Math. & Comput. Sci., 7(3) (2026), 21–31
Online: ISSN 2717-2708
Mathematics and Computational Sciences
Journal Homepage: http://mcs.qut.ac.ir/mcs

Block hybrid method with off-step points for numerical solutions of classical Blasius Equation

Oludare Adedire[a,b,*], Paul C.Mordi[c]

[a]Department of Mathematics, University of Jos, Plateau State, Nigeria.
[b]Department of Statistics, Federal College of Forestry, Jos, Plateau State, Nigeria.
[c]Department of Mathematics and Statistics, University of Windsor, Ontario Canada.

Abstract

Changes in conditions and parameters of equations in fluid dynamics can place a significant burden on procedures that lead to their approximate analytic solutions. While some classes of numerical methods can easily accommodate such changes, explicit linear multistep methods (LMM) have problems of instability. In this research in order to address problem of instability associated with explicit LMM when used to solve nonlinear differential equations a hybrid form of explicit LMM incorporating two off-step points in block form was derived using multistep collocation and matrix inversion technique. The two off-step points involved an interpolation point and a collocation point. Furthermore, the derived block method was shown to be convergent; investigation of its region of absolute stability indicated an $A(\alpha)$-stable method having large region of absolute stability with $\alpha = 88.7^0$ and capable of handling nonlinear systems. For implementation purpose, it was tested on classical Blasius problem in fluid dynamics and results compared well with Matlab ode23s known for solving stiff and nonlinear problems.

Keywords: Approximate, collocation, interpolation, multistep, block.

2020 MSC: 5C20, 65L07, 65L20.

1. Introduction

In fluid dynamics, nonlinear mechanics of uniform flow past a semi-infinite plate have been modelled using differential equations (DEs). One of such DEs is a third order differential equation popularly known as classical Blasius equation [37]. It can be obtained from simplification of Navier-Stokes equations under similarity transform and certain prescribed assumptions [35]. The classical Blasius equation is given by Blasius[6];

$$y'''_{xxx} + \alpha y y''_{xx} = 0 \tag{1.1}$$

subject to initial-boundary conditions;

$$y(0) = 0, y'(0) = 0, \lim_{x\to\infty} y'(x) = 1 \tag{1.2}$$

*Corresponding author
Email addresses: dharenss@gmail.com (Oludare Adedire), chukscharles.mpc@gmail.com (Paul C.Mordi)

doi: 10.30511/mcs.2026.2055296.1317

Received: 07 March 2025 Accepted: 29 May 2026

where α is a constant and the notations $y'_x = \frac{dy}{dx}, y''_{xx} = \frac{d^2y}{dx^2}, y'''_{xxx} = \frac{d^3y}{dx^3}$ are first, second and third order derivatives of y with respect to x respectively. Existence and uniqueness of (1.1) are guaranteed when adequate initial-boundary conditions are used. Equivalently (1.1) and (1.2) can be written as a system of ordinary differential equations [22];

$$\left.\begin{array}{l} u'_x - v_x = 0, u_x(0) = 0, \\ v'_x - w_x = 0, v_x(0) = 0, \\ u'_x + \alpha u_x w_x = 0, w_x(0) = 0.46957. \end{array}\right\} \tag{1.3}$$

There has not been simple closed-form solution to the Blasius problem despite its simple outlook. Several attempts have been made, for example perturbation method used for solution by Blasius[6] was shown to be inadequate due to restricted radius of convergence of the first power series applied to obtain approximate series solution [36]. Afterwards, many other methods of solutions have been developed for the classical Blasius equation. Among them are Adomians decomposition [1], homotopy perturbation [13, 22], analytic approximate solution [20], Runge-Kutta [10], generalized Laguerre function collocation [27]; Variational iteration [14], differential transformation, Shooting and parallel shooting methods [7]. Readers may consult other numerous methods of solutions to Blasius equation in literature [2, 3, 16, 18, 21, 28, 29, 30, 34, 39, 40].

It has been noted that analytical solutions to complicated problems in fluid dynamics can be time-consuming and tiresome [29]. Slight changes in some system factors and boundary conditions of such problems can induce remarkable challenges leading to more complications. However, use of numerical computation for simulation of the problems can offer easier approach to solutions. Motivation to embark on this study came from possibility of deriving a block hybrid method capable of solving complex systems often commonly found in fluid dynamics. Particularly by creating a hybrid method through modification of a class of explicit linear multistep method which in time past was considered unfit for such purpose due to some barriers. This will be achieved by leveraging on advantages found in multistep block methods where all step values are acquired in a single block advance thereby speeding up process of computation of solutions [9]. There are some block methods in literature whose structure, class and capability differ from the block hybrid method proposed in this study. Some of such methods have specifically targeted block backward differentiation formulae (BDF) due to consideration of stability [4, 8, 17, 19, 31, 32, 33]. Okuonghae and Ozobokeme [25] introduced an optimal Lstable timeefficient hybrid block method based on interpolation and collocation, which is particularly effective for stiff systems and AlKhaled et al. [5] developed a reliable numerical algorithm for solving the Blasius boundary value problem, offering a recent comparison for accuracy and computational efficiency.

In this study, objectives are to derive block hybrid explicit method (BHEM) with two off-grid points based on multistep collocation and to implement the block method for numerical solutions of classical Blasius problems commonly found in fluid dynamics. Development of the block hybrid method will be achieved particularly by making the first off-grid point an interpolation point and the second off-grid point a collocation point. The advantage of using multistep block method over single-step method includes better computational speed, efficiency and accuracy of solutions. Other objectives of this study include analysis of the block hybrid method and its numerical test on classical Blasius problem.

The remaining part of the paper is arranged as follows: section 2 considers derivation of the block hybrid method, section 3 centres on analysis of the derived method. section 4 considers results from numerical simulations of classical Blasius problem and conclusion comes up in section 5.

2. Derivation of block hybrid method

Derivation of the block hyrid method for this study follows Onumanyi et al. [26] hence define a polynomial of degree $p = t + m - 1, t > 0, m > 0$ by

$$y(x) - \sum_{j=0}^{t-1} \alpha_j(x) y(x_{n+j}) = h \sum_{j=0}^{m-1} \beta_j(x) f(\bar{x}_j, y(\bar{x}_j)), \qquad x_n \leqslant x \leqslant x_{n+k} \tag{2.1}$$

where $\alpha_j(x)$ and $\beta_j(x)$ are continuous coefficients defined by

$$\alpha_j(x) = \sum_{i=0}^{t+m-1} \alpha_{j,i+1}x^i; j \in \{0, ..., t-1\} \tag{2.2}$$

and

$$h\beta_j(x) = h\sum_{i=0}^{t+m-1} \beta_{j,i+1}x^i; j \in \{0, ..., m\} \tag{2.3}$$

In (2.1), m are collocation points and x_{n+j} are interpolation points $t(0 < t \leqslant k)$ which are arbitrarily chosen from $\{x_n, x_{n+1}, ..., x_{n+k-1}\}$.

Let $k = 2, t = 2, m = 2$ and two off-grid points $x = x_{n+\frac{5}{4}}$ and $x = x_{n+\frac{8}{5}}$ be incorporated as interpolation and collocation points respectively hence (2.1) becomes

$$y(x) = \alpha_1(x)y_{n+1} + \alpha_{\frac{5}{4}}(x)y_{n+\frac{5}{4}} + h\left[\beta_1(x)f_{n+1}+\beta_{\frac{8}{5}}(x)f_{n+\frac{8}{5}}\right] \tag{2.4}$$

In this procedure, it is necessary to obtain multistep collocation matrix D. However, to avoid repetition, readers may check Onumanyi et al.[26] for its derivation in general form and matrix inversion technique. Hence for this study, multistep collocation matrix D is defined by;

$$D = \begin{bmatrix} 1 & x_{n+1} & x_{n+1}^2 & x_{n+1}^3 \\ 1 & x_{n+\frac{5}{4}} & x_{n+\frac{5}{4}}^2 & x_{n+\frac{5}{4}}^3 \\ 0 & 1 & 2x_{n+1} & 3x_{n+1}^2 \\ 0 & 1 & 2x_{n+\frac{8}{5}} & 3x_{n+\frac{8}{5}}^2 \end{bmatrix} \tag{2.5}$$

such that $C = D^{-1}$ is a matrix where C represents continuous coefficients $\alpha_j(x)$ and $\beta_j(x)$ in (2.1). With aid of Maple software $C = D^{-1}$ can be obtained and its subsequent substitution into (2.1) give the following continuous formulation;

$$\begin{aligned} y(x) = &\left(-\frac{476\,x_n+595\,h-476\,x}{13\,h^3}\left(h^2+\left(-\frac{212\,x}{119}+\frac{212\,x_n}{119}\right)h+\frac{80\,(x-x_n)^2}{119}\right)\right)y_{n+1} \\ &+\left(\frac{(608\,h-320\,x+320\,x_n)(h-x+x_n)^2}{13\,h^3}\right)y_{n+\frac{5}{4}}+\left(-\frac{(4\,x_n+5\,h-4\,x)(h-x+x_n)(95\,x_n+173\,h-95\,x)}{78\,h^2}\right)y_{n+1} \\ &+\left(-\frac{25\,(h-x+x_n)^2(4\,x_n+5\,h-4\,x)}{78\,h^2}\right)y_{n+\frac{8}{5}} \end{aligned} \tag{2.6}$$

Evaluation of (2.6) at $x = x_n$, $x = x_{n+\frac{8}{5}}$ and its derivative at $x = x_{n+\frac{5}{4}}$ give the following;

$$\left.\begin{aligned} y_{n+1} &= -\frac{13}{595}y_n + \frac{608}{595}y_{n+\frac{5}{4}} - \frac{173}{714}hf_{n+1} - \frac{25}{714}hf_{n+\frac{8}{5}} \\ y_{n+\frac{5}{4}} &= y_{n+1} + \frac{31}{288}hf_{n+1} + \frac{13}{84}hf_{n+\frac{5}{4}} - \frac{25}{2016}hf_{n+\frac{8}{5}} \\ y_{n+\frac{8}{5}} &= -\frac{539}{325}y_{n+1} + \frac{864}{325}y_{n+\frac{5}{4}} - \frac{147}{650}hf_{n+1} + \frac{21}{130}hf_{n+\frac{8}{5}} \end{aligned}\right\} \tag{2.7}$$

Equation (2.7) is the derived block hybrid scheme with off-grid interpolation and collocation points. Convergence analysis of (2.7) follows subsequently.

3. Analyses of the block method (2.7)

In this section, convergence analysis and plots of regions of absolute stability of (2.7) will be considered.

3.1. Convergence analysis of the block method (2.7)

Using the approach of Chu and Hamilton [9] and Fatunla [12], reformulate (2.7) to obtain;

$$M^{(1)}Y_{n+i} = M^{(0)}Y_n + hB^{(1)}f_{n+i}, \ i \in \mathbb{R} \tag{3.1}$$

where

$$M^{(1)} = \begin{pmatrix} 1 & -\frac{608}{595} & 0 \\ -1 & 1 & 0 \\ \frac{539}{325} & -\frac{864}{325} & 1 \end{pmatrix}, M^{(0)} = \begin{pmatrix} 0 & 0 & -\frac{13}{595} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{3.2}$$

and

$$B^{(1)} = \begin{pmatrix} -\frac{173}{714} & 0 & -\frac{25}{714} \\ \frac{31}{288} & \frac{13}{84} & -\frac{25}{2016} \\ -\frac{147}{650} & 0 & \frac{21}{130} \end{pmatrix} \tag{3.3}$$

so that the first characteristics polynomial of (2.7) given by;

$$\begin{aligned} \rho(\lambda) &= \det(\lambda M^{(1)} - M^{(0)}) \\ &= |\lambda M^{(1)} - M^{(0)}| \\ &= 0 \end{aligned}$$

gives

$$\begin{aligned} \rho(\lambda) &= \left| \lambda \begin{pmatrix} 1 & -\frac{608}{595} & 0 \\ -1 & 1 & 0 \\ \frac{539}{325} & -\frac{864}{325} & 1 \end{pmatrix} - \begin{pmatrix} 0 & 0 & -\frac{13}{595} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \right| \\ &= -\frac{13}{595}\lambda^2(\lambda - 1) \end{aligned}$$

which implies that

$$\lambda_1 = 1, \ \lambda_2 = 0, \lambda_3 = 0 \tag{3.4}$$

According to Fatunla [11], since $|\lambda_i| \leqslant 1$, $i = 1, 2$ & 3 the block method (2.7) is zero stable.

The block hybrid method (2.7) consists of three difference equations and order p is determined by expanding each equation in Taylor series about x_n and requiring the local truncation error (LTE) to be $\mathcal{O}(h^{p+1})$.

Let $y(x)$ be the exact solution. Use the notation $y_n = y(x_n)$, $y_{n+1} = y(x_n + h)$, $y_{n+5/4} = y(x_n + \frac{5}{4}h)$, $f_{n+1} = y'(x_n + h)$, $f_{n+8/5} = y'(x_n + \frac{8}{5}h)$. Expand:

$$\begin{aligned} y_{n+1} &= y_n + hy_n' + \frac{h^2}{2}y_n'' + \frac{h^3}{6}y_n''' + \frac{h^4}{24}y_n^{(4)} + \frac{h^5}{120}y_n^{(5)} + \cdots, \\ y_{n+\frac{5}{4}} &= y_n + \frac{5}{4}hy_n' + \frac{25}{32}h^2y_n'' + \frac{125}{384}h^3y_n''' + \frac{625}{6144}h^4y_n^{(4)} + \cdots, \\ f_{n+1} &= y_n' + hy_n'' + \frac{h^2}{2}y_n''' + \frac{h^3}{6}y_n^{(4)} + \frac{h^4}{24}y_n^{(5)} + \cdots, \\ f_{n+\frac{8}{5}} &= y_n' + \frac{8}{5}hy_n'' + \frac{32}{25}h^2y_n''' + \frac{256}{375}h^3y_n^{(4)} + \cdots. \end{aligned}$$

Substitute into the first equation of the block method:

$$L_1 = y_{n+1} + \frac{13}{595}y_n - \frac{608}{595}y_{n+\frac{5}{4}} + \frac{173}{714}hf_{n+1} + \frac{25}{714}hf_{n+\frac{8}{5}}.$$

After collecting terms, the coefficients of h^0, h^1, h^2, h^3 vanish identically because of the coefficients chosen in the method. The first nonzero term occurs at h^4:

$$\mathrm{LTE}_1 = \frac{229}{114240}\, h^4 y^{(4)}(x_n) + \mathcal{O}(h^5).$$

Hence the first equation has order $p = 3$ with error constant $C_1 = \frac{229}{114240}$. Analogous expansions for the second and third equations give

$$\mathrm{LTE}_2 = \frac{19}{29160}\, h^4 y^{(4)}(x_n) + \mathcal{O}(h^5), \qquad \mathrm{LTE}_3 = -\frac{441}{520000}\, h^4 y^{(4)}(x_n) + \mathcal{O}(h^5).$$

Thus the entire block method is of order $p = 3$ and error constants $\left(\frac{229}{114240}, \frac{19}{29160}, -\frac{441}{520000}\right)$ respectively. Following Henrici [15], the block method (2.7) is convergent since it is zero stable and has order $p > 1$

3.2. Stability polynomial of the block method (2.7)

Definition 3.1. Let S be region of absolute stability in a complex plane and r_i, $i = 1, 2, ..., k$ be roots of stability polynomial $\pi(r, z)$ defined by

$$\pi(r, z) = \rho(r) - z\sigma(r) = 0 \tag{3.5}$$

block method (2.7) is said to be absolutely stable with region of absolute stability S if all roots r_i of the stability polynomial $\pi(r, z)$ satisfy $|r_i| < 1$ for a fixed $z = h\lambda$, $r = e^{i\theta}$, where $\rho(r)$ and $\sigma(r)$ are first and second characteristics polynomials respectively ([4], [23]).

Definition 3.2. A numerical integration scheme is said to be $A(\alpha)$ stable for some $\alpha \in \left[0, \frac{\pi}{2}\right]$ if the wedge

$$S_{\alpha^*} = \{z \,:\, |\mathrm{Arg}(-z)| < \alpha^*,\ z \neq 0\} \tag{3.6}$$

is contained in its region of absolute stability. The largest $\alpha^*_{\max}$ is called an angle of absolute stability or an argument of stability (Okuonghae and Ikhile, 2013).

Using (3.5), the (3.1) has the stability polynomial $\pi(r, z)$ for the block method (2.7) given by;

$$\pi(r, z) = M^{(1)} Y_{n+i} - M^{(0)} Y_n - h\, B^{(1)} f_{n+i},\ i \in \mathbb{R}$$
$$\pi(r, z) = M^{(1)} r^1 - M^{(0)} r^0 - z\, B^{(1)} r^1$$
$$\pi(r, z) = M^{(1)} r - M^{(0)} - z\, B^{(1)} r$$
$$\pi(r, z) = r\left(M^{(1)} - zB^{(1)}\right) - M^{(0)} \tag{3.7}$$

so that (3.7) for values of $M^{(1)}, B^{(1)}), M^{(0)}$ in (3.2) and (3.3) for
$r = \begin{pmatrix} r & 0 & 0 \\ 0 & r^{\frac{5}{4}} & 0 \\ 0 & 0 & r^{\frac{8}{5}} \end{pmatrix}$ give the following matrix obtained with the aid of Maple software version 2022.0 as;

$$\pi(r, z) = \begin{pmatrix} r\left(1 + \frac{173}{714} z\right) & -\frac{608}{595} r & \frac{25}{714} rz + \frac{13}{595} \\ r^{\frac{5}{4}}\left(-1 - \frac{31}{288} z\right) & r^{\frac{5}{4}}\left(1 - \frac{13}{84} z\right) & \frac{25}{2016} r^{\frac{5}{4}} z \\ r^{\frac{8}{5}}\left(\frac{539}{325} + \frac{147}{650} z\right) & -\frac{864}{325} r^{\frac{8}{5}} & r^{\frac{8}{5}}\left(1 - \frac{21}{130} z\right) \end{pmatrix}, \tag{3.8}$$

The determinant of(3.8) gives the stability polynomial;

$$\begin{aligned} \pi(r, z) = &-\frac{13}{595} r^{\frac{77}{20}} + \frac{143}{5100} z r^{\frac{77}{20}} - \frac{1261}{71400} z^2 r^{\frac{77}{20}} + \frac{13}{1785} z^3 r^{\frac{77}{20}} + \frac{13}{595} r^{\frac{57}{20}} \\ &+ \frac{247}{35700} z r^{\frac{57}{20}} + \frac{13}{17000} z^2 r^{\frac{57}{20}} \end{aligned} \tag{3.9}$$

and the plot of (3.9) using MATLAB software version R2018a gives;

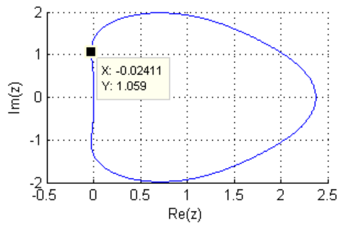

Figure 1: **Plot of stability polynomial (3.9)**

4. Results and Discussion

From Figure 1, it can be obtained that the derived block method (2.7) is $A(\alpha)$ stable with $\alpha^*_{max} = 88.7^0$. Hence implementation of (2.7) on (1.3) gives results which are compared with results from Matlab ode23s and are shown in Figure 2 with α in (1.3) set equal to 0.5.

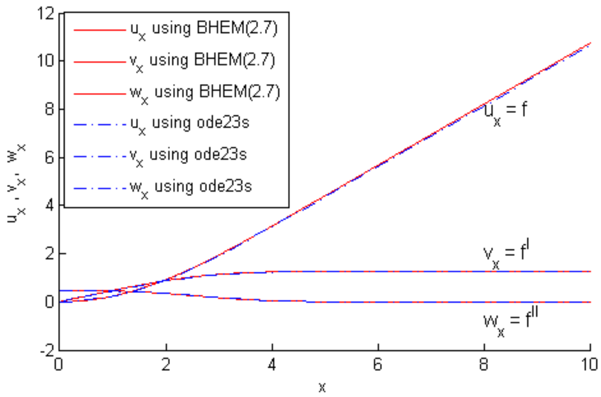

Figure 2: **Comparison of BHEM (2.7) with Matlab ode23s for numerical simulation of (1.3)**

From Figure 2, it can be observed that the newly derived block hybrid method (2.7) compete with Matlab odes23s well. This shows that despite using fixed step size of (2.7) compared to variable step size of Matlab ode23s, the close matching of the results is an indication that the newly derived block hybrid method performed well for the solutions of Blasius problem. Also, observation that the newly derived block hybrid method (2.7) performed well prompted its use for numerical simulation of (1.3) using various values of parameters α and results are displayed in Figures 3, 4 and 5.

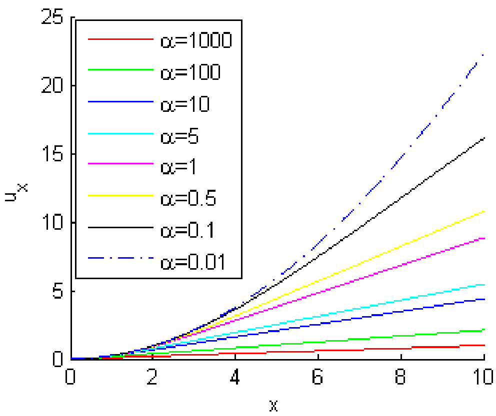

Figure 3: **Results showing changes in $u_x = y$ using (2.7) for different α**

In Figure 3, there is consistent increase in velocity profile u_x for each α along non dimensional x-axis. However, it can be observed that increase in α brings about decrease in the profile of $u_x = y$ along the x-axis and vice versa. For instance observe that $\alpha = 0.01$ induced higher velocity profile $u_x = y$ at a faster rate than for $\alpha = 1000$. Analogously, results from numerical simulation of (1.3) for $v_x = y'_x$ for various values of α are displayed in Figure 4.

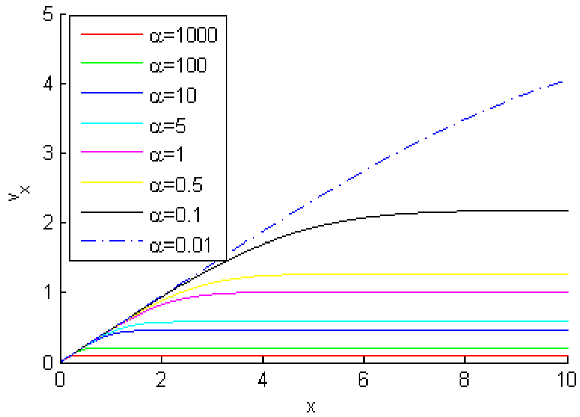

Figure 4: **Results showing changes in $v_x = y'$ using (2.7) for different α .**

For each value of α in Figure 4, the $v_x = y'_x$ attain equilibrium at some points as x tends to infinity. When results in Figure 4 is contrasted with that of Figure 3 it can be observed that $u_x = y$ in Figure 3 does not tend to equilibrium for each value of α. Essentially, the behavior of v_x relative to α is similar to the behavior of u_x relative to α because lower values of α induced higher value of $v_x = y'_x$ along the non dimensional x-axis for both Figures 3 and 4.
While results for Figure 5 tend to equilibrium for $w_x = y''_{xx}$, it can be observed that the higher values of α induced lower values of $w_x = y''_{xx}$ along non dimensional x-axis.

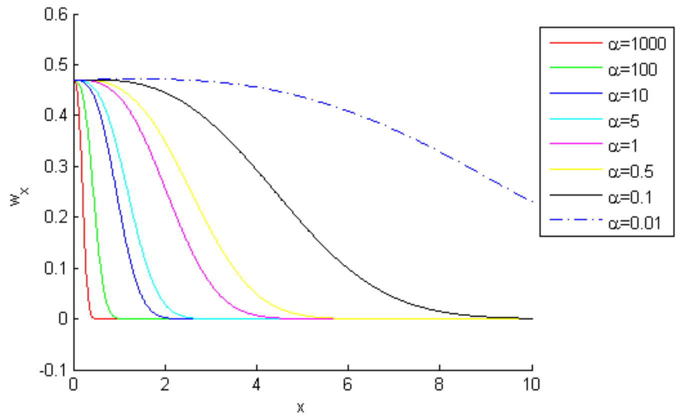

Figure 5: **Results showing changes in $w_x = y''_{xx}$ using (2.7) for different α .**

When solving (1.3) for initial conditions $u(0) = 0, v(0) = 0$ and $w(0) = 0.46957$ the α has to be set equal to 1 so that boundary condition $\lim_{x\to\infty} y'(x) = v_x = 1$ prescribed in (1.2) will be satisfied as shown in Figure 4. Some studies have used initial conditions different from the one used in this research ([10], [38]). However, obtaining initial condition $y''(0) = w(0)$ for (1.3) has been a challenging task and some authors have used initial guess to arrive at a value which satisfies the boundary value $\lim_{x\to\infty} y'(x) = v_x = 1$ in (1.2) ([10], [22]).

The infinite boundary condition $y'(\infty) = 1$ is handled numerically by replacing the semiinfinite interval $[0, \infty)$ by a finite interval $[0, X_{max}]$. The X_{max} is chosen large enough so that $v(x)$ essentially reached its asymptotic value 1. From Figures 2, 3, 4 and 5, domain length $X_{max} = 10$ is used, the plots extend to $x = 10$. Using fixed step size: $h = 0.1$, number of steps (subintervals): $N = \frac{X_{max}}{h} = \frac{10}{0.1} = 100$; smaller step size $h = 0.05$ can also be used with more computational efforts where $N = 200$.

This study considers effects of initial conditions on dynamics of the system (1.3) by making arbitrary choices of $\alpha = 0.1$ and $\alpha = 100$, the results are shown in Figures 6, 7, 8, 9, 10 and 11.

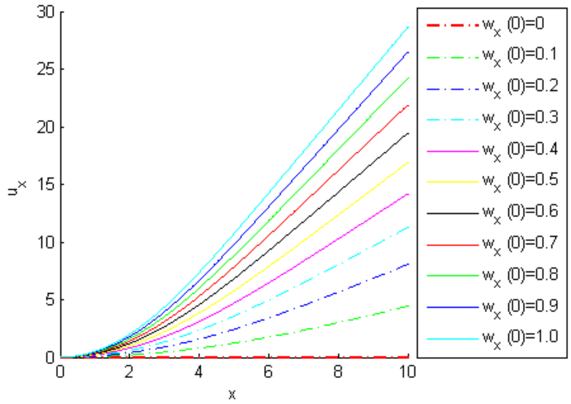

Figure 6: **Changes in $u_x = y$ with $\alpha = 0.1$ for different initial conditions of w_x using (2.7).**

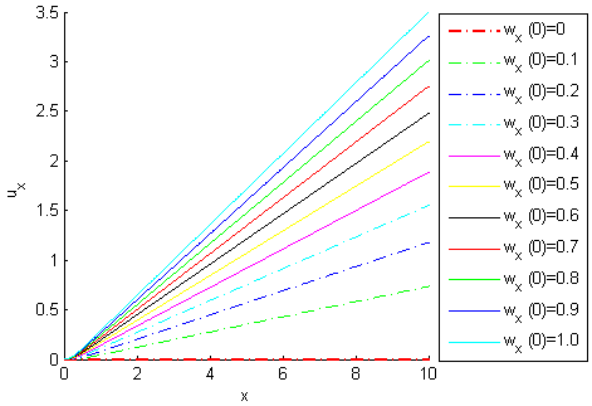

Figure 7: **Changes in $u_x = y$ with $\alpha = 100$ for different initial conditions of w_x using (2.7).**

For $\alpha = 0.1$, the u_x showed a curved profile in Figure 6 and for $\alpha = 100$, the u_x showed a straightened profile in Figure 7 for different initial conditions of w_x . It can be seen that highest initial condition brought about highest profile of u_x . Also, profiles of v_x are shown in Figures 8 and 9 for different values of α and different initial conditions of w_x.

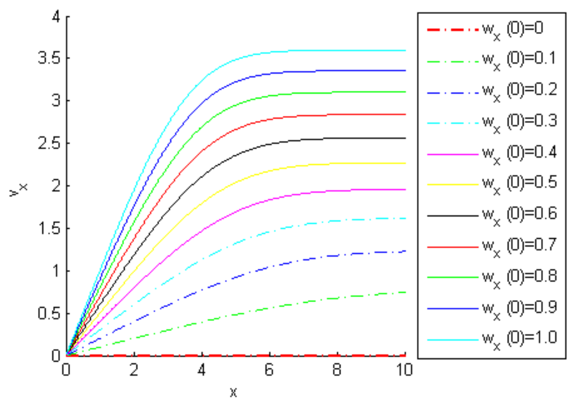

Figure 8: **Changes in $v_x = y'_x$ with $\alpha = 0.1$ for different initial conditions of w_x using (2.7).**

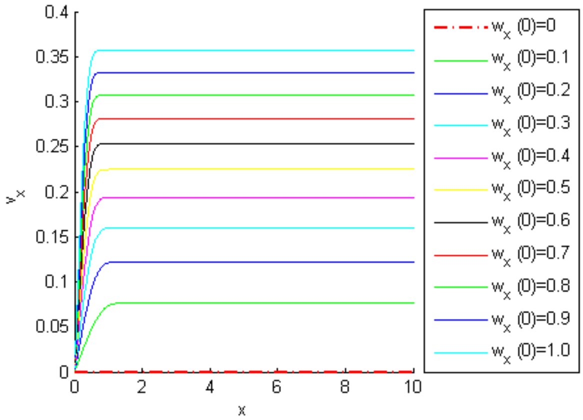

Figure 9: **Changes in $v_x = y'_x$ with $\alpha = 100$ for different initial conditions of w_x using (2.7).**

Numerical simulation of (1.3) for $v_x = y'_x$ showed that for $\alpha = 0.1$, equilibrium can be attained at a slower rate as depicted in Figure 8 compared to attainment of equilibrium at a faster rate for $\alpha = 100$ shown in Figure 9. There is direct proportionality between v_x and each initial condition of w_x. This means that high value of initial condition of w_x brings high value of $v_x = y'_x$. Effects of different initial conditions on dynamics of $w_x = y''_{xx}$ in system (1.3) for $\alpha = 0.1$ and $\alpha = 100$ are also shown in Figures 10 and 11.

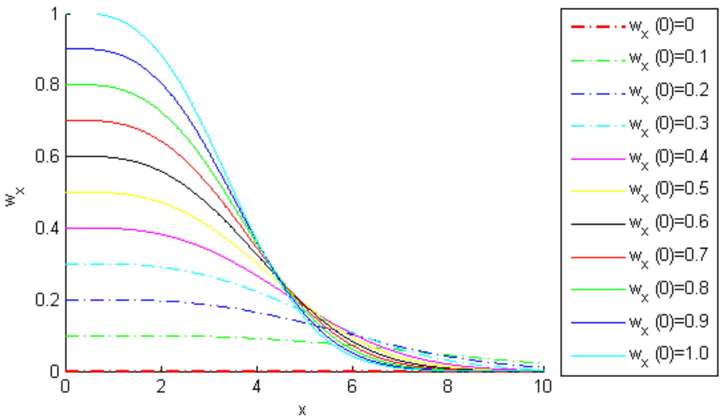

Figure 10: **: Changes in $w_x = y''_{xx}$ with $\alpha = 0.1$ for different initial conditions w_x of (1.3) using (2.7).**

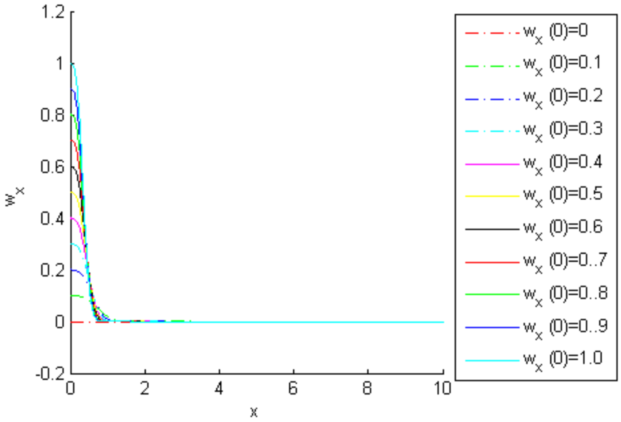

Figure 11: **Changes in $w_x = y''_{xx}$ with $\alpha = 100$ for different initial conditions w_x of (1.3) using (2.7).**

It can be observed that the $w_x = y''_{xx}$ attained a steady state at much slower rate when $\alpha = 0.1$ in Figure 10 than when $\alpha = 100$ in Figure 11 for each initial condition. Hence there is direct proportionality between $w_x = y''_{xx}$ and each initial condition.

5. Conclusion

In this study, a new block hybrid method with an off-step interpolation point and an off-step collocation point was derived. It was shown to be zero stable and consistent hence a convergent method. To display efficiency of its performance, the block hybrid explicit method was applied to initial value problem representing classical Blasius flow and results were graphically compared with results obtained from Matlab ode23s. It was confirmed that the newly derived block hybrid explicit method compete well with Matlab ode23s. Further investigations of the classical Blasius equation were carried out for different values of parameter α and different initial conditions using the derived block hybrid method. Hence the derived block hybrid method was found to be suitable for solving classical Blasius problem in fluid dynamics. Consequently, it can be applied to solve other similar problems in fluid dynamics.

References

[1] Abbasbandy, S. , (2007). A numerical solution of Blasius equation by Adomians decomposition method and comparison with homotopy perturbation method, Chaos, Soliton and Fractals, 31(1) , 257-260. 1

[2] Ahmad, I., Bilal, M. , (2014). Numerical Solution of Blasius Equation through Neural Networks Algorithm, American J. Compu. Math. 4 , 223-232. 1

[3] Ahmad, I., Bilal, M. , (2017). A novel method for the solution of Blasius equation in semi-infinite domains, An Int. J. of Opt. and Control:Theories and Apps. 7(2) , 225-233. 1

[4] Akinfenwa, O.A., Jator, S.N., Yao, N.M. , (2012). On the stability of continuous block backward differentiation formula for solving stiff ordinary differential equations, J. Modern Meth. Num. Meth., 3(2) , 50-58. 1, 3.1

[5] Al-Khaled, K.,Ajeel, M. S., Darweesh, A., & Al-Khalid, H., (2024). A reliable algorithm for solving Blasius boundary value problem, Results in Nonlinear Anal., 7(4), 1-8. 1

[6] Blasius, P. H.(1908). Grenzschichten in Flussigkeiten mit kleiner Reibung. Zeitschrift fur Mathematik und Physik, 56(1). 1, 1

[7] Cebeci, T., Keller, H.B., (1971). Shooting and parallel shooting methods for solving the Falkner-Skan boundary-layer equation, J. Comput. Phy., 7(2) , 289-300. 1

[8] Celaya, E. A., Anza, J. J. (2013). BDF-α: A multistep method with numerical damping control. Universal Journal of Computational Mathematics, 1(3), 96-108. 1

[9] Chu, M.T., Hamilton, H. , (1987). Parallel solution of ODEs by multi-block methods, Siam J. Sci. Stat. Comput, 8(3) , 342-353. 1, 3.1

[10] Cortell, R., (2005). Numerical solutions of the classical Blasius flat-plate problem, Appl. Math. Comput. 170 , 706-710. 1, 4

[11] Fatunla, S.O., (1995). A Class of Block Methods for Second Order IVPs, International J. Comput. Math. 55 , 119-133. 3.1

[12] Fatunla, S.O., (1991). Block Methods for Second Order ODEs, Int. J. Compu. Math. 41 , 55-63. 3.1

[13] Ganji, D.D., Babazadeh, H., Noori, F., Pirouz, M.M., Janipour, M., (2009). An application of homotopy perturbation method for nonlinear blasius equation to boundary layer flow over a flat plate, International Journal of Nonlinear Science. 7 , 399-404. 1

[14] He, J.H., (1997). A new approach to nonlinear partial differential equations, Comm. Nonlinear Sci. Nume. Simulation. 2 , 230-235. 1

[15] Henrici, P., (1962). Discrete variable methods in ODEs, John Wiley, New York. 3.1

[16] Howarth, L., (1938). On the solution of the laminar boundary layer equations, Proc. London Math. Soc. 2 , 547-579. 1

[17] Ibrahim, Z.B., Othman, K.I., Suleiman, M., (2007). Implicit r-point block backward differentiation formula for solving first-order stiff ODEs, Appl. Math. Comput. 186 , 558-565. 1

[18] , M.K., Molla, M.R., Sultana, S., (2011). Numerical approximations of Blasius boundary layer equation, Dhaka University J. Sci. 59 , 87-90. 1

[19] Jator, S.N., Sahi, R., Akinyemi, M., Nyonna, D., (2021). Exponentially fitted block backward differentiation formulas for pricing options, Cogent Economics & Fin. 9(1), 1875565. 1

[20] Liao, S.J., (1999). An explicit, totally analytic approximate solution for Blasius viscous flow problems, Int. J. Non-Linear Mech. 34 , 759-778. 1

[21] Liu, Y., Kurra, S.N., (2012). Solving Blasius equation using HVIM, World Acad. Sci. 65, 578-581. 1

[22] Miansari, M.O., Miansari, M.E., Barari, A., Domairry, G., (2010). Analysis of Blasius Equation for Flat-plate Flow with Infinite Boundary Value, Int. J. Comput. Meth. Engnr. Sci. & Mech. 11(2) , 79-84. 1, 1, 4

[23] Okuonghae, R.I., Ikhile, M.N.O., (2010). Stiffly stable second derivative linear multistep methods with two hybrid points, Num. Analysis & Appl. 8, 248-259. 3.1

[24] Okuonghae, R.I., Ikhile, M.N.O., (2013). A class of hybrid linear multistep methods with $A(\alpha)$-stability properties for stiff IVPs in ODEs, J. Num. Math. 21(2), 157-172, doi: 10.1515/jnum-2013-0006.

[25] Okuonghae, R. I., & Ozobokeme, J. K., (2024). Falkner Hybrid Block Methods for Second-Order IVPs: A Novel Approach to Enhancing Accuracy and Stability Properties, J. Numer. Anal. Approx. Theory, 53(2), 324-342. 1

[26] Onumanyi, P., Sirisena, U.W., Jator, S.N., (1999). Continuous finite difference approximations for solving differential equations, Int. J. Comput. Math. 72(1) , 15-27. 2, 2

[27] Parand, K., Taghavi, A., (2009). Rational scaled generalized Laguerre function collocation method for solving the Blasius equation, J. Comput. & Appl. Math. 233(4) , 980-989. 1

[28] Rahman, M.M., (2018). The Comparison between analytical and numerical solution of Blasius equation on boundary layer parallel flow over a flat plate, J. Sci. Engg. Res. 6(2), 1-12. 1

[29] Rahman, M.M., Khan, S., Akbar, M.A., (2018). Numerical and analytical solutions of new Blasius equation for turbulent flow, Heliyon. 9(2), e14319, 1-13. 1

[30] Selamat, M.S., Halmi, N.A., Ayob, N.A., (2019). A semi analytic iterative method for solving two forms of Blasius equation, J. Academia. 7(2) , 76-85. 1

[31] Shateyi, S., (2023). On the application of the block hybrid methods to solve linear and non-linear first order differential equations, Axioms. 12(2), 189. 1

[32] Suleiman, M., Musa, H., Ismail, F., Senu, N., (2013). A new variable step size block backward differentiation formula for solving stiff initial value problems, Int. J. Comput. Math. 90, 2391-2408. 1

[33] Voss, D., Abbas, S., (1997). Block predictorcorrector schemes for the parallel solution of ODEs, Comput. & Math. Appl. 33(6) , 65-72. 1

[34] Wang, L., (2004). A new algorithm for solving classical Blasius equation, Appl. Math. & Comput. 157(1) , 1-9. 1

[35] Wang, L., (1991). Exact solutions of the steady-state Navier-Stokes equations, Ann. Review. Fluid Mech. 23(1) , 159-177. 1

[36] Weyl, H., (1942). On the differential equations of the simplest boundary-layer problems, Annal. Math. 43 , 381-407. 1

[37] White, F.M., (1991). Viscous Fluid Flow. McGraw-Hill, New York. 1

[38] Xu, D., Xu, J., Xie, G., (2014). Revisiting Blasius flow by fixed point method, Abstract & Appl. Analysis, Article ID 953151, 1-9. 4

[39] Yasuri, A.K., (2023). An analytical self-consistent method for differential forms of the Blasius equation, Math. Meth. Appl. Sci. 46, 5836-5849. 1

[40] Yao, B., Chen, J., (2009). A new analytical solution branch for the Blasius equation with a shrinking sheet, Appl. Math. & Comput. 215(3) , 1146-1153. 1