-20	0.008	1.3208	5.7148	2335.11
η	+20	0.24	0.9922	5.6063	2371.39
	+10	0.22	1.1557	5.6556	2357.01
	-10	0.18	1.4836	5.7872	2305.32
	-20	0.16	1.6482	5.8745	2266.68

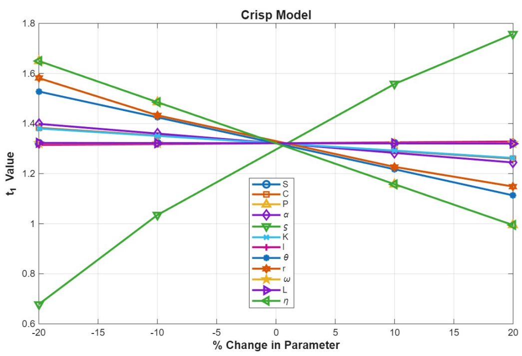

Figure 4: t_1^* with parameter variations

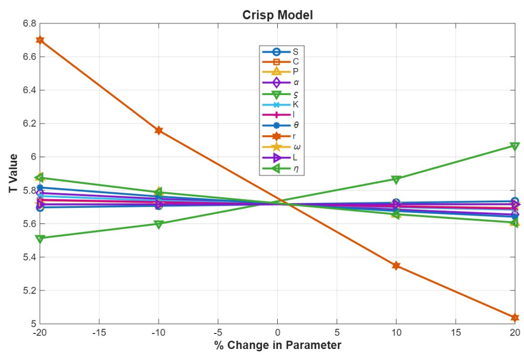

Figure 5: T^* with parameter variations

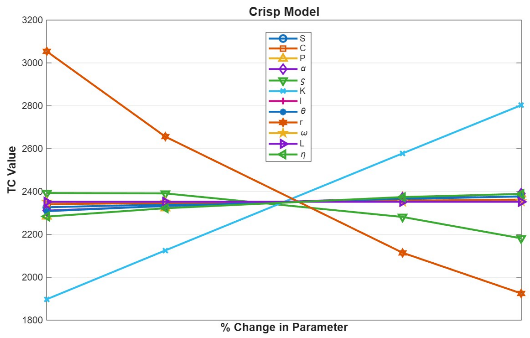

Figure 6: Impact of parameter on TC*

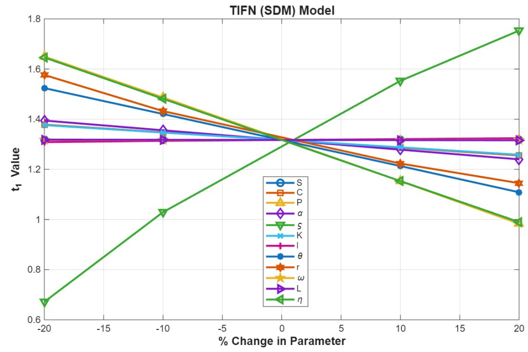

Figure 7: t_1^* with parameter variation

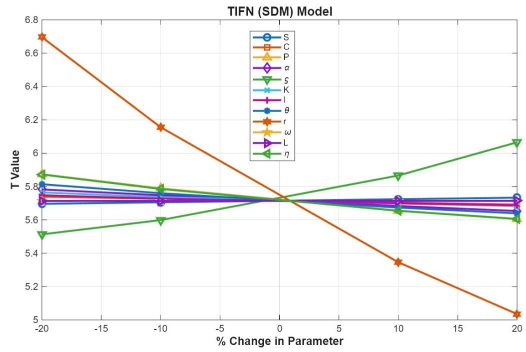

Figure 8: T* with parameter variations

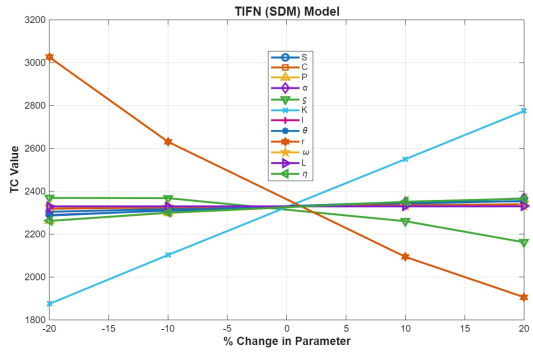

Figure 9: Impact of parameter on TC*

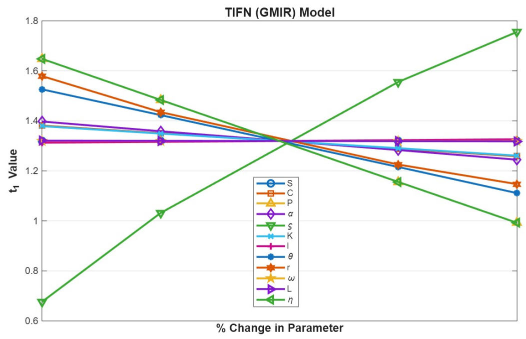

Figure 10: t_1^* with parameter variation

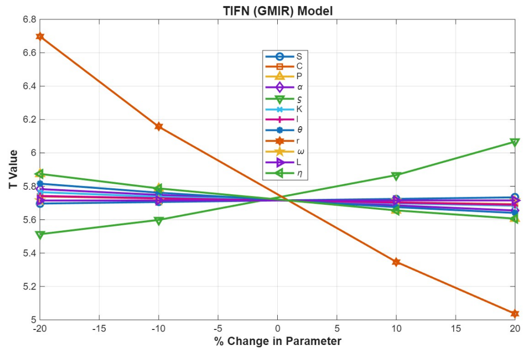

Figure 11: T* with parameter variations

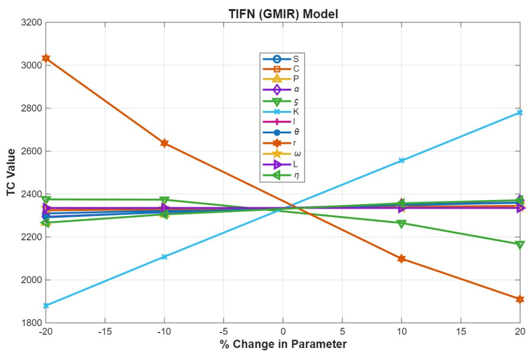

Figure 12: Impact of parameter on TC*

Observations Based on Sensitivity Analysis

1. Figure 5 shows the variation of t_1 under $\pm 20\%$ changes in the parameters. The parameters $P, \sigma, r, \alpha, \theta$ and η are highly sensitive. The parameters S, C, and K show medium sensitivity. The parameters l,ω and L are low sensitive.
2. Figure 6 shows how T responds to $\pm 20\%$ changes in the parameters. The parameters r, σ, P, and α are highly sensitive. The parameters S, C, K, θ, and η show medium sensitivity. The remaining parameters L, ω and l are low sensitive.
3. Figure 7 shows how TC responds to $\pm 20\%$ changes in the parameters. The parameters $r, K, \alpha, \sigma, \theta$, and η are highly sensitive. The parameters P, C, and S show medium sensitivity. The parameters l, ω, and L are low sensitive.
4. Figure 8 shows the response of t_1 to $\pm 20\%$changes in the parameters. The parameters $\tilde{P}, r, \sigma, \tilde{\alpha}, \theta$ and η are highly sensitive. The parameters $\tilde{C}, \tilde{S}, \tilde{K}$ and $\tilde{l}$ exhibit moderate sensitivity. The parameters L and ω are low sensitive.
5. Figure 9 illustrates the response of T to $\pm 20\%$ changes in the parameters. The parameters $\tilde{P}, r, \sigma, \tilde{\alpha}, \theta$, and η are highly sensitive. The parameters $\tilde{C}, \tilde{S}, \tilde{K}$,, and $\tilde{l}$ exhibit moderate sensitivity. The parameters ω and L are low sensitive.
6. Figure 10 illustrates the response of TC to $\pm 20\%$ changes in the parameters. The parameters $\tilde{K}, r, \sigma, \tilde{\alpha}, \theta$, and η are highly sensitive. The parameters $\tilde{l}, \tilde{C}, \tilde{P}$, and $\tilde{S}$ exhibit moderate sensitivity. The parameters ω and L are low sensitive.
7. Figure 11 illustrates the sensitivity of t_1 to $\pm 20\%$ changes in the parameters. The parameters $\tilde{P}, \tilde{\alpha}, \tilde{K}, r, \sigma, \theta$ and η are highly sensitive. The parameters $\tilde{C}, \tilde{S}$ and $\tilde{l}$ exhibit moderate sensitivity. The parameters ω and L are low sensitive.
8. Figure 12 shows how T responds to $\pm 20\%$ changes in the parameters. The parameters $\tilde{P}, \tilde{\alpha}, r, \sigma, \theta$, and η are highly sensitive. The parameters $\tilde{C}, \tilde{S}, \tilde{K}$ and $\tilde{l}$ show medium sensitivity. The parameters ω and L are low sensitive.
9. Figure 13 illustrates the response of TC to $\pm 20\%$ changes in the parameters. The parameters $\tilde{K}, \tilde{\alpha}, r, \sigma, \theta$, and η are highly sensitive. The parameters $\tilde{P}, \tilde{C}, \tilde{S}$, and $\tilde{l}$ show moderate sensitivity. The parameters ω and L are low sensitive.

4.3. Managerial Insights

1. An increase in ordering costs leads to a slight increase in both the optimal total cost and the optimal replenishment cycle time. Managers can adopt bulk-ordering policies and implement an e-procurement system to reduce ordering frequency and spread fixed costs more efficiently.
2. An increase in deterioration expenses leads to a slight rise in both the optimal total cost and the optimal replenishment cycle time. Therefore, managers need to focus on improving storage conditions and adopting PT to reduce deterioration losses and maintain cost efficiency.
3. Higher purchasing cost results in an increase in the optimal total cost and induces adjustment in the optimal replenishment cycle. Managers should focus on supplier, bulk, and contract procurement methods to reduce costs and enhance system efficiency.
4. Increasing holding cost parameters raises the optimal total cost and shortens the optimal replenishment cycle time. Managers can invest in an efficient storage system to minimize costs.
5. A higher shortage cost increases the optimal total cost and leads to adjustments in the optimal cycle time. To avoid stockouts, managers should focus on improving demand forecasting and maintaining safety stock to ensure smooth inventory flow.
6. An increase in the cost of lost sales leads to higher optimal total costs and longer replenishment cycle times. Managers should focus on improving service levels and maintaining sufficient inventory to reduce lost sales.

7. Higher inflation rates increase the optimal total cost and shorten the optimal replenishment cycle time. Therefore, managers should revise ordering policies, reduce cycle times, and control costs under inflationary conditions.
8. An increase in PT cost leads to higher optimal total costs and longer replenishment cycle times. Managers should balance PT costs with savings from reduced spoilage to ensure the inventory system remains efficient.
9. A higher advance payment results in a marginally increased optimal total cost and replenishment cycle time. Managers should aim to reduce lead times by improving coordination with suppliers and creating efficient payment and delivery processes to enhance overall system performance.
10. Higher interest rates lead to increased optimal total costs and longer ideal replenishment cycle times. Managers should strategically plan inventory levels to minimize excess capital, enhance turnover rates, and implement effective financial planning strategies.

5. Conclusion

This study proposed an EOQ inventory model under an IFN environment by integrating advance payment, deterioration, preservation technology, inflation, time-dependent holding cost, and partial backlogging. The demand is driven by advertisement, selling price, and stock level. To capture uncertainty in cost parameters, TIFN were formulated, and SDM and GMIR were used for defuzzification.

The primary conclusion is as follows

1. The numerical results determine the optimal positive inventory period, replenishment cycle time, and minimum total inventory cost under crisp, TIFN(SDM), and TIFN(GMIR). Among the three approaches, the TIFN(SDM) yields the lowest total cost, followed by TIFN(GMIR) and the crisp model.
2. Sensitivity analysis showed that shortage cost, inflation rate, purchasing cost, holding cost parameters, deterioration rate, and interest rate significantly affect total inventory cost and replenishment cycle time. Minor adjustments to these parameters led to significant changes in the optimal solution.
3. Preservation technology cost, lead time, and lost-sales cost showed relatively low sensitivity, indicating that moderate fluctuations in these factors do not significantly affect system performance.
4. From a managerial perspective, firms should prioritize controlling purchasing and holding costs, minimizing shortages through improved forecasting, and reducing deterioration through appropriate investments in preservation technology.
5. In inflationary environments, shorter replenishment cycles and quicker inventory turnover are more cost-effective, helping companies reduce both carrying and financing costs.
6. The results indicate that an advance payment policy can be advantageous if paired with supplier discounts. However, firms need to carefully balance the benefits of discounts against potential capital blocking and interest costs.

Future research could expand on this work by exploring multi-item inventory systems, as well as neutrosophic, interval-valued, or cloudy fuzzy systems, and addressing them with metaheuristic optimization techniques.

Appendix I

The necessary conditions for determining the optimal values of t_1 and T are obtained from the following first-order conditions:

$$\frac{\partial TC}{\partial t_1} = 0$$

$$\begin{bmatrix} \frac{A^{x}(a-bp)}{\theta+A^{x}\psi}\left(2\alpha+2\alpha\big(\theta+A^{x}\psi-\tfrac{3r}{2}\big)t_1-\sigma t_1+\frac{3\sigma t_1^2 r(\theta+A^{x}\psi)}{2(\theta+A^{x}\psi+r)}\right) \\ +C\theta A^{x}(a-bp)t_1+P\delta\eta L A^{x}(a-bp)\left((\theta+A^{x}\psi)t_1+1-\pi\right) \\ +P\delta\eta t_1 A^{x}(a-bp)+\pi K A^{x}(a-bp)\left(T+\frac{rT^2}{2}+t_1\right) \\ +l(1-\pi)(a-bp)A^{x}\left(1-\frac{rt_1}{2}\right) \end{bmatrix}=0$$

$$\frac{\partial TC}{\partial T}=0$$

$$-\frac{1}{T^2}\begin{bmatrix} O+\frac{A^{x}(a-bp)}{\theta+A^{x}\psi}\left(2\alpha t_1+\alpha\big(\theta+A^{x}\psi-\tfrac{3r}{2}\big)t_1^2-\frac{\sigma t_1^2}{2}+\frac{\sigma t_1^3 r(\theta+A^{x}\psi)}{2(\theta+A^{x}\psi+r)}+\frac{\sigma}{2}\right) \\ +\frac{C\theta A^{x}(a-bp)t_1^2}{2}+P\delta\eta L A^{x}(a-bp)\left(\frac{(\theta+A^{x}\psi)t_1^2}{2}+t_1-\pi(t_1-T)\right) \\ +\frac{P\delta\eta t_1^2 A^{x}(a-bp)}{2} \\ +\pi K A^{x}(a-bp)\left(Tt_1+\frac{2}{r^2}+\frac{3T^2}{2}+\frac{rT^2t_1}{2}+\frac{t_1^2}{2}-\frac{2T}{r}-\frac{rT^3}{2}\right) \\ +l(1-\pi)(a-bp)A^{x}\left(t_1-T-\frac{rt_1^2}{2}+\frac{rT^2}{2}\right)+\omega T \end{bmatrix} +\frac{1}{T}\begin{bmatrix} P\delta\eta L A^{x}(a-bp)\pi+\pi K A^{x}(a-bp)\left(t_1+3T+rTt_1+\frac{2}{r}-\frac{3rT^2}{2}\right) \\ +l(1-\pi)(a-bp)A^{x}(rT-1)+\omega \end{bmatrix}=0$$

For convexity, the following sufficient conditions must be satisfied:

$$\left(\frac{\partial^2 TC}{\partial t_1^2}\right)\left(\frac{\partial^2 TC}{\partial T^2}\right)-\left(\frac{\partial^2 TC}{\partial t_1\,\partial T}\right)^2>0,\quad \frac{\partial^2 TC}{\partial t_1^2}>0,\quad \frac{\partial^2 TC}{\partial T^2}>0.$$

$$\frac{\partial^2 TC}{\partial t_1^2}=\begin{bmatrix} \frac{A^{x}(a-bp)}{\theta+A^{x}\psi}\left(2\alpha+2\alpha\big(\theta+A^{x}\psi-\tfrac{3r}{2}\big)-\sigma+\frac{3\sigma t_1 r(\theta+A^{x}\psi)}{(\theta+A^{x}\psi+r)}\right) \\ +C\theta A^{x}(a-bp)+P\delta\eta L A^{x}(a-bp)(\theta+A^{x}\psi)+P\delta\eta A^{x}(a-bp) \\ +\pi K A^{x}(a-bp)+l(1-\pi)(a-bp)A^{x}\left(1-\frac{r}{2}\right) \end{bmatrix}$$

$$\frac{\partial^2 TC}{\partial T^2} = \frac{2}{T^3}\left[\begin{array}{l} O + \frac{A^x(a-bp)}{\theta + A^x\psi}\left(2\alpha t_1 + \alpha(\theta + A^x\psi - \frac{3r}{2})t_1^2 - \frac{\sigma t_1^2}{2} + \frac{\sigma t_1^3 r(\theta + A^x\psi)}{2(\theta + A^x\psi + r)} + \frac{\sigma}{2}\right) \\ + \frac{C\theta A^x(a-bp)t_1^2}{2} + P\delta\eta L A^x(a-bp)\left(\frac{(\theta + A^x\psi)t_1^2}{2} + t_1 - \pi(t_1 - T)\right) \\ + \frac{P\delta\eta t_1^2 A^x(a-bp)}{2} + \pi K A^x(a-bp)\left(Tt_1 + \frac{2}{r^2} + \frac{3T^2}{2} + \frac{rT^2 t_1}{2} + \frac{t_1^2}{2} - \frac{2T}{r} - \frac{rT^3}{2}\right) \\ + l(1-\pi)(a-bp)A^x\left(t_1 - T - \frac{rt_1^2}{2} + \frac{rT^2}{2}\right) + \omega T \end{array}\right]$$
$$- \frac{1}{T^2}\left[\begin{array}{l} -P\pi\delta\eta L A^x(a-bp) + \pi K A^x(a-bp)\left(t_1 + 3T + rTt_1 - \frac{2}{r} - \frac{r3T^2}{2}\right) \\ + l(1-\pi)(a-bp)A^x(t_1 - 1 + rT) + \omega \end{array}\right]$$
$$+ \frac{-1}{T^2}\left[\begin{array}{l} P\delta\eta L A^x(a-bp)\pi + \pi K A^x(a-bp)\left(t_1 + 3T + rTt_1 + \frac{2}{r} - \frac{3rT^2}{2}\right) \\ \quad + l(1-\pi)(a-bp)A^x(rT - 1) + \omega \end{array}\right]$$
$$+ \frac{1}{T}\left[\pi K A^x(a-bp)(3 + rT - 3rT) + l(1-\pi)(a-bp)A^x(r-1)\right]$$

$$\frac{\partial TC^2}{\partial t_1 \partial T} = \left[\pi K A^x(a-bp)(1 + rT)\right]$$

Appendix II

Definition (Fuzzy set) [52]: Consider a universe of discourse X. A fuzzy set B in X is represented by

$$B = \{(x, \mu_B(x)) : x \in X\},$$

where $\mu_B : X \to [0,1]$ denotes the membership function, which specifies the extent to which each $x \in X$ belongs to the fuzzy set B.

Definition (Fuzzy number) [52]: A fuzzy number on $\mathbb{R}$ is described by a membership function $\mu_A : \mathbb{R} \to [0,1]$ that fulfills:

1. Normality: at least one element $x \in \mathbb{R}$ has $\mu_A(x) = 1$.
2. Convexity: for all $x, y \in \mathbb{R}$ and $\lambda \in [0,1]$,

$$\mu_A(\lambda x + (1-\lambda)y) \geqslant \min\{\mu_A(x), \mu_A(y)\}.$$

3. Upper semi-continuity: each α-cut $\{x \in \mathbb{R} : \mu_A(x) \geqslant \alpha\}$ is a closed interval.
4. Bounded support: the set $\{x \in \mathbb{R} : \mu_A(x) > 0\}$ is bounded.

Definition (Triangular Intuitionistic Fuzzy Number (TIFN)) [35]: A TIFN $\tilde{B}$ on $\mathbb{R}$ is specified by a membership function $\mu_{\tilde{B}}(x)$ and a non-membership function $\nu_{\tilde{B}}(x)$, defined as

$$\mu_{\tilde{B}}(x) = \begin{cases} \frac{x - \phi_1}{\phi_2 - \phi_1}, & \phi_1 \leqslant x \leqslant \phi_2, \\ \frac{\phi_3 - x}{\phi_3 - \phi_2}, & \phi_2 \leqslant x \leqslant \phi_3, \\ 0, & \text{otherwise}, \end{cases}$$

$$\nu_{\tilde{B}}(x) = \begin{cases} \dfrac{\phi_1' - x}{\phi_2' - \phi_1'}, & \phi_1' \leqslant x \leqslant \phi_2, \\ \dfrac{x - \phi_2'}{\phi_3' - \phi_2'}, & \phi_2 \leqslant x \leqslant \phi_3', \\ 0, & \text{otherwise.} \end{cases}$$

The parameters satisfy $\phi_1' \leqslant \phi_1 \leqslant \phi_2 \leqslant \phi_3 \leqslant \phi_3'$. For each $x \in \mathbb{R}$, the condition $0 \leqslant \mu_{\tilde{B}}(x) + \nu_{\tilde{B}}(x) \leqslant 1$ holds. A TIFN is concisely denoted by

$$\tilde{B} = (\phi_1, \phi_2, \phi_3; \phi_1', \phi_2, \phi_3').$$

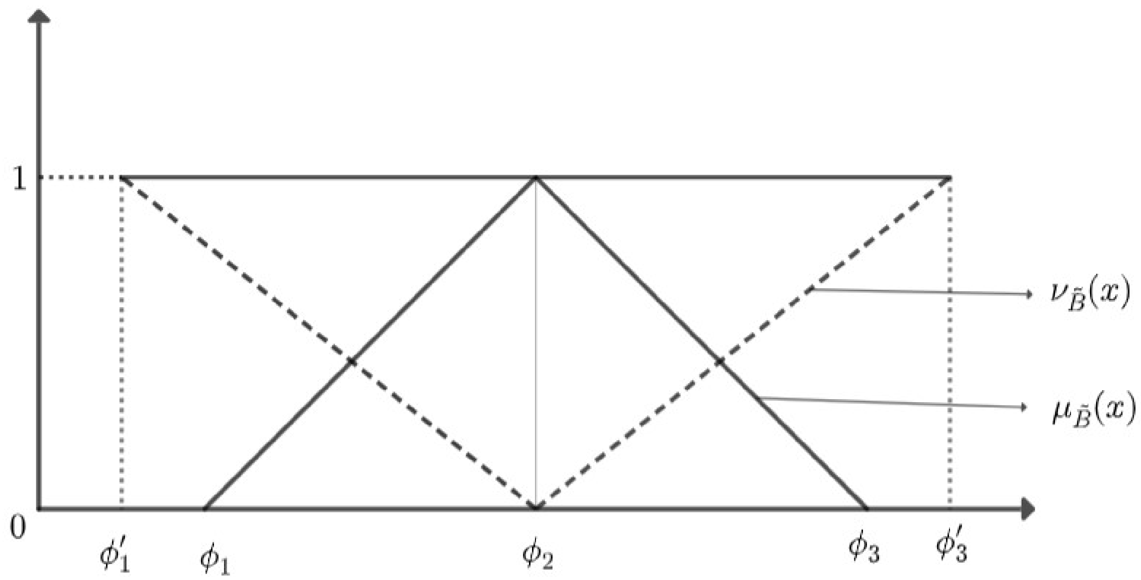

Figure .13: Triangular Intuitionistic Fuzzy Number

Transformation rules for TIFN:

- To a TFN $\tilde{B} = (\phi_1, \phi_2, \phi_3)$: set $\phi_1 = \phi_1'$, $\phi_3 = \phi_3'$ and $\nu_{\tilde{B}}(x) = 1 - \mu_{\tilde{B}}(x)$, $\forall x \in \mathbb{R}$.
- To a crisp interval $[\phi_1, \phi_3]$: set $\phi_1 = \phi_1'$, $\phi_3 = \phi_3'$.
- To a real number ϕ: set $\phi_1' = \phi_1 = \phi_2 = \phi_3 = \phi_3'$.

Arithmetic operation of TIFN based on (α, σ)-cuts:

For a TIFN $\tilde{B}$ defined on $\mathbb{R}$, the (α, σ)-cut is given by

$$A_1(\alpha) = \phi_1 + \alpha(\phi_2 - \phi_1), A_2(\alpha) = \phi_3 - \alpha(\phi_3 - \phi_2),$$
$$A_1(\sigma) = \phi_2 - \sigma(\phi_2 - \phi_1'), A_2(\sigma) = \phi_2 + \sigma(\phi_3' - \phi_2).$$

Hence,

$$\begin{aligned} \tilde{B}_{\alpha,\sigma} &= [A_1(\alpha), A_2(\alpha)]\,;[A_1(\sigma), A_2(\sigma)] \\ &= [\phi_1 + \alpha(\phi_2 - \phi_1),\ \phi_3 - \alpha(\phi_3 - \phi_2)]\,;\left[\phi_2 - \sigma(\phi_2 - \phi_1'),\ \phi_2 + \sigma(\phi_3' - \phi_2)\right], \end{aligned}$$

where $\alpha, \sigma \in [0, 1]$ and $\alpha + \sigma \leqslant 1$.

Signed Distance Method (SDM) [9]:

The SDM is a ranking method that determines the representative crisp value of a TIFN by measuring its average distance from the origin. It combines the membership and non-membership values of IFN into a crisp value for effective comparison and optimization.

$$d_{\mu}(\tilde{B},0) = \frac{1}{2}\int_0^1 [A_1(\alpha) + A_2(\alpha)]\, d\alpha = \frac{1}{2}\int_0^1 [\phi_1 + \alpha(\phi_2 - \phi_1) + \phi_3 - \alpha(\phi_3 - \phi_2)]\, d\alpha = \frac{1}{4}(\phi_1 + 2\phi_2 + \phi_3),$$

$$d_{\nu}(\tilde{B},0) = \frac{1}{2}\int_0^1 [A_1(\sigma) + A_2(\sigma)]\, d\sigma = \frac{1}{2}\int_0^1 \left[\phi_1' + \sigma(\phi_2 - \phi_1') + \phi_3' - \sigma(\phi_3' - \phi_2)\right] d\sigma = \frac{1}{4}(\phi_1' + 2\phi_2 + \phi_3'),$$

$$d_A(\tilde{B},0) = \frac{d_{\mu}(\tilde{B},0) + d_{\nu}(\tilde{B},0)}{2} = \frac{1}{8}(\phi_1 + 4\phi_2 + \phi_3 + \phi_1' + \phi_3').$$

Graded Mean Integration Representation (GMIR) [8]:
The GMIR method gives greater weight to the central value of a TIFN to determine its representative crisp value. It combines the membership and non-membership values in a weighted form for comparison and optimization.

$$d_{\mu}(\tilde{B},0) = \frac{\int_0^1 \frac{\alpha}{2}[A_1(\alpha) + A_2(\alpha)]d\alpha}{\int_0^1 \alpha d\alpha} = \frac{1}{6}(\phi_1 + 4\phi_2 + \phi_3),$$

$$d_{\nu}(\tilde{B},0) = \frac{\int_0^1 \frac{\sigma}{2}[A_1(\sigma) + A_2(\sigma)]d\sigma}{\int_0^1 \sigma d\sigma} = \frac{1}{6}(\phi_1' + 4\phi_2 + \phi_3'),$$

$$d_A(\tilde{B},0) = \frac{d_{\mu}(\tilde{B},0) + d_{\nu}(\tilde{B},0)}{2} = \frac{1}{12}(\phi_1 + 8\phi_2 + \phi_3 + \phi_1' + \phi_3').$$

Appendix III

The conditions necessary and sufficient for finding the optimal values of t_1 and T that minimize the $\Delta\left(\widetilde{TC}_{SDM}(t_1,T)\right)$, as well as verifying the solution's optimality, are given below:

$$\frac{\partial\Delta\left(\widetilde{TC}_{SDM}(t_1,T)\right)}{\partial t_1} \quad \text{and} \quad \frac{\partial\Delta\left(\widetilde{TC}_{SDM}(t_1,T)\right)}{\partial T}$$

The following condition ensures the convexity of $\widetilde{(TC)}$:

$$\left(\frac{\partial^2\Delta\left(\widetilde{TC}_{SDM}(t_1,T)\right)}{\partial t_1^2}\right)\left(\frac{\partial^2\Delta\left(\widetilde{TC}_{SDM}(t_1,T)\right)}{\partial T^2}\right) - \left(\frac{\partial^2\Delta\left(\widetilde{TC}_{SDM}(t_1,T)\right)}{\partial t_1 \partial T}\right)^2 > 0$$

and

$$\left(\frac{\partial^2\Delta\left(\widetilde{TC}_{SDM}(t_1,T)\right)}{\partial t_1^2}\right) > 0, \quad \left(\frac{\partial^2\Delta\left(\widetilde{TC}_{SDM}(t_1,T)\right)}{\partial T^2}\right) > 0$$

Appendix IV

The conditions necessary and sufficient for finding the optimal values of t_1 and T that minimize the $\Delta\left(\widetilde{\widetilde{TC}}_{GMIR}(t_1, T)\right)$, as well as verifying the solution's optimality, are given below:

$$\frac{\partial \Delta\left(\widetilde{\widetilde{TC}}_{GMIR}(t_1, T)\right)}{\partial t_1} \quad \text{and} \quad \frac{\partial \Delta\left(\widetilde{\widetilde{TC}}_{GMIR}(t_1, T)\right)}{\partial T}$$

The following condition ensures the convexity of $\widetilde{\widetilde{TC}}$:

$$\left(\frac{\partial^2 \Delta\left(\widetilde{\widetilde{TC}}_{GMIR}(t_1, T)\right)}{\partial t_1^2}\right)\left(\frac{\partial^2 \Delta\left(\widetilde{\widetilde{TC}}_{GMIR}(t_1, T)\right)}{\partial T^2}\right) - \left(\frac{\partial^2 \Delta\left(\widetilde{\widetilde{TC}}_{GMIR}(t_1, T)\right)}{\partial t_1 \partial T}\right)^2 > 0$$

and

$$\left(\frac{\partial^2 \Delta\left(\widetilde{\widetilde{TC}}_{GMIR}(t_1, T)\right)}{\partial t_1^2}\right) > 0, \quad \left(\frac{\partial^2 \Delta\left(\widetilde{\widetilde{TC}}_{GMIR}(t_1, T)\right)}{\partial T^2}\right) > 0$$

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