Math. & Comput. Sci., 7(3) (2026), 130–141

Online: ISSN 2717-2708

Mathematics and Computational Sciences

Journal Homepage: http://mcs.qut.ac.ir/mcs

Expansion formulas for q-Hypergeometric and I_q-functions: An application of the q-Leibniz rule

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Abstract

In this paper, we develop novel expansion formulas for the product of the basic analogue of the hypergeometric function and the I_q-function. Applying the q-Leibniz rule to the q-Riemann-Liouville and q-Weyl fractional derivative operators, we have found that these formulas can be derived more efficiently, potentially leading to new avenues of research within q-calculus and mathematical analysis. Furthermore, this study delves into specific cases of our main result, thereby elucidating the importance and utility of these derived expressions within a broader mathematical context.

Keywords: q-hypergeometric function, I_q-function, q-Fox's H-function, q-Riemann-Liouville derivative, q-Weyl derivative, q-Leibniz rule.

2020 MSC: 33D15, 33C60, 45P05.

1. Introduction and Preliminaries

The study of special functions has long been a foundation of mathematical analysis, with numerous applications in physics, engineering, and computer mathematics. The hypergeometric function and I_q-function are especially important in the setting of q-calculus. q-calculus, a generalization of classical calculus, has gained popularity in recent decades due to its capacity to find unique solutions to hard problems, particularly in fractional calculus and non-integer order derivatives. Despite its increasing popularity, many elements of q-calculus remain unexplored, particularly the efficient manipulation and application of q-special functions.

In this study, we want to close this gap by creating new expansion formulas for the product of the fundamental counterpart of the hypergeometric function and the I_q-function. The impetus for this study stems from the need to increase the computing efficiency and application of these functions in a range of settings. By applying the q-Leibniz rule to the q-Riemann-Liouville and q-Weyl fractional derivative operators, we demonstrate that these expansion formulae may be generated more efficiently, making them more useful in both theoretical and applied mathematics issues.

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doi: 10.30511/mcs.2026.2068119.1434

Received: 08 August 2025 Accepted: 02 June 2026

The consequences of this work are tremendous. Not only do the derived formulas provide new tools for working with q-special functions, but they also point to new directions for future research in q-calculus and fractional derivative theory. By going into specific examples of the primary conclusion, we hope to highlight the broader utility and significance of these expressions, shining light on their applicability in diverse domains of mathematical research. Thus, this study contributes to the theoretical foundation of q-calculus while also providing practical methods for handling complex fractional problems in mathematics and related disciplines.

Jackson [13] pioneered the development of q-calculus. As this field has evolved, various mathematical functions have been recently introduced, including the q-Mittag-Leffler function [7, 8], q analogue of Bessel function [5], q analogue of M-function [23, 24], q-hypergeometric series [28, 29], q-gamma function, and several q-calculus operators and inequalities (see, [4, 9, 19, 20, 25]). The wide-ranging applications of q-calculus in statistics, applied mathematics, and physics, have attracted the attention of numerous researchers (see, for example, [10]). As outlined q-fractional calculus represents a significant and novel generalization of traditional fractional calculus. Several recent works have been published on q-calculus and the properties and applications of special functions (see [2, 12, 14, 17, 26])

We first give some definitions and notations. For a real or complex number h and $|q| < 1$, the q-shifted factorial in q-series theory (see [6]) is defined as follows:

$$(h;q)_k \equiv (q^h;q)_k = \begin{cases} 1 & (k=0), \\ \prod_{s=0}^{k-1}(1-hq^s) & (k \in \mathbb{N}), \end{cases} \tag{1.1}$$

here, $h, q \in \mathbb{C}$ and $h \neq q^{-l} (l \in \mathbb{N}_0)$ and treating the parameter h as an exponential q-index, essentially defining the symbol's input as q^h.

Then, (1.1) can be expressed in terms of q-gamma function as follows

$$(h;q)_k = \frac{\Gamma_q(h+k)(1-q)^k}{\Gamma_q(h)}, \tag{1.2}$$

where the q-gamma function is defined as [11]:

$$\Gamma_q(h) = \frac{(q;q)_\infty}{(q^h;q)_\infty (1-q)^{h-1}}. \tag{1.3}$$

The definition of q-shifted factorial via negative subscript for $k \in \mathbb{N}_0$ is as follows.

$$(h;q)_{-k} = \frac{1}{(1-hq^{-1})(1-hq^{-2})(1-hq^{-3})\cdots(1-hq^{-k})}, \tag{1.4}$$

which yields

$$(h;q)_{-k} = \frac{1}{(hq^{-k};q)_k} = \frac{(-q/h)^k q^{\frac{k(k+1)}{2}}}{(q/h;q)_k}. \tag{1.5}$$

We also write that

$$(h;q)_\infty = \prod_{s=0}^{\infty}(1-hq^s), \quad (h,q \in \mathbb{C}). \tag{1.6}$$

From (1.1), (1.4) and (1.5), we can stated that

$$(h;q)_k = \frac{(h;q)_\infty}{(hq^k;q)_\infty}, \qquad (k \in \mathbb{Z}). \tag{1.7}$$

The Reimann-Liouville type fractional q-derivative introduced in [1] and given as follows:

$$D^u_{y,q}\{f(y)\} = \frac{1}{\Gamma_q(-u)} \int_0^y (y-tq)_{-u-1} f(t)\, d_q t, \tag{1.8}$$

The above equation (1.8), exist $\forall$ values of u.
From [11], the q-integral of a function $T(z)$ is defined as:

$$\int_0^b T(z)d_q z = b(1-q)\sum_{l=0}^{\infty} q^l T(zq^l), \tag{1.9}$$

$$\int_b^{\infty} T(z)d_q z = b(1-q)\sum_{l=0}^{\infty} q^{-l} T(zq^{-l}), \tag{1.10}$$

$$\int_0^{\infty} T(z)d_q z = (1-q)\sum_{l=-\infty}^{\infty} q^l T(zq^l). \tag{1.11}$$

From the equation (1.9), the above operator (1.8) can be stated as:

$$D^u_{y,q}\{f(y)\} = \frac{y^{-u}(1-q)}{\Gamma_q(-u)}\sum_{l=0}^{\infty} q^l(1-q^{l+1})_{-u-1} f(yq^l). \tag{1.12}$$

For $f(y) = y^{\eta-1}$, the above equation yields to

$$D^u_{y,q}\left\{y^{\eta-1}\right\} = \frac{\Gamma_q(\eta)}{\Gamma_q(\eta-u)} y^{\eta-u-1}, \tag{1.13}$$

where, $\Re(\eta) > 0$, and exist $\forall$ values of u.

The q-extension of the Leibniz formula involving type of Riemann-Liouville fractional q-derivative is defined as follows by [1]:

$$D^u_{y,q}\{W_1(y)W_2(y)\} = \sum_{r=0}^{\infty} \frac{(-1)^r q^{r(r+1)/2}(q^{-u};q)_r}{(q;q)_r} D^{u-r}_{y,q}\{W_1(yq^r)\} D^r_{y,q}\{W_2(y)\}, \tag{1.14}$$

where, $W_1(y)$ and $W_2(y)$ are two regular functions with power series representation

$$W_1(y) = \sum_{s=0}^{\infty} a_s y^s,\ |y| < S_1;\ W_2(y) = \sum_{s=0}^{\infty} b_s y^s,\ |y| < S_2,$$

then for above result (1.14), $|y| < S = \min(S_1, S_2)$.
The q-Weyl fractional derivative operator introduced by [6] is as follows:

$${}_yD^u_{\infty,q}\{f(y)\} = \frac{q^{-u(1+u)/2}}{\Gamma_q(-u)}\int_y^{\infty}(t-y)_{-u-1} f(tq^{1+u})d_q t, \tag{1.15}$$

where, $\Re(u) < 0$ and from the equation (1.10), the above operator (1.15), can be stated as follows:

$${}_yD^u_{\infty,q}\{f(y)\} = \frac{q^{u(1-u)/2}y^{-u}(1-q)}{\Gamma_q(-u)}\sum_{l=0}^{\infty} q^{ul}(1-q^{l+1})_{-u-1} f(yq^{u-l}) \tag{1.16}$$

where, $\Re(u) < 0$.
For, $f(y) = y^{-a}$, the above equation reduces to

$${}_yD^u_{\infty,q}\left\{y^{-a}\right\} = \frac{\Gamma_q(a+u)}{\Gamma_q(a)} q^{-ua+u(1-u)/2} y^{-a-u}. \tag{1.17}$$

The q-extension of the Leibniz formula involving type of Weyl fractional q-derivatives is defined by [18] as follows:

$$ {}_yD^{u}_{\infty,q}\{W_1(y)W_2(y)\} = \sum_{r=0}^{u} \frac{(-1)^r q^{r(r+1)/2}(q^{-u};q)_r}{(q;q)_r} \times {}_yD^{u-r}_{\infty,q}\{W_1(y)\}\,{}_yD^{r}_{\infty,q}\left\{W_2(yq^{u-r})\right\}. \tag{1.18} $$

To, drive our main results, we need following q-analogue of Special functions.

Generalized q-hypergeometric abnormal type series ${}_r\phi_s$ is defined in [27] as follows:

$$ {}_r\phi_s\left[\begin{array}{c} c_1,\cdots,c_r;q;z \\ d_1,\cdots,d_s;q^{\sigma} \end{array}\right] = \sum_{m=0}^{\infty} \frac{(c_1,\cdots,c_r;q)_m}{(q,d_1,\cdots,d_s;q)_m} z^m q^{\sigma m(m+1)/2}, \tag{1.19} $$

where, $\sigma > 0$.
For $\sigma = 0$, the above series (1.19), becomes the generalized q-hypergeometric series ${}_r\phi_s$ as in [27]:

$$ {}_r\phi_s\left[\begin{array}{c} c_1,\cdots,c_r; \\ d_1,\cdots,d_s; \end{array} q,z\right] = \sum_{m=0}^{\infty} \frac{(c_1,\cdots,c_r;q)_m}{(q,d_1,\cdots,d_s;q)_m} z^m, \tag{1.20} $$

[27] provides information on the convergence conditions of series (1.20).
As indicated in paper of [22], for $0 \leqslant \mathfrak{I} \leqslant Q_i, 0 \leqslant \wp \leqslant P_i, i = 1,2,...r, r$ is finite; $w = \sqrt{-1}$ and a_j, b_j, a_{ji}, b_{ji} are complex numbers; $\vartheta_j, \xi_j, \vartheta_{ji}, \xi_{ji}$ are real and positive, the q-analogue of I-function (called I_q-function) in term of Mellin-Barnes type basic contour integral is defined as:

$$ I^{\mathfrak{I},\wp}_{P_i,Q_i}\left[x;q \left| \begin{array}{c} (a_j,\vartheta_j)_{1,\wp}\,,\,(a_{ji},\vartheta_{ji})_{\wp+1,P_i} \\ (b_j,\xi_j)_{1,\mathfrak{I}}\,,\,(b_{ji},\xi_{ji})_{\mathfrak{I}+1,Q_i} \end{array}\right.\right] = \frac{1}{2\pi w}\int_C \psi(s;q)x^s d_q s, \tag{1.21} $$

where

$$ \psi(s;q) = \frac{\left\{\prod_{j=1}^{\mathfrak{I}} G(q^{b_j-\xi_j s})\right\}\left\{\prod_{j=1}^{\wp} G(q^{1-a_j+\vartheta_j s})\vartheta\right\}\pi}{\sum_{i=1}^{r}\left\{\prod_{j=\mathfrak{I}+1}^{Q_i} G(q^{1-b_{ji}+\xi_{ji}s})\right\}\left\{\prod_{j=\wp+1}^{P_i} G(q^{a_{ji}-\vartheta_{ji}s})\right\} G(q^{1-s})\sin\pi s}, \tag{1.22} $$

and

$$ G(q^a) = \left\{\prod_{\varepsilon=0}^{\infty}(1-q^{a+\varepsilon})\right\}^{-1} = \frac{1}{(q^a;q)_\infty}. \tag{1.23} $$

The contour C extends from $-w\infty$ to $w\infty$ with the poles of $G(q^{1-a_j+\vartheta_j s}), 1 \leqslant j \leqslant \wp$ to the left, and those $G(q^{b_j-\xi_j s}), 1 \leqslant j \leqslant \mathfrak{I}$ are to the right of C. The integral converges for large values of $|s|$, if $\Re[s\log(x) - \log\sin\pi s] < 0$ on the contour C, that means if $|\arg(x)| < \pi$. It is possible to see that the integration contour C can be substituted by other appropriately indented contours parallel to the imaginary axis.

Interestingly, for $r = 1$, $P_1 = P$; $Q_1 = Q$; hence, (1.21) gives the q-analogue of the H-function attributed to [21], exactly.

$$ H^{\mathfrak{I},\wp}_{P,Q}\left[x;q \left| \begin{array}{c} (a_1,\vartheta_1),...,(a_P,\vartheta_P) \\ (b_1,\xi_1),...,(b_Q,\xi_Q) \end{array}\right.\right] = \frac{1}{2\pi w}\int_C \psi^{\star}(s;q)x^s d_q s, \tag{1.24} $$

where

$$ \psi^{\star}(s;q) = \frac{\left\{\prod_{j=1}^{\mathfrak{I}} G(q^{b_j-\xi_j s})\right\}\left\{\prod_{j=1}^{\wp} G(q^{1-a_j+\vartheta_j s})\right\}\pi}{\left\{\prod_{j=\mathfrak{I}+1}^{Q} G(q^{1-b_j+\xi_j s})\right\}\left\{\prod_{j=\wp+1}^{P} G(q^{a_j-\vartheta_j s})\right\} G(q^{1-s})\sin\pi s}. \tag{1.25} $$

where $G(.)$ is given in (1.23), $\vartheta_i's$ and $\xi_j's$ are all positive integers and $0 \leqslant \mathfrak{I} \leqslant Q, 0 \leqslant \wp \leqslant P$. The contour C is a line parallel to $\Re(ws) = 0$ with any required indentations placed so that the poles of both $G(q^{b_j-\xi_j s}), 1 \leqslant j \leqslant \mathfrak{I}$ and $G(q^{1-a_j+\vartheta_j s}), 1 \leqslant j \leqslant \wp$ are to the left and right of C, respectively. The integral converges for large values of $|s|$, if $\Re[s\log(x) - \log \sin\pi s] < 0$ on the contour C, i.e. if $\left|\arg(x) - w_2 w_1^{-1} \log|x|\right| < \pi$, where $\log q = -w = -(w_1 + iw_2), w_1$ and w_2 being real and $0 < |q| < 1$.

Further, $r = 1$, $P_1 = P; Q_1 = Q; \vartheta_j = \xi_j = 1$, $j = 1, ...P; i = 1, ...Q$, (1.21) relates to the q-analogue of Meijer's G-function according to [21], precisely

$$G^{\mathfrak{I},\wp}_{P,Q}\left[x; q \,\middle|\, \begin{matrix} a_1, a_2, ..., a_P \\ b_1, b_2, ..., b_Q \end{matrix}\right] = \frac{1}{2\pi i}\int_C \psi^{\star\star}(s; q) x^s \, d_q s, \tag{1.26}$$

where

$$\psi^{\star\star}(s; q) = \frac{\left\{\prod_{j=1}^{\mathfrak{I}} G(q^{b_j - s})\right\}\left\{\prod_{j=1}^{\wp} G(q^{1-a_j+s})\right\}\pi}{\left\{\prod_{j=\mathfrak{I}+1}^{Q} G(q^{1-b_j+s})\right\}\left\{\prod_{j=\wp+1}^{P} G(q^{a_j - s})\right\} G(q^{1-s})\sin\pi s} \tag{1.27}$$

where $G(.)$ is given in (1.23).
Research monographs by [15, 16] provide a complete characterization of Fox's H-function, Meijer's G-function, and many functions accessible in terms of Fox's H-function. Furthermore, the articles of [28] and [29] also provide the basic functions of one variable defined in terms of the functions $G^{\mathfrak{I},\wp}_{P,Q}(.)$.
Our recent work applies the q-Leibniz rule to analyze functions with multiple variables, specifically focusing on the q-analogue of I-function, as defined by Saxena and Kumar [22]. We've discovered new transformations and expansions for basic hypergeometric functions. Building upon this, and similar to Agrawal et al.'s [3] use of the q-Leibniz rule to expand products of basic hypergeometric and Fox's H-functions, our results enhance the understanding of these mathematical functions and their related transformations.
Leveraging the q-Leibniz rule in conjunction with Riemann-Liouville and Weyl-type q-derivative operators, we aim to establish a novel q-expansion formula that incorporates the q-analogue of hypergeometric function and the q-analogue of the basic I-function, thus enriching the existing body of research.

2. Main Result

Now, use the Reimann-Liouville type fractional q-derivative to construct expansion formulas related to the product of I_q-function and q-analogue of generalized hypergeometric abnormal type function. This is achieved by substituting appropriate values to the functions $W_1(y)$ and $W_2(y)$ in the q-Leibniz rule.

Theorem 2.1. *The following expansion formulae pertaining to the product of I_q-function and q-analogue of generalized hypergeometric abnormal type function as under*

$$\begin{aligned}
&\sum_{m=0}^{\infty} \frac{(c_1, \cdots, c_r; q)_m}{(q, d_1, \cdots, d_s; q)_m} (\rho y)^m q^{\sigma m(m+1)/2} \\
&\times I^{\mathfrak{I}+1,\wp}_{P_i+1,Q_i+1}\left[\rho(yq^u)^k; q \,\middle|\, \begin{matrix} (1-\lambda-m, k), (a_j, \vartheta_j)_{1,\wp}, (a_{ji}, \vartheta_{ji})_{\wp+1,P_i} \\ (1-\lambda-m+u, k), (b_j, \xi_j)_{1,\mathfrak{I}}, (b_{ji}, \xi_{ji})_{\mathfrak{I}+1,Q_i} \end{matrix}\right] \\
= &\sum_{r=0}^{\infty} \frac{(q^{-u}; q)_r (q^{1-\lambda-r}; q)_u q^{(\lambda+u)r}}{(q; q)_r (q^\lambda; q)_r} {}_{r+1}\phi_{s+1}\left[\begin{matrix} c_1, \cdots, c_r, \lambda; q; \rho(yq^r) \\ d_1, \cdots, d_s, \lambda + r - u; q^\sigma \end{matrix}\right] \\
&\times I^{\mathfrak{I}+1,\wp}_{P_i+1,Q_i+1}\left[\rho(yq^r)^k; q \,\middle|\, \begin{matrix} (0, k), (a_j, \vartheta_j)_{1,\wp}, (a_{ji}, \vartheta_{ji})_{\wp+1,P_i} \\ (r, k), (b_j, \xi_j)_{1,\mathfrak{I}}, (b_{ji}, \xi_{ji})_{\mathfrak{I}+1,Q_i} \end{matrix}\right].
\end{aligned} \tag{2.1}$$

where $|\rho y^k| < |q|^{-\frac{1}{2k}(|\sigma - k^2| + \sum\vartheta_j - \sum\xi_j)}$ and $\Re\left(\sum_{j=1}^{\mathfrak{I}} \xi_j b_j - \sum_{j=1}^{\wp} \vartheta_j a_j + \frac{k}{2}\right) > 0$.

Proof. To prove the theorem, we apply the q-Leibniz rule stated in equation (1.14). By addressing the expressions obtained from both the right-hand and left-hand sides separately, we establish the desired result. To do this, start from equation (1.14), we obtain

$$D^{u}_{y,q}W_1(y)W_2(y)=\sum_{r=0}^{\infty}\frac{(-1)^r q^{r(r+1)/2}(q^{-u};q)_r}{(q;q)_r}D^{u-r}_{y,q}W_1(yq^r)D^{r}_{y,q}W_2(y). \tag{2.2}$$

To, prove our result, we begin with $W_1(y)=y^{\lambda-1}{}_r\phi_s\left[\begin{array}{c}c_1,\cdots,c_r;q;\rho y\\ d_1,\cdots,d_s;q^{\sigma}\end{array}\right]$ and

$$W_2(y)=I^{\mathfrak{I},\wp}_{P_i,Q_i}\left[\rho y^k;q\left|\begin{array}{c}(a_j,\vartheta_j)_{1,\wp},\,(a_{ji},\vartheta_{ji})_{\wp+1,P_i}\\ (b_j,\xi_j)_{1,\mathfrak{I}},\,(b_{ji},\xi_{ji})_{\mathfrak{I}+1,Q_i}\end{array}\right.\right]$$

in the above equation to obtain

$$\begin{aligned}&D^{u}_{y,q}\,y^{\lambda-1}{}_r\phi_s\left[\begin{array}{c}c_1,\cdots,c_r;q;\rho y\\ d_1,\cdots,d_s;q^{\sigma}\end{array}\right]I^{\mathfrak{I},\wp}_{P_i,Q_i}\left[\rho y^k;q\left|\begin{array}{c}(a_j,\vartheta_j)_{1,\wp},\,(a_{ji},\vartheta_{ji})_{\wp+1,P_i}\\ (b_j,\xi_j)_{1,\mathfrak{I}},\,(b_{ji},\xi_{ji})_{\mathfrak{I}+1,Q_i}\end{array}\right.\right]\\
&=\sum_{r=0}^{\infty}\frac{(-1)^r q^{r(r+1)/2}(q^{-u};q)_r}{(q;q)_r}D^{u-r}_{y,q}(yq^r)^{\lambda-1}{}_r\phi_s\left[\begin{array}{c}c_1,\cdots,c_r;q;\rho y\\ d_1,\cdots,d_s;q^{\sigma}\end{array}\right]\\
&\times D^{r}_{y,q}I^{\mathfrak{I},\wp}_{P_i,Q_i}\left[\rho y^k;q\left|\begin{array}{c}(a_j,\vartheta_j)_{1,\wp},\,(a_{ji},\vartheta_{ji})_{\wp+1,P_i}\\ (b_j,\xi_j)_{1,\mathfrak{I}},\,(b_{ji},\xi_{ji})_{\mathfrak{I}+1,Q_i}\end{array}\right.\right].\end{aligned} \tag{2.3}$$

Denote the left-hand of the above equation by L, then

$$L\equiv D^{u}_{y,q}\,y^{\lambda-1}{}_r\phi_s\left[\begin{array}{c}c_1,\cdots,c_r;q;\rho y\\ d_1,\cdots,d_s;q^{\sigma}\end{array}\right]I^{\mathfrak{I},\wp}_{P_i,Q_i}\left[\rho y^k;q\left|\begin{array}{c}(a_j,\vartheta_j)_{1,\wp},\,(a_{ji},\vartheta_{ji})_{\wp+1,P_i}\\ (b_j,\xi_j)_{1,\mathfrak{I}},\,(b_{ji},\xi_{ji})_{\mathfrak{I}+1,Q_i}\end{array}\right.\right]. \tag{2.4}$$

Then view of definitions (1.19), (1.21), the above equation becomes

$$\begin{aligned}L=&\sum_{m=0}^{\infty}\frac{(c_1,\cdots,c_r;q)_m}{(q,d_1,\cdots,d_s;q)_m}\rho^m q^{\sigma m(m+1)/2}\frac{1}{2\pi w}\int_C\\
&\times\frac{\left\{\prod_{j=1}^{\mathfrak{I}}G(q^{b_j-\xi_j s})\right\}\left\{\prod_{j=1}^{\wp}G(q^{1-a_j+\vartheta_j s})\vartheta\right\}\pi}{\sum_{i=1}^{r}\left\{\prod_{j=\mathfrak{I}+1}^{Q_i}G(q^{1-b_{ji}+\xi_{ji}s})\right\}\left\{\prod_{j=\wp+1}^{P_i}G(q^{a_{ji}-\vartheta_{ji}s})\right\}G(q^{1-s})\sin\pi s}\\
&\times D^{u}_{y,q}y^{\lambda+m+ks-1}d_q s.\end{aligned} \tag{2.5}$$

On using the equation (1.13), we get

$$D^{u}_{y,q}\,y^{\lambda+m+ks-1}=\frac{\Gamma_q(\lambda+m+ks)}{\Gamma_q(\lambda+m+ks-u)}y^{\lambda+m+ks-u-1}. \tag{2.6}$$

Then using the identities (1.2),(1.5),(1.7) and (1.23), we have

$$D^{u}_{y,q}\,y^{\lambda+m+ks-1}=\frac{G(q^{1-\lambda-m+u-ks})}{(-1)^u q^{(1-\lambda-m-ks)u}q^{u(u-1)/2}(1-q)^u G(q^{1-\lambda-m-ks})}y^{\lambda+m+ks-u-1}. \tag{2.7}$$

Then substitute value of (2.7) in the equation (2.5), we have

$$L = \sum_{m=0}^{\infty} \frac{(c_1, \cdots, c_r; q)_m}{(q, d_1, \cdots, d_s; q)_m} \rho^m q^{\sigma m(m+1)/2} \frac{1}{2\pi i} \int_{\ell}$$

$$\times \frac{\left\{\prod_{j=1}^{\mathfrak{I}} G(q^{b_j - \xi_j s})\right\} \left\{\prod_{j=1}^{\wp} G(q^{1-a_j+\vartheta_j s})\vartheta\right\} \pi}{\sum_{i=1}^{r} \left\{\prod_{j=\mathfrak{I}+1}^{Q_i} G(q^{1-b_{ji}+\xi_{ji} s})\right\} \left\{\prod_{j=\wp+1}^{P_i} G(q^{a_{ji}-\vartheta_{ji} s})\right\} G(q^{1-s}) \sin \pi s}$$

$$\times \frac{G(q^{1-\lambda-m+u-ks})}{(-1)^u q^{(1-\lambda-m-ks)u} q^{u(u-1)/2} (1-q)^u G(q^{1-\lambda-m-ks})} y^{\lambda+m+ks-u-1} d_q s. \tag{2.8}$$

On re-arranging the terms, we get

$$L = \frac{(-1)^{-u} y^{\lambda-u-1} q^{(\lambda-1)u} q^{u(1-u)/2}}{(1-q)^u} \sum_{m=0}^{\infty} \frac{(c_1, \cdots, c_r; q)_m}{(q, d_1, \cdots, d_s; q)_m} \rho^m y^m q^{\sigma m(m+1)/2}$$

$$\times \frac{1}{2\pi i} \int_{\ell} \frac{\left\{\prod_{j=1}^{\mathfrak{I}} G(q^{b_j - \xi_j s})\right\} \left\{\prod_{j=1}^{\wp} G(q^{1-a_j+\vartheta_j s})\vartheta\right\} \pi}{\sum_{i=1}^{r} \left\{\prod_{j=\mathfrak{I}+1}^{Q_i} G(q^{1-b_{ji}+\xi_{ji} s})\right\} \left\{\prod_{j=\wp+1}^{P_i} G(q^{a_{ji}-\vartheta_{ji} s})\right\}}$$

$$\times \frac{G(q^{1-\lambda-m+u-ks})}{G(q^{1-s}) \sin \pi s \, q^{(-m-ks)u} G(q^{1-\lambda-m-ks})} y^{ks} d_q s.$$

Following simplification and rewriting in terms of the I_q-function, it reduces to:

$$L = \frac{(-1)^{-u} y^{\lambda-u-1} q^{(\lambda-1)u} q^{u(1-u)/2}}{(1-q)^u} \sum_{m=0}^{\infty} \frac{(c_1, \cdots, c_r; q)_m}{(q, d_1, \cdots, d_s; q)_m} (\rho y)^m q^{\sigma m(m+1)/2}$$

$$\times I_{P_i+1, Q_i+1}^{\mathfrak{I}+1, \wp} \left[\rho (y q^u)^k; q \,\middle|\, \begin{matrix} (1-\lambda-m, k), (a_j, \vartheta_j)_{1,\wp}, (a_{ji}, \vartheta_{ji})_{\wp+1, P_i} \\ (1-\lambda-m+u, k), (b_j, \xi_j)_{1,\mathfrak{I}}, (b_{ji}, \xi_{ji})_{\mathfrak{I}+1, Q_i} \end{matrix} \right]. \tag{2.9}$$

Now denote right-hand side of (2.3), by Ω

$$\Omega \equiv \sum_{r=0}^{\infty} \frac{(-1)^r q^{r(r+1)/2} (q^{-u}; q)_r}{(q; q)_r} D_{y,q}^{u-r} (y q^r)^{\lambda-1} {}_r\phi_s \left[\begin{matrix} c_1, \cdots, c_r; q; \rho(y q^r) \\ d_1, \cdots, d_s; q^{\sigma} \end{matrix} \right]$$

$$\times D_{y,q}^{r} I_{P_i, Q_i}^{\mathfrak{I}, \wp} \left[\rho y^k; q \,\middle|\, \begin{matrix} (a_j, \vartheta_j)_{1,\wp}, (a_{ji}, \vartheta_{ji})_{\wp+1, P_i} \\ (b_j, \xi_j)_{1,\mathfrak{I}}, (b_{ji}, \xi_{ji})_{\mathfrak{I}+1, Q_i} \end{matrix} \right]. \tag{2.10}$$

Now, from definition of (1.19) and applying (1.13), some identities (1.3), and

$$(h; q)_{n-k} = \frac{(h; q)_n}{(h^{-1} q^{1-n}; q)_k} (-q b^{-1})^k q^{\frac{k(k+1)}{2} - nk}, \quad (n, k \in \mathbb{Z}), \tag{2.11}$$

$$(h q^{-n}; q)_k = \frac{(h; q)_k (q h^{-1}; q)_n}{(h^{-1} q^{1-k}; q)_n} q^{-nk}, \quad (n \in \mathbb{Z}). \tag{2.12}$$

we get

$$D_{y,q}^{u-r}\left\{{}_r\phi_s\left[\begin{array}{c} c_1,\cdots,c_r;q;\rho(yq^u)\\ d_1,\cdots,d_s;q^\sigma\end{array}\right]\right\}$$

$$=\sum_{m=0}^{\infty}\frac{(c_1,\cdots,c_r;q)_m}{(q,d_1,\cdots,d_s;q)_m}\rho^m q^{\sigma m(m+1)/2}q^{rm}q^{(\lambda-1)r}$$

$$\times\frac{q^{-um}(1-q)^{r-u}(q^\lambda;q)_m(q^{1-\lambda-r};q)_u}{(-1)^u(q^\lambda;q)_r(q^{\lambda+r-u};q)_m q^{u-\lambda u-um}q^{u(u-1)/2-ur}}. \tag{2.13}$$

Then from definition (1.21) and applying (1.13) and some identities (1.3), (1.5), (1.7) and (1.23), we get

$$D_{y,q}^{r}I_{P_i,Q_i}^{\mathfrak{I},\wp}\left[\rho y^k;q\left|\begin{array}{c}(a_j,\vartheta_j)_{1,\wp},\,(a_{ji},\vartheta_{ji})_{\wp+1,P_i}\\(b_j,\xi_j)_{1,\mathfrak{I}},\,(b_{ji},\xi_{ji})_{\mathfrak{I}+1,Q_i}\end{array}\right.\right]$$

$$=\frac{1}{2\pi i}\int_\ell\frac{\left\{\prod_{j=1}^{\mathfrak{I}}G(q^{b_j-\xi_j s})\right\}\left\{\prod_{j=1}^{\wp}G(q^{1-a_j+\vartheta_j s})\vartheta\right\}\pi}{\sum_{i=1}^{r}\left\{\prod_{j=\mathfrak{I}+1}^{Q_i}G(q^{1-b_{ji}+\xi_{ji}s})\right\}\left\{\prod_{j=\wp+1}^{P_i}G(q^{a_{ji}-\vartheta_{ji}s})\right\}G(q^{1-s})\sin\pi s}$$

$$\times\frac{G(q^{r-ks})(1-q)^{-r}y^{ks-r}}{(-1)^r q^{-ksr}q^{r(r-1)/2}G(q^{-ks})}d_q s. \tag{2.14}$$

Now, using (2.14), (2.13) in (2.10) and re-arranging some terms, we get

$$\Omega=\frac{y^{\lambda-u-1}q^{(\lambda-1)u}(-1)^u q^{u(1-u)/2}}{(1-q)^u}\sum_{r=0}^{\infty}\frac{(q^{-u};q)_r(q^{1-\lambda-r};q)_u q^{(\lambda+u)r}}{(q;q)_r(q^\lambda;q)_r}$$

$$\times\,{}_{r+1}\phi_{s+1}\left[\begin{array}{c}c_1,\cdots,c_r,\lambda;q;\rho(yq^r)\\d_1,\cdots,d_s,\lambda+r-u;q^\sigma\end{array}\right]$$

$$\times\, I_{P_i+1,Q_i+1}^{\mathfrak{I}+1,\wp}\left[\rho(yq^r)^k;q\left|\begin{array}{c}(0,k),(a_j,\vartheta_j)_{1,\wp},\,(a_{ji},\vartheta_{ji})_{\wp+1,P_i}\\(r,k),(b_j,\xi_j)_{1,\mathfrak{I}},\,(b_{ji},\xi_{ji})_{\mathfrak{I}+1,Q_i}\end{array}\right.\right]. \tag{2.15}$$

So, from (2.9) and (2.15), we have our desired result.

$$\sum_{m=0}^{\infty}\frac{(c_1,\cdots,c_r;q)_m}{(q,d_1,\cdots,d_s;q)_m}(\rho y)^m q^{\sigma m(m+1)/2}$$

$$\times\, I_{P_i+1,Q_i+1}^{\mathfrak{I}+1,\wp}\left[\rho(yq^u)^k;q\left|\begin{array}{c}(1-\lambda-m,k),(a_j,\vartheta_j)_{1,\wp},\,(a_{ji},\vartheta_{ji})_{\wp+1,P_i}\\(1-\lambda-m+u,k),(b_j,\xi_j)_{1,\mathfrak{I}},\,(b_{ji},\xi_{ji})_{\mathfrak{I}+1,Q_i}\end{array}\right.\right]$$

$$=\sum_{r=0}^{\infty}\frac{(q^{-u};q)_r(q^{1-\lambda-r};q)_u q^{(\lambda+u)r}}{(q;q)_r(q^\lambda;q)_r}$$

$$\times\,{}_{r+1}\phi_{s+1}\left[\begin{array}{c}c_1,\cdots,c_r,\lambda;q;\rho(yq^r)\\d_1,\cdots,d_s,\lambda+r-u;q^\sigma\end{array}\right]$$

$$\times\, I_{P_i+1,Q_i+1}^{\mathfrak{I}+1,\wp}\left[\rho(yq^r)^k;q\left|\begin{array}{c}(0,k),(a_j,\vartheta_j)_{1,\wp},\,(a_{ji},\vartheta_{ji})_{\wp+1,P_i}\\(r,k),(b_j,\xi_j)_{1,\mathfrak{I}},\,(b_{ji},\xi_{ji})_{\mathfrak{I}+1,Q_i}\end{array}\right.\right]. \tag{2.16}$$

□

If $\sigma = 0$ in 2.1, we get the following corollary which is the expansion formula containing the q-analogue of the basic hypergeometric function and I-function.

Corollary 2.2.

$$\sum_{m=0}^{\infty} \frac{(c_1, \cdots, c_r; q)_m}{(q, d_1, \cdots, d_s; q)_m} (\rho y)^m$$
$$\times I^{\mathfrak{I}+1,\wp}_{P_i+1,Q_i+1}\left[\rho(yq^u)^k; q \left| \begin{matrix} (1-\lambda-m, k), (a_j, \vartheta_j)_{1,\wp}, (a_{ji}, \vartheta_{ji})_{\wp+1,P_i} \\ (1-\lambda-m+u, k), (b_j, \xi_j)_{1,\mathfrak{I}}, (b_{ji}, \xi_{ji})_{\mathfrak{I}+1,Q_i} \end{matrix} \right.\right]$$

$$= \sum_{r=0}^{\infty} \frac{(q^{-u}; q)_r (q^{1-\lambda-r}; q)_u q^{(\lambda+u)r}}{(q; q)_r (q^{\lambda}; q)_r}$$
$$\times {}_{r+1}\phi_{s+1}\left[\begin{matrix} c_1, \cdots, c_r, \lambda; q; \rho(yq^r) \\ d_1, \cdots, d_s, \lambda+r-u \end{matrix}\right]$$
$$\times I^{\mathfrak{I}+1,\wp}_{P_i+1,Q_i+1}\left[\rho(yq^r)^k; q \left| \begin{matrix} (0, k), (a_j, \vartheta_j)_{1,\wp}, (a_{ji}, \vartheta_{ji})_{\wp+1,P_i} \\ (r, k), (b_j, \xi_j)_{1,\mathfrak{I}}, (b_{ji}, \xi_{ji})_{\mathfrak{I}+1,Q_i} \end{matrix} \right.\right]. \tag{2.17}$$

Theorem 2.3. *The following expansion formulae pertaining to the product of I_q-function and q-analogue of generalized hypergeometric abnormal type function as under*

$$\sum_{m=0}^{\infty} \frac{(c_1, \cdots, c_r; q)_m}{(q, d_1, \cdots, d_s; q)_m} \left(\frac{y\rho}{q^u}\right)^m q^{\sigma m(m+1)/2}$$
$$\times I^{\mathfrak{I}+1,\wp}_{P_i+1,Q_i+1}\left[(\rho q^u y)^k; q \left| \begin{matrix} (\lambda+m, k), (a_j, \vartheta_j)_{1,\wp}, (a_{ji}, \vartheta_{ji})_{\wp+1,P_i} \\ (\lambda+m+u, k), (b_j, \xi_j)_{1,\mathfrak{I}}, (b_{ji}, \xi_{ji})_{\mathfrak{I}+1,Q_{i+1}} \end{matrix} \right.\right]$$
$$= \sum_{r=0}^{u} \frac{q^{2r}(q^{-u}; q)_r}{(q; q)_r (q^{1-\lambda-u}; q)_r} {}_{r+1}\phi_{s+1}\left[\begin{matrix} c_1, \cdots, c_r, q^{\lambda}+u-r; q; \frac{\rho}{yq^{u-r}} \\ d_1, \cdots, d_s, q^{\lambda}; q^{\sigma} \end{matrix}\right]$$
$$\times I^{\mathfrak{I}+1,\wp}_{P_i+1,Q_i+1}\left[\rho y^k q^{u-r-rk}; q \left| \begin{matrix} (0, k), (a_j, \vartheta_j)_{1,\wp}, (a_{ji}, \vartheta_{ji})_{\wp+1,P_i} \\ (r, k), (b_j, \xi_j)_{1,\mathfrak{I}}, (b_{ji}, \xi_{ji})_{\mathfrak{I}+1,Q_{i+1}} \end{matrix} \right.\right] \tag{2.18}$$

where

$$|\rho y| < |q|^{u+k\left(\sum_{j=1}^{\mathfrak{I}} \xi_j - \sum_{j=1}^{\wp} \vartheta_j\right) - \frac{\sigma}{2}}$$

and

$$\Re\left(\sum_{j=1}^{\mathfrak{I}} \xi_j b_j - \sum_{j=1}^{\wp} \vartheta_j a_j + \frac{k}{2}\right) > 0.$$

Proof. To prove the above theorem choose

$$W_1(y) = y^{-\lambda}{}_r\phi_s\left[\begin{matrix} c_1, \cdots, c_r; q; \rho/y \\ d_1, \cdots, d_s; q^{\sigma} \end{matrix}\right]$$

and

$$W_2(y) = I^{\mathfrak{I},\wp}_{P_i,Q_i}\left[\rho y^k; q \left| \begin{matrix} (a_j, \vartheta_j)_{1,\wp}, (a_{ji}, \vartheta_{ji})_{\wp+1,P_i} \\ (b_j, \xi_j)_{1,\mathfrak{I}}, (b_{ji}, \xi_{ji})_{\mathfrak{I}+1,Q_i} \end{matrix} \right.\right]$$

in the Leibniz rule given by (1.18), and by employing the same methodology and steps outlined in the proof of Theorem 2.1, we can replicate the derivation and arrive at the desired result. □

If $\sigma = 0$ in 2.1, we get the following corollary which is the expansion formula containing the q-analogue of the basic hypergeometric function and I-function.

Corollary 2.4.

$$\sum_{m=0}^{\infty}\frac{(c_1,\cdots,c_r;q)_m}{(q,d_1,\cdots,d_s;q)_m}\left(\frac{y\rho}{q^u}\right)^m$$
$$\times I^{\mathfrak{I}+1,\wp}_{P_i+1,Q_i+1}\left[(\rho q^u y)^k;q\left|\begin{matrix}(\lambda+m,k),(a_j,\vartheta_j)_{1,\wp},(a_{ji},\vartheta_{ji})_{\wp+1,P_i}\\(\lambda+m+u,k),(b_j,\xi_j)_{1,\mathfrak{I}},(b_{ji},\xi_{ji})_{\mathfrak{I}+1,Q_{i+1}}\end{matrix}\right.\right]$$
$$=\sum_{r=0}^{u}\frac{q^{2r}(q^{-u};q)_r}{(q;q)_r(q^{1-\lambda-u};q)_r}{}_{r+1}\phi_{s+1}\left[\begin{matrix}c_1,\cdots,c_r,q^{\lambda}+u-r;q;\frac{\rho}{yq^{u-r}}\\d_1,\cdots,d_s,q^{\lambda}\end{matrix}\right]$$
$$\times I^{\mathfrak{I}+1,\wp}_{P_i+1,Q_i+1}\left[\rho y^k q^{u-r-rk};q\left|\begin{matrix}(0,k),(a_j,\vartheta_j)_{1,\wp},(a_{ji},\vartheta_{ji})_{\wp+1,P_i}\\(r,k),(b_j,\xi_j)_{1,\mathfrak{I}},(b_{ji},\xi_{ji})_{\mathfrak{I}+1,Q_{i+1}}\end{matrix}\right.\right]. \tag{2.19}$$

3. Special cases

Letting $r=1,P_1=P$ and $Q_1=Q$ in Theorem 2.1 and Theorem 2.3 we have the following results which are the q-Leibniz expansion formula for the q-analogue of the hypergeometric abnormal type function and Fox's H-function given by (3.1) and (3.2) respectively.

$$\sum_{m=0}^{\infty}\frac{(c_1,\cdots,c_r;q)_m}{(q,d_1,\cdots,d_s;q)_m}(\rho y)^m q^{\sigma m(m+1)/2}$$
$$\times H^{\mathfrak{I}+1,\wp}_{P+1,Q+1}\left[\rho(yq^u)^k;q\left|\begin{matrix}(1-\lambda-m,k),(a_j,(a_1,\vartheta_1),...,(a_P,\vartheta_P)\\(1-\lambda-m+u,k),(b_1,\xi_1),...,(b_Q,\xi_Q)\end{matrix}\right.\right]$$
$$=\sum_{r=0}^{\infty}\frac{(q^{-u};q)_r(q^{1-\lambda-r};q)_u q^{(\lambda+u)r}}{(q;q)_r(q^{\lambda};q)_r}$$
$$\times{}_{r+1}\phi_{s+1}\left[\begin{matrix}c_1,\cdots,c_r,\lambda;q;\rho(yq^r)\\d_1,\cdots,d_s,\lambda+r-u;q^{\sigma}\end{matrix}\right]$$
$$\times H^{\mathfrak{I}+1,\wp}_{P+1,Q+1}\left[\rho(yq^r)^k;q\left|\begin{matrix}(0,k),(a_1,\vartheta_1),...,(a_P,\vartheta_P)\\(r,k),(b_1,\xi_1),...,(b_Q,\xi_Q)\end{matrix}\right.\right]. \tag{3.1}$$

And

$$\sum_{m=0}^{\infty}\frac{(c_1,\cdots,c_r;q)_m}{(q,d_1,\cdots,d_s;q)_m}\left(\frac{y\rho}{q^u}\right)^m q^{\sigma m(m+1)/2}$$
$$\times H^{\mathfrak{I}+1,\wp}_{P+1,Q+1}\left[(\rho q^u y)^k;q\left|\begin{matrix}(\lambda+m,k),(a_1,\vartheta_1),\ldots,(a_P,\vartheta_P)\\(\lambda+m+u,k),(b_1,\xi_1),\ldots,(b_Q,\xi_Q),\end{matrix}\right.\right]$$
$$=\sum_{r=0}^{u}\frac{q^{2r}(q^{-u};q)_r}{(q;q)_r(q^{1-\lambda-u};q)_r}{}_{r+1}\phi_{s+1}\left[\begin{matrix}c_1,\cdots,c_r,q^{\lambda}+u-r;q;\frac{\rho}{yq^{u-r}}\\d_1,\cdots,d_s,q^{\lambda};q^{\sigma}\end{matrix}\right]$$
$$\times H^{\mathfrak{I}+1,\wp}_{P+1,Q+1}\left[\rho y^k q^{u-r-rk};q\left|\begin{matrix}(0,k),(a_1,\vartheta_1),\ldots,(a_P,\vartheta_P)\\(r,k),(b_1,\xi_1),\ldots,(b_Q,\xi_Q)\end{matrix}\right.\right]. \tag{3.2}$$

Further, making $r=1,P_1=P$ and $Q_1=Q$ in corollaries 2.2 and 2.4, we have the following result.

$$\sum_{m=0}^{\infty}\frac{(c_1,\cdots,c_r;q)_m}{(q,d_1,\cdots,d_s;q)_m}(\rho y)^m$$
$$\times H^{\mathfrak{I}+1,\wp}_{P+1,Q+1}\left[\rho(yq^u)^k;q\left|\begin{matrix}(1-\lambda-m,k),(a_j,(a_1,\vartheta_1),...,(a_P,\vartheta_P)\\(1-\lambda-m+u,k),(b_1,\xi_1),...,(b_Q,\xi_Q)\end{matrix}\right.\right]$$

$$= \sum_{r=0}^{\infty} \frac{(q^{-u};q)_r (q^{1-\lambda-r};q)_u q^{(\lambda+u)r}}{(q;q)_r (q^{\lambda};q)_r}$$

$$\times {}_{r+1}\phi_{s+1} \left[\begin{array}{c} c_1, \cdots, c_r, \lambda; q; \rho(yq^r) \\ d_1, \cdots, d_s, \lambda + r - u \end{array} \right]$$

$$\times H^{\mathfrak{I}+1,\wp}_{P+1,Q+1} \left[\rho(yq^r)^k; q \middle| \begin{array}{c} (0,k), (a_1, \vartheta_1), ..., (a_P, \vartheta_P) \\ (r,k), (b_1, \xi_1), ..., (b_Q, \xi_Q) \end{array} \right].$$

And

$$\sum_{m=0}^{\infty} \frac{(c_1, \cdots, c_r; q)_m}{(q, d_1, \cdots, d_s; q)_m} \left(\frac{y\rho}{q^u}\right)^m$$

$$\times H^{\mathfrak{I}+1,\wp}_{P+1,Q+1} \left[(\rho q^u y)^k; q \middle| \begin{array}{c} (\lambda+m,k), (a_1, \vartheta_1), \ldots, (a_P, \vartheta_P) \\ (\lambda+m+u,k), (b_1, \xi_1), \ldots, (b_Q, \xi_Q), \end{array} \right]$$

$$= \sum_{r=0}^{u} \frac{q^{2r}(q^{-u};q)_r}{(q;q)_r (q^{1-\lambda-u};q)_r} {}_{r+1}\phi_{s+1} \left[\begin{array}{c} c_1, \cdots, c_r, q^{\lambda} + u - r; q; \frac{\rho}{yq^{u-r}} \\ d_1, \cdots, d_s, q^{\lambda} \end{array} \right]$$

$$\times H^{\mathfrak{I}+1,\wp}_{P+1,Q+1} \left[\rho y^k q^{u-r-rk}; q \middle| \begin{array}{c} (0,k), (a_1, \vartheta_1), \ldots, (a_P, \vartheta_P) \\ (r,k), (b_1, \xi_1), \ldots, (b_Q, \xi_Q) \end{array} \right].$$

4. Conclusion

This article presents the derivation of novel summation formulas for the product of the q-analogue of a generalized hypergeometric function of abnormal type function and the q-analogue of the I-function. These derivations are accomplished by employing the q-Leibniz rule in conjunction with Riemann-Liouville and Weyl-type q-fractional derivative operators. Furthermore, we investigate specific instances of the obtained results, illustrating their applications and implications. This research contributes new and significant insights to the domain of q-calculus. This research contributes significant and novel insights to the domain of q-calculus and q-special functions. It extends the classical theory of fractional calculus to the q-framework in a product-specific context and provides a powerful toolbox for generating identities that were previously inaccessible. The established formulas can serve as foundational tools for solving complex q-integral equations, analyzing q-series transforms, and modeling phenomena in quantum calculus, statistical mechanics, and other fields where q-deformations are natural.

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